



Stream: Computer Science & Engineering
Subject Name: Circuit Theory & Networks
Subject Code: EE (CSE) 301
Contacts Hour/Week: (3L+0T)
Credit: 3
Total Contact Hours: 34

Course Outline

Subject Name: Circuit Theory & Networks

Subject Code: EE (CSE) 301

Credit Value: 3.0

Length: 34.0 classes

Co requisite: Basic Electrical Engineering, Physics, Chemistry and Mathematics

Contact Hours: (3L + 0T)

Course Objective:

This course is intended for CSE Engineering students to facilitate the student's development into computer engineering investigation. The course has been designed to introduce fundamental principles of electrical circuits as well as the technical skills to analyze such simple circuits. It is a course suitable for students pursuing computer engineering as well as other related engineering disciplines.

Learning Outcomes:

Upon successful completion of this course, the student will have demonstrated:

1. Define various kinds of sources and their symbols, identify and use Kirchoff's Laws and Networks theorem for simple circuit analyses
2. To introduce the concept of DC & AC transient analysis
3. Deduce expressions and perform calculations relating to the Transient response, Laplace transform, Two port network and Graph theory
4. The Laplace's transform students will be able to understand initial & final value theorem and its applications both time & s domain.

5. Ability to solve circuits using node, branch, cutset & tie set and tree
6. Ability to express circuits in terms of Z, Y parameter model solve the circuit

Lecture Schedule:

Module No.	Total No of Lectures	No. of Lecture	Topics	Applications	Book(s) Follow	Topics for Self Study
Module-I Introduction	3L	1L	Continuous & Discrete, Fixed & Time varying, Linear and Nonlinear, Lumped and Distributed	Control, Communication s, Signal Processing	<ol style="list-style-type: none"> 1. Sudhakar:Circuits & Networks:Analysis & Synthesis 2/e TMH New Delhi 2. Valkenburg M. E. Van, “Network Analysis”, Prentice Hall 3. Roy Choudhury D., “Networks and Systems”, New Age International Publishers. 4. A. Chakrabarti: Circuit Theory Analysis & Synthesis 	Application of algorithms to modify signals in a way to make them more useful.
		1L	Passive and Active networks and systems. Independent & Dependent sources, Step, Ramp, Impulse			
		1L	Sinusoidal, Square, Saw tooth signals			

Module-II Network Equations	6L	2L	Formulation of network equations, Source transformation, Loop variable analysis, Node variable analysis.	Analysis of various types of DC voltages and circuits forwarded by resistance elements the voltages across, and the currents through, every component in the network.	<ol style="list-style-type: none"> 1. Basic Electrical Engineering by J.B. Gupta 2. Principles of Electrical Engineering by S. Chand and Company Ltd. 3. Circuits and Networks Analysis and Synthesis by Sudhakar 4. Roy Choudhury D., "Networks and Systems", New Age International Publishers. 5. A. Chakrabarti: Circuit Theory Analysis & Synthesis 	Network theorems
		2L	Network theorem: Superposition, Thevenin's, Norton's & Maximum power transfer theorem			
		2L	Millman's theorem. Solution of Problems with DC & AC sources.			
Module-III Resonance circuits	4L	1L	Series and parallel resonance-their frequency response, Impedance and Admittance Characteristics	Application of resonance is tuning, series resonant is used as voltage amplifier as well as parallel resonant is used as current amplifier	<ol style="list-style-type: none"> 1. Basic Electrical Engineering by J.B. Gupta 2. Basic Electrical Engineering by Edward Hughes 3. Basic Electrical Engineering by T.K. Nagasarkar 4. Roy Choudhury D., "Networks and Systems", New Age International Publishers. 5. A. Chakrabarti: 	Understanding series and parallel resonance
		2L (Page 60 to Page 61)	Quality factor, Half Power Points, and bandwidth. Phasor diagrams, Transform diagrams			
		1L	Practical			

			resonant and series circuits, Solution of Problems		Circuit Theory Analysis & Synthesis	
Module-IV Coupled circuits	3L	1L	Magnetic coupling, polarity of coils, polarity of induced voltage	Magnetic circuits application in electrical equipments	<ol style="list-style-type: none"> 1. Basic Electrical Engineering by J.B. Gupta 2. Principles of Electrical Engineering by S. Chand and Company Ltd. 3. Basic Electrical Engineering by T.K. Nagasarkar 	Faraday's laws
		1L	Concept of Self and mutual inductance, Coefficient of coupling			
		1L	Solution of Problems.			
Module-V Graph of Network	3L	1L	Concept of Tree and Branch, tree link, junctions	Graph theory is applied to areas of mathematics, science and technology. It is being actively used in field communication networks and coding theory and algorithms	<ol style="list-style-type: none"> 1. Basic Electrical Engineering by J.B. Gupta 2. Basic Electrical Engineering by Oxford 3. Basic Electrical Engineering by T.K. Nagasarkar 	Graph Theory applications in CSE
		1L	Incident matrix, Tie set matrix			
		1L	Determination of loop current and node voltages			
Module-VI Circuit transients	2L	1L	DC transients in R-L and R-C Circuits with and without initial charge, R-L-C Circuits	Transient analysis solves the complete nonlinear algebraic-differential equations of a circuit	<ol style="list-style-type: none"> 1. Basic Electrical Engineering by J.B. Gupta 2. Roy Choudhury D., "Networks and Systems", New Age International Publishers. 3. A. Chakrabarti: Circuit Theory Analysis & 	Ist order and 2 nd order time and s domain
		1L	AC Transients in sinusoidal R-L, R-C and R-L-C Circuits, Solution of Problems			

					Synthesis	
Module-VII Laplace transforms	8L	2L	Concept of Complex frequency , transform of $f(t)$ into $F(s)$ transform of step, exponential	Data mining, machine learning. Laplace Transform is heavily used in signal processing	<ol style="list-style-type: none"> 1. Basic Electrical Engineering by J.B. Gupta 2. Principles of Electrical Engineering by S. Chand and Company Ltd. 3. Basic Electrical Engineering by T.K. Nagasarkar 	Signal processing
		2L	Over damped surge, critically damped surge, damped and un-damped sine functions , properties of Laplace transform , linearity, real differentiation , real integration			
		2L	Initial value theorem and final value theorem , inverse Laplace transform			
		2L	Application in circuit analysis, Partial fraction expansion, Solution of problems.			

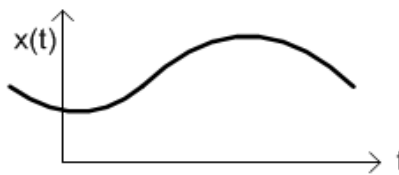
Module-VIII Two Port Networks Analysis	4L	2L	Relationship of Two port network variables, short circuit admittance parameters, open circuit impedance parameters, transmission parameters	Communication s, networking	<ol style="list-style-type: none"> 1. Basic Electrical Engineering by J.B. Gupta 2. Principles of Electrical Engineering by S. Chand and Company Ltd. 3. Basic Electrical Engineering by T.K. Nagasarkar 	All Parameters
		2L	relationship between parameter sets, network functions for ladder network and general network			

MODULE 1: INTRODUCTION

LECTURE1

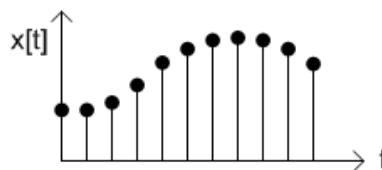
Continuous Signal: A continuous-time signal is a quantity of interest that depends on an independent variable, where we usually think of the independent variable as time. A continuous-time signal is a function $x(t)$ of the real variable t defined for $-\infty < t < \infty$

If the independent variable (t) is continuous, then the corresponding signal is continuous time signal.

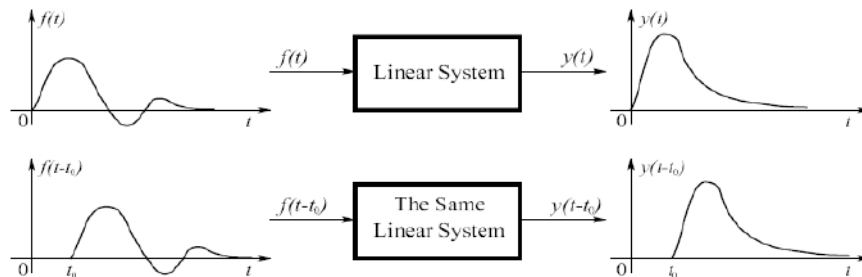


Discrete Signal: Discrete signal is well known as Discrete time signal which holds a specific value at a specific time instant. It can be obtained by sampling a continuous signal.

If the independent variable (t) takes on only discrete values, for example $t = \pm 1, \pm 2, \pm 3, \dots$, we get a plot of the dependent signal $x(t)$

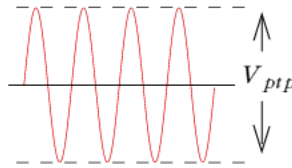


Time Invariant or Fixed Signal: A signal is time-invariant or fixed if the characteristics of the system do not change with time.



If input $f(t)$ gives output $y(t)$, then after a time delay of t_0 units it is found that the input $f(t-t_0)$ provides the same output $y(t-t_0)$ only by a time shift represented in the above figure.

Time Varying signals: A signal that changes continuously with time is a time varying signal. Eg sinusoidal voltage or current.



Linear System: A system is said to be linear if it follows both the Homogeneity and superposition principles.

Homogeneity: If the input is multiplied by a constant, the output shall also be multiplied by the same.

Superposition: If the input is superposed by two signals, the out put shall also be superposition.

The *principle of superposition* is another name for the additivity property of Linearity:

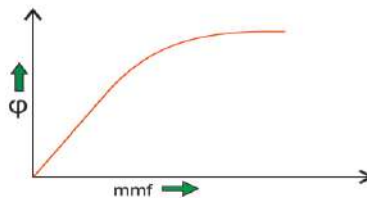
$$f(x_1+x_2)=f(x_1)+f(x_2)$$

Eg. Let us consider a purely resistive network with a constant DC source. This circuit follows the principle of homogeneity and additivity. Assuming ideal behaviour of each element in the network, we say that we will get linear voltage and current characteristic.

Non-linear Systems:

A **non linear control system** is simply defined as all those system which do not follow the principle of homogeneity. In practical life all the systems are non-linear system

Eg. A well known example of non-linear system is magnetization curve or no load curve of a DC machine.



LECTURE2

Lumped Parameter System:

In these types of systems the various parameters (like inductor, resistor and capacitor) are assumed to be concentrated at a point. Analysis of such type of system is very easy which includes differential equations.

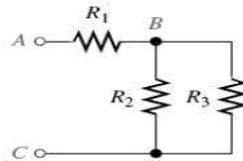
Distributed Parameter System:

In these **systems** the various parameters (like resistors inductor and capacitor) are assumed to be distributed uniformly along the length and that's why these are called distributed parameter type of system. Analysis of such type of system is slightly difficult which includes partial differential equations.

Active Network: It is a network which contains one or more than one sources of emf. An active network consists of an active element like a battery or a transistor.



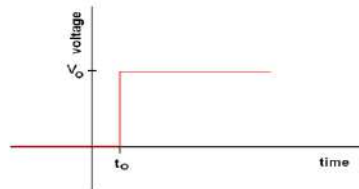
Passive Network: When the network does not contain any source of emf, it is called passive network. A passive network consists of resistance, inductance or capacitance as passive elements. A passive network does not contain any energy source.



LECTURE3

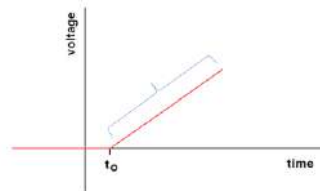
Unit Step Function:

$$u(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$



Unit Ramp Function:

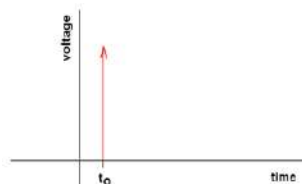
A unit ramp function is the result of the integration of a unit step function



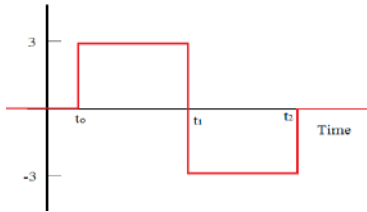
Unit Impulse Function:

An impulse function is the derivative of a unit step function

$$\delta(t-t_0) = \begin{cases} 0 & t < t_0 \\ \infty & t = t_0 \\ 0 & t > t_0 \end{cases}$$

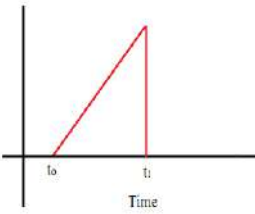


Square Waveform:



$$3[u(t-t_0) - 2u(t-t_1) + u(t-t_2)]$$

Sawtooth Waveform:



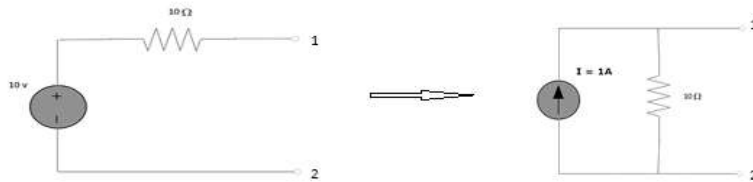
Sawtooth Waveform is formed by multiplying the Ramp Function times the Unit Step Function

MODULE II: NETWORK EQUATIONS

LECTURE 4

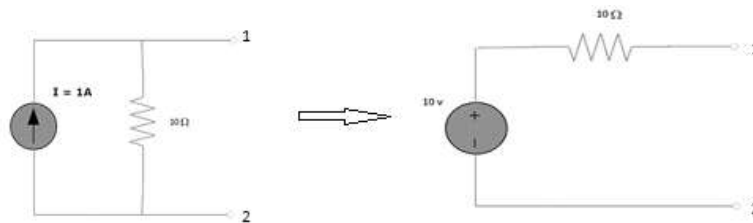
Source transformation is simplifying a circuit solution by transforming a voltage into a current source, and vice versa.

Conversion of voltage source to current source:



Conversion of current source to voltage source:

Convert the constant current source shown in figure 1.12 to voltage source.



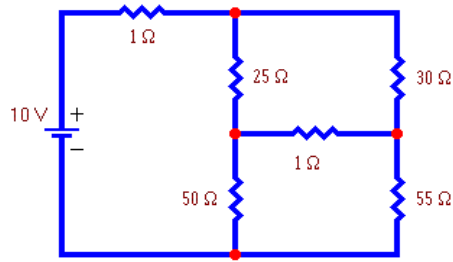
Loop Variable Analysis:

In this method, we set up and solve a system of equations in which the unknowns are **loop currents**. The currents in the various branches of the circuit are then easily determined from the loop currents.

Loop analysis is a special application of KVL on a circuit. We use a special kind of loop called a 'mesh' which is a loop that does not have any other loops inside of it. A mesh starts at a node and traces a path around a circuit, returning to the original node without encountering any nodes more than once.

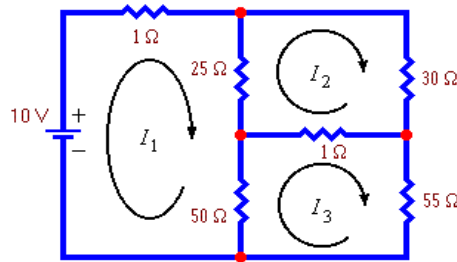
Let us consider the following example:

Example 1: Find the current flowing in each branch of this circuit.



By observation, we find the number of meshes= 3 .Thus the number of equations will be 3.

Let us assign the direction of the loop currents as shown in figure below.



Applying KVL in each loop we get the following equations:

$$1I_1 + 25(I_1 - I_2) + 50(I_1 - I_3) = 10 \dots\dots\dots(1)$$

$$25(I_2 - I_1) + 30I_2 + 1(I_2 - I_3) = 0 \dots\dots\dots(2)$$

$$50(I_3 - I_1) + 1(I_3 - I_2) + 55I_3 = 0 \dots\dots\dots(3)$$

Rearranging the equations we get:

$$76I_1 - 25I_2 - 50 I_3 = 10 \dots\dots\dots(I)$$

$$-25I_1 + 56I_2 - I_3 = 0 \dots\dots\dots(II)$$

$$-50I_1 - I_2 + 106I_3 = 0 \dots\dots\dots(III)$$

Solving the equations by elimination process we get :

$$I_1 = 0.245 \text{ Amps} , I_2 = 0.111 \text{ Amps} \text{ and } I_3 = 0.117 \text{ Amps.}$$

LECTURES

Nodal Analysis:

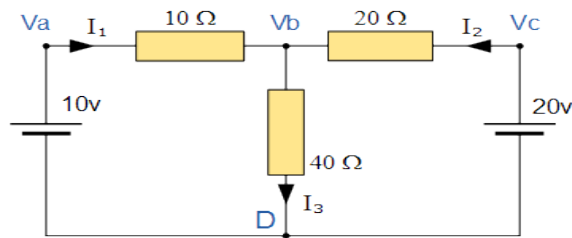
Nodal Analysis uses the “Nodal” equations of Kirchoff’s first law to find the voltage potentials around the circuit.

A junction point in an electrical circuit is called a node. A potential drop can be measured with respect to this point and another node acting as reference node. If there are N nodes, then number of independent KCL equations in nodal analysis is N-1.

At each node point , Kirchoff’s first law equation, is written: *“The currents entering a node are exactly equal in value to the currents leaving the node”*

In this method all the branch currents are computed. In writing the current expression the assumption is made that the *node potential* is always higher than the other voltages appearing in the equation.

Let us consider a circuit as:



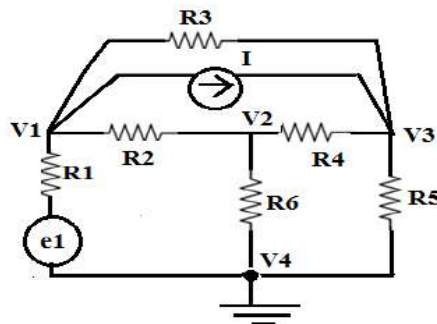
In the above circuit, node D is chosen as the reference node and the other three nodes are assumed to have voltages, Va, Vb and Vc with respect to node D.

$$\frac{V_A - V_B}{10} + \frac{V_C - V_B}{20} = \frac{V_B}{40}$$

From the circuit we find : **VA = 10V, VB =20V**

Substituting We in get **VB =80/7 [Volts]** and **I3 =2/7 [Amps]**.

Example: Develop the nodal equations for the circuit shown below:



Soultion:

Considering the node voltages as V1,V2,V3,using nodal method we get:

At Node 1(A)

$$\frac{V1 - V2}{R2} + \frac{V1 - e1}{R1} + \frac{V1 - V3}{R3} + I = 0$$

At Node 2,

$$\frac{V2 - V1}{R2} + \frac{V2}{R6} + \frac{V2 - V3}{R4} = 0 \quad \dots\dots\dots(B)$$

At Node 3,

$$\frac{V3 - V2}{R4} + \frac{V3}{R5} + \frac{V3 - V1}{R3} = I \quad \dots\dots\dots(C)$$

Equations (A),(B),(C) give the nodal equations.

LECTURE6

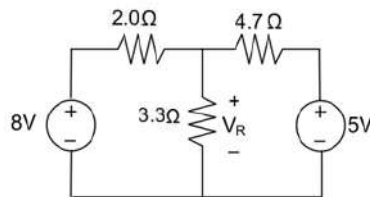
Super Position Theorem:

The superposition theorem states that in any linear bilateral network that consisting of two or more independent sources, current through (or voltage across) an element is the algebraic sum of the currents through (voltages across) that element caused by each independent source acting alone with all other sources is replaced by their internal resistances.

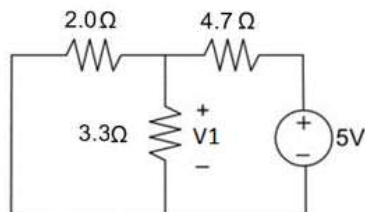
If the circuit consists of N independent sources, it is requires to analyses N circuits, each circuit will produce a result with respect to each individual source. Finally the individual results are added to get the whole response of the circuit.

Example:

Using the superposition theorem, determine the voltage drop and current across the resistor 3.3K as shown in figure below.



Step 1: Replacing the 8V power supply, the circuit is drawn as :

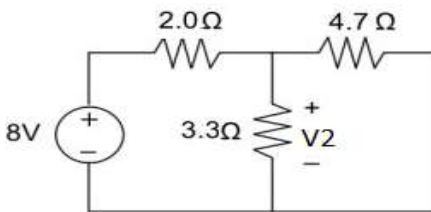


Here Resistances 3.3Ω and 2Ω are in parallel, therefore their equivalent resistance will be $3.3\Omega \parallel 2\Omega = 1.245\Omega$.

Using voltage divider rule voltage across 1.245Ω will be:

$$V_1 = [1.245 / (1.245 + 4.7)] * 5 = 1.047 \text{ Volts}$$

Step 2: Replacing the 5V power supply is by a short circuit, the circuit is drawn as



It is required to calculate the voltage V_2 appearing across 3.3Ω .

It is observed that the resistances 3.3Ω and 4.7Ω are in parallel, therefore equivalent resistance of the parallel combination will be $3.3 \parallel 4.7\Omega = 1.938\Omega$

Using voltage divider rule voltage across 1.938Ω will be

$$V_2 = [1.938 / (1.938 + 2)] * 8 = 3.9377 \text{ Volts}$$

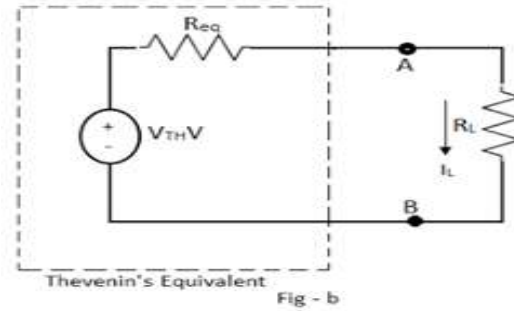
Step 3: By the principle of the voltage drop across $3.3K$ resistor is $V_1 + V_2$

$$= 1.047 + 3.9377 = 4.9847 \text{ Volts}$$

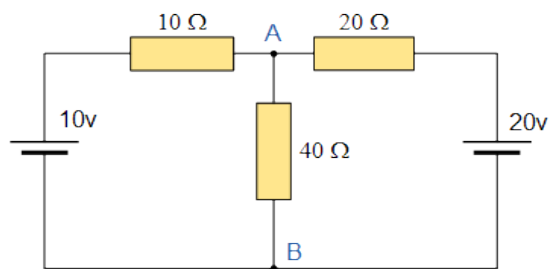
Thevenin's Theorem:

Thevenin's theorem states that "For Any two terminal bilateral linear circuit consisting of energy sources and impedances can be replaced with an equivalent circuit consisting of a single Thevenin's equivalent voltage source V_{TH} in series with a thevenin's equivalent resistance R_{TH} in DC circuits and impedance Z_{TH} in AC circuits.

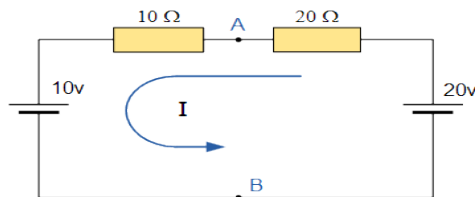
Thevenin's theorem in DC System:



Explanation: Let us consider a circuit where It is required to find out the current through load resistance $R_L=40\Omega$



Step 1: To Find the Equivalent Voltage (V_{TH}): The load resistance R_L is removed and open circuit voltage is calculated:



The total resistance in the circuit is $10\Omega + 20\Omega = 30\Omega$

The circuit current $I = (20V - 10V)/30 = 0.33$ amps.

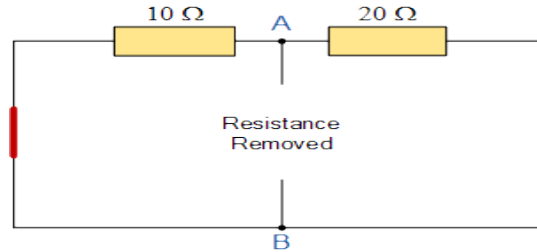
Open circuit voltage $V_{OC} = V_{TH} = V_{AB} = 20 - (0.33 \times 20\Omega) = 13.33$ Volts.

Step 2: To Find the Equivalent Resistance (R_{TH}):

To find out R_{TH} it is needed to remove the centre 40Ω load resistor connected across the terminals A-B, and replace the voltage sources by their internal impedances.

This is done by shorting out all the voltage sources connected to the circuit, that is $V = 0$.

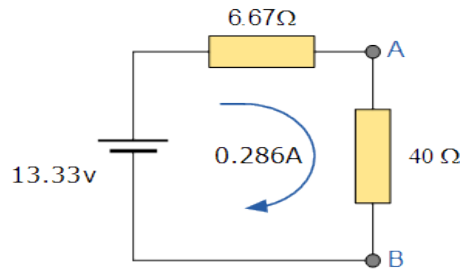
Voltage sources are replaced by short circuits and current sources are replaced by open circuits.



$$R_{TH} = 10\Omega + 20\Omega = 30\Omega$$

Step 3: Formation Of Thevenin's equivalent circuit.

Thus the Thevenin's Equivalent circuit would consist of a series resistance of 6.67Ω and a voltage source of $13.33V$. With the 40Ω load resistor connected back into the circuit we get:



$$\text{The load current } I_L = \frac{V_{TH}}{(R_{TH} + R_L)} = \frac{13.33}{(6.67 + 40)} = 0.286 \text{ Amps.}$$

The basic procedure for solving a circuit using **Thevenin's Theorem** is as follows:

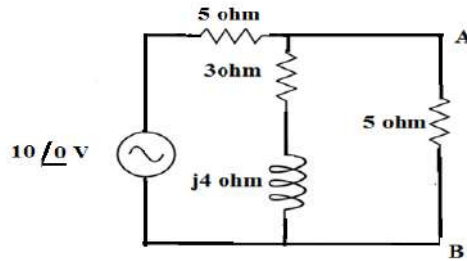
- Remove the load resistor R_L or component concerned.
- Find V_{TH} by the usual circuit analysis methods.
- Find R_{TH} by shorting all voltage sources or by open circuiting all the current sources.
- Determination of load current I_L from Thevenin's equivalent circuit.

LECTURE 7

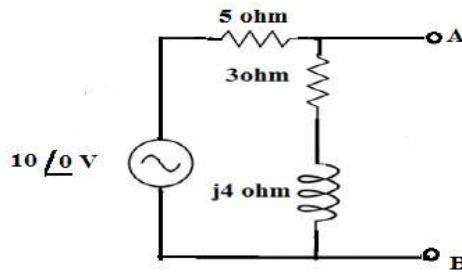
Thevenin's theorem in AC System:

Example: Find the current through the branch AB of the network

Solution:



Step 1: To Find the Equivalent Voltage (V_{TH}): The load resistance of 5 ohm R_L is removed and open circuit voltage is calculated:



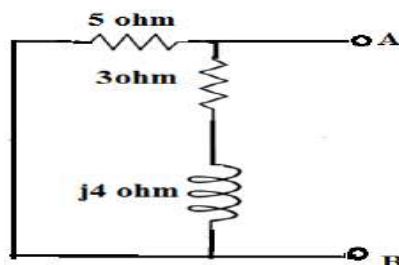
Applying KVL in the circuit we get:

$$I(5+3+j4)=10$$

$$V_{TH}=V_{AB} = \frac{10(3+j4)}{8+j4}$$

$$V_{TH} = 5.59 \angle 26.56 \text{ Volts}$$

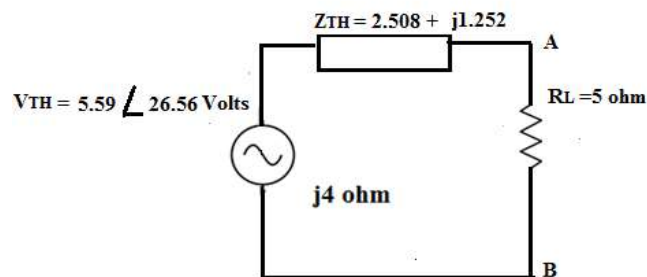
Step 2: To Find the Equivalent Resistance (Z_{TH}):



$$Z_{TH} = \frac{5(3+j4)}{3+j4+5}$$

$$Z_{TH} = 2.508 + j1.252\Omega$$

Step 3: Formation of Thevenin's equivalent circuit.



$$\begin{aligned} \text{The current through branch AB} &= \frac{5.59 \angle 26.56}{2.5 + j1.25 + 5} \\ &= 0.735 \angle 7.097 \text{ Amp} \end{aligned}$$

Norton's Theorem:

Norton's theorem states that "Any two terminal bilateral linear circuit consisting of energy sources and impedances can be replaced with a Norton's equivalent circuit with a single Norton's Current source I_N in parallel with a Norton's resistance R_N in DC circuits and Norton's impedance Z_N connected to a load impedance Z_L .

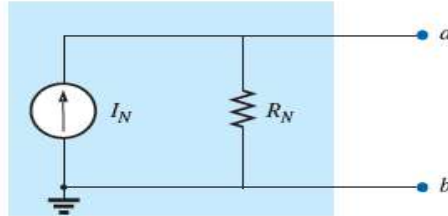
The basic procedure for solving a circuit using **Norton's Theorem** is as follows:

1. The load resistance R_L must be removed.
2. The Norton's equivalent resistance R_N is to be calculated by shorting all voltage sources or by open circuiting all the current sources.
3. The Norton's Equivalent current I_N is to be calculated by placing a shorting link on the output terminals.

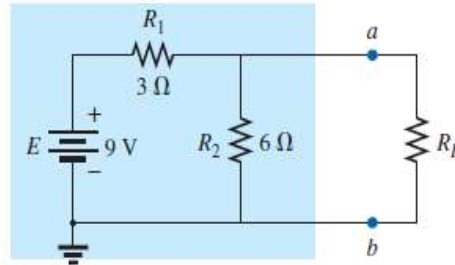
4. The load current I_L flowing through the load resistor R_L is to be calculated from Norton's Equivalent circuit.

Norton's equivalent circuit:

Explanation:
Find the Norton network in the fig.



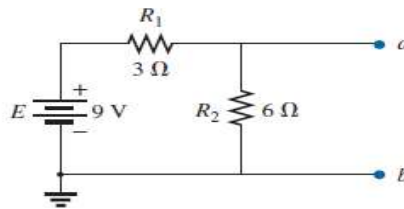
equivalent circuit for the



Solution:

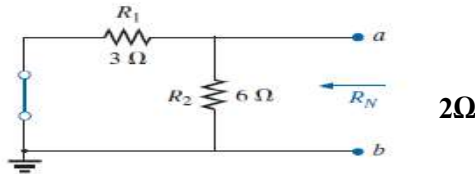
STEP 1: Opening the load

terminals, we get the figure as:



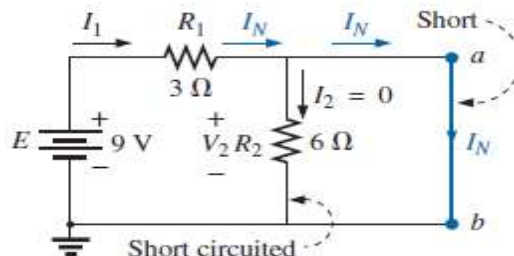
STEP 2: Replacing the voltage source by short circuit we get the circuit as:

$R_N = R_1 \parallel R_2 = 3\Omega \parallel 6\Omega =$



STEP 3: Calculation Of I_N a,b.

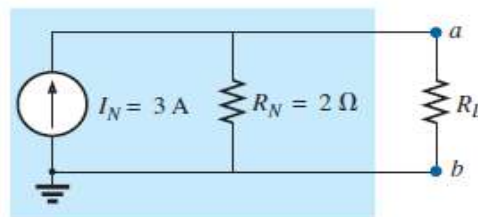
after shorting the terminals



From the fig we find that the short-circuit connection between terminals a and b is in parallel with R_2 and for this there will be no current flowing through R_2 I_N is the same current as through R_1 i.e I_1

$$I_N = E/R_1 = I_1 = 9V/3\Omega = 3 \text{ Amps}$$

STEP 4: Calculation Of load current I_L from Norton's equivalent circuit.



By current division rule we get the value of load current as $I_L = I_N * [(R_N)/(R_N + R_L)]$.

LECTURE 8

Maximum Power Transfer Theorem:

This theorem is to find the value of load resistance for which there would be maximum amount of power transfer from source to load.

In DC circuits , the Condition of maximum power transfer is : $R_L = R_{TH}$

In AC circuits, the condition of maximum power transfer is : $Z_L = Z_{TH} = R_{TH} - jX_{TH}$.

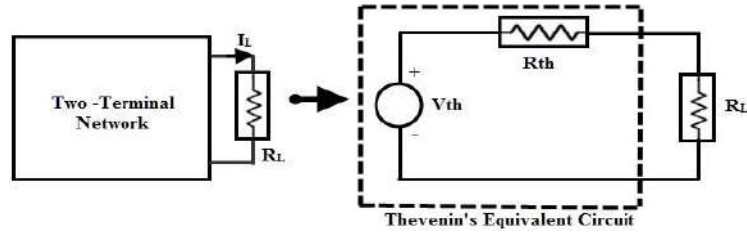
Maximum Power Transfer Theorem for DC Circuits:

The maximum power transfer theorem states that in a linear, bilateral DC network, maximum power is delivered to the load when the load resistance is equal to the internal resistance of a source.

A load will receive maximum power from a network when its resistance is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$R_L = R_{Th}$$

Proof of Maximum Power Transfer Theorem:



The original two terminal circuits are replaced with a Thevenin's equivalent circuit across the variable load resistance. The current through the load for any value of load resistance is calculated as:

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

The power absorbed by the load is

$$\begin{aligned}
 P_L &= I_L^2 \times R_L \\
 &= \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 \times R_L
 \end{aligned}$$

The power delivered from this equivalent source to the load entirely depends on the load resistance R_L . To find the exact value of R_L , we apply differentiation to P_L with respect to R_L and equating it to zero as:

$$\frac{dP(R_L)}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L \times (R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] = 0$$

$$\Rightarrow (R_{Th} + R_L) - 2R_L = 0$$

$$\Rightarrow R_L = R_{Th}$$

(Proved)

The maximum power delivered to the load is,

$$\begin{aligned}
 P_{\max} &= \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 \times R_L \Bigg|_{R_L = R_{Th}} \\
 &= \frac{V_{Th}^2}{4 R_{Th}}
 \end{aligned}$$

Power Transfer Efficiency

Total power transferred from source is

$$\begin{aligned}
 P_T &= I_L^2 (R_{TH} + R_L) \\
 &= 2 I_L^2 R_L \dots\dots\dots(2)
 \end{aligned}$$

$$\text{Efficiency} = (\text{Output}) / (\text{Input}) \times 100 = P_L / P_T \times 100$$

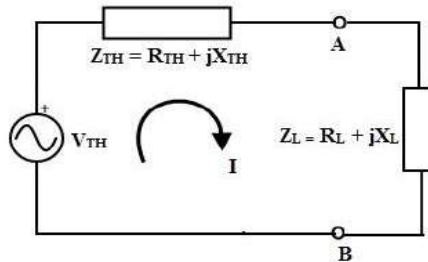
$$= I_L^2 R_L / 2 I_L^2 R_L \times 100 = 50 \%$$

LECTURE9

Maximum Power Transfer Theorem For Ac Circuits:

It can be stated as in an active network, the maximum power is transferred to the load when the load impedance is equal to the complex conjugate of an equivalent impedance of a given network as viewed from the load terminals.

Let us consider the following circuit:



Consider the above Thevenin's equivalent circuit across the load terminals in which the current flowing through the circuit is given as

$$\text{The current in the circuit is given as } I = V_{TH} / (Z_{TH} + Z_L)$$

Where $Z_L = R_L + j X_L$ is the Load impedance

$Z_{TH} = R_{TH} + jX_{TH}$ is the Thevenin's equivalent impedance of the network

Thus the Load Current, $I = V_{TH} / (R_L + jX_L + R_{TH} + jX_{TH})$

$$= V_{TH} / ((R_L + R_{TH}) + j(X_L + X_{TH}))$$

The power delivered to the load, $P_L = I^2 R_L$

$$P_L = V_{TH}^2 \times R_L / ((R_L + R_{TH})^2 + (X_L + X_{TH})^2) \dots\dots(1)$$

For maximum power the derivative of the above equation must be zero. For power to be maximized, the above equation must be differentiated with respect to X_L and equates it to zero.

$$\frac{dP_L}{dX_L} = 0$$

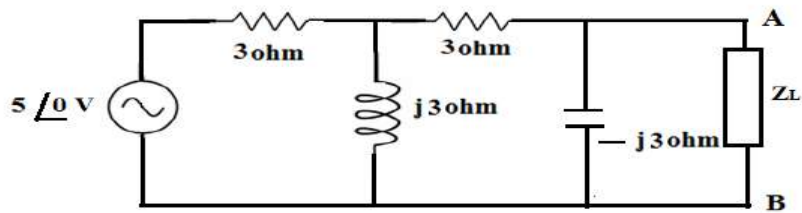
After differentiation we get $X_L + X_{TH} = 0$

$$\text{or } X_L = -X_{TH}$$

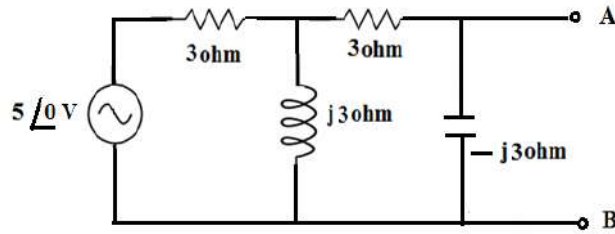
Hence, the maximum power will transferred to the load from source, if

$R_L = R_{TH}$ and $X_L = -X_{TH}$ in an AC circuit. This means that the load impedance should be equal to the complex conjugate of equivalent impedance of the circuit, $Z_L = \text{Complex Conjugate of } Z_{TH} = R_{TH} - jX_{TH}$.

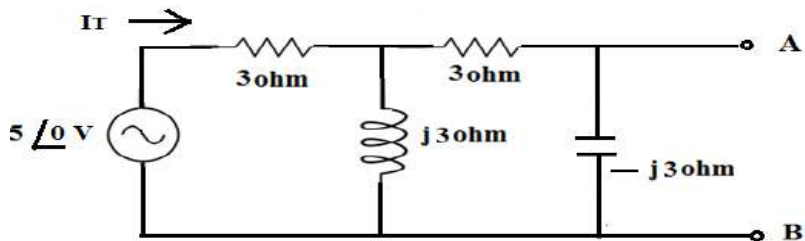
Example: Find the value of Z_L for which maximum power can be delivered?



Let us remove the load Z_L . The circuit is drawn as :



The total current supplied by the voltage source is calculated as:



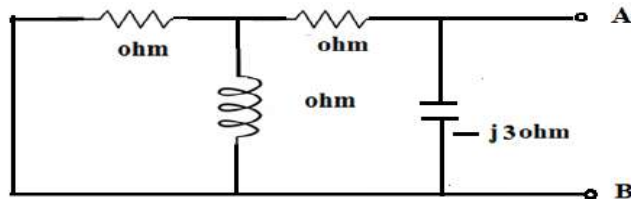
Total current from source is:

$$I_T = \frac{5 \angle 0^\circ \text{ V}}{3 + [j3 \parallel (3 - j3)]}$$

The value of $I_T = 5 / (6 + 3j)$ Amps

The value of $V_{TH} = (-3j) \times [5 / (6 + 3j)]$ Volts = $5 / (2 + j)$ Volts

The Thevenin's impedance Z_{TH} is calculated as :



From the above circuit, Z_{TH} is calculated as:

$$Z_{TH} = [(3 \parallel j3) + 3] \parallel (-j3)$$

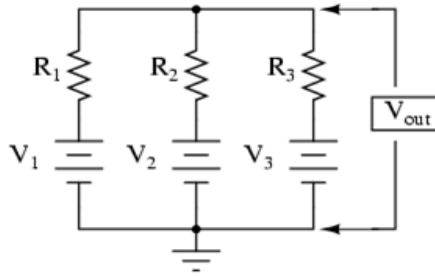
Simplifying the above expression, we get $Z_{TH} = (1.8 - j2.4) \Omega$

For maximum power, $Z_L = Z_{TH}^* = (1.8 + j2.4) \Omega$

Millman's Theorem:

Statement: When a number of voltage sources (V_1, V_2, \dots, V_n) are in parallel having internal resistances (R_1, R_2, \dots, R_n) respectively, the arrangement can be replaced by a single equivalent series resistance R as given below:

Explanation: Let us consider three voltage sources connected in parallel:



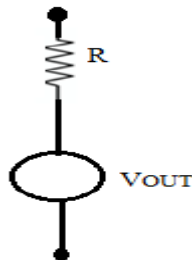
As per Millman's theorem: $V_{OUT} =$

$$\frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3}$$

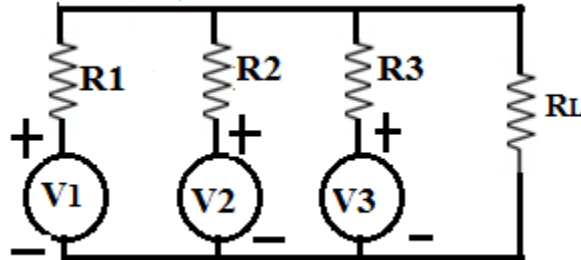
Equivalent Resistance R is given as :

$$R = \frac{1}{G}$$

Equivalent voltage and resistance of the network is:



Example: Using Millman's theorem find the current through load resistance R_L



Given: $V_1 = 4V$, $V_2 = 2V$, $V_3 = 10V$, $R_1 = R_2 = R_3 = 4\Omega$, $R_L = 10\Omega$.

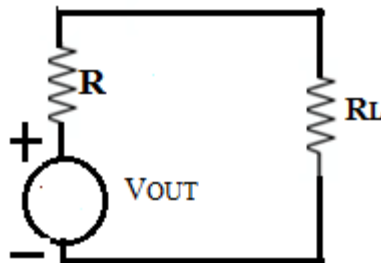
Solution:

By Millman's theorem, we get $V_{OUT} = (V_1G_1 + V_2G_2 + V_3G_3) / (G_1 + G_2 + G_3)$

$$= 1 + 0.5 + 2.5 = 4 \text{ Volts.}$$

$$R = (1/4) + (1/4) + (1/4) = (3/4) \Omega$$

The equivalent network of the



circuit is:

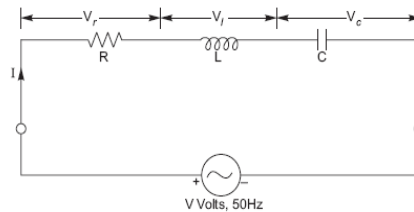
The circuit current $I = (V_{OUT}) / (R_L + R) = (4) / (10 + 3/4) = 0.37209 \text{ Amps}$

MODULE III: RESONANCE CIRCUITS

LECTURE10

Series Resonance:

A sinusoidal voltage V sends a current I through the circuit. The circuit is said to be resonant when the resultant reactance is zero, (*i.e.*, the circuit is purely resistive or we can say the imaginary part should be equal to zero).



The impedance Z of the circuit is given by

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

where

Z and R in ohms

L is in Henrys

C is in Farads

and ω is the angular frequency of the applied voltage in radians/sec.

The current I is calculated as:

$$I = \frac{V}{Z} = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

At resonance, the imaginary part of Z should be equal to zero

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega L = \frac{1}{\omega \cdot C}$$

Let ω_r is the frequency of resonance in radians/seconds or the resonant frequency.

So, we get

$$\omega_r = \frac{1}{\sqrt{LC}}$$

Or

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

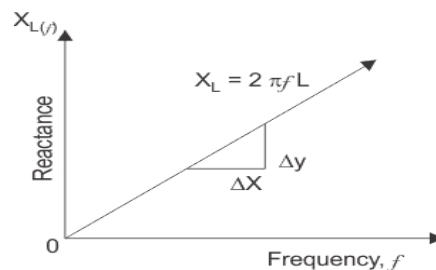
Effects of Series Resonance:

The following effects of the series resonance condition are given below

- At Resonance condition, $X_L = X_C$ the impedance of the circuit is minimum and is reduced to the resistance of the circuit. i.e $Z_r = R$.
- At the resonance condition, as the impedance of the circuit is minimum, the current in the circuit is maximum. i.e $I_r = V/Z_r = V/R$
- As the value of resonant current I_r is maximum hence, the power drawn by the circuit is also maximized. i.e $P_r = I^2 R_r$
- At the resonant condition, the current drawn by the circuit is very large or we can say that the maximum current is drawn. Therefore the voltage drop across the inductance L i.e ($V_L = IX_L = I \times 2\pi f_r L$) and the capacitance C i.e ($V_C = IX_C = I \times 1/2\pi f_r C$) will also be very large.

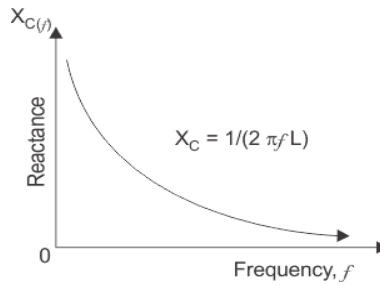
A series resonant circuit has the capability to draw heavy current and power from the mains; it is also called as **Acceptor Circuit**.

Variation of Inductive Reactance Vs Frequency



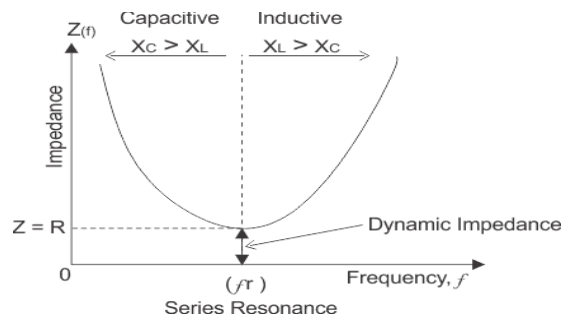
Inductive reactance $X_L = 2\pi fL$ means inductive reactance is directly proportional to frequency (X_L and prop f). When the frequency is zero or, inductive reactance is also zero, the circuit acts as a short circuit; but when frequency increases; inductive reactance also increases. At infinite frequency, inductive reactance becomes infinity and circuit behaves as open circuit. It means that, when frequency increases inductive reactance also increases and when frequency decreases, inductive reactance also decreases. So, if a graph is plotted between inductive reactance and frequency, it is a straight line linear curve passing through origin as shown in the figure above.

Variation Of Capacitive Reactance Vs Frequency



Capacitive reactance $X_C = 1 / 2\pi fC$ that, frequency and capacitive reactance are inversely proportional to each other. When frequency is zero, capacitive reactance becomes infinity and circuit behaves as open circuit and when frequency increases and becomes infinite, capacitive reactance decreases and becomes zero at infinite frequency, at that point the circuit acts as short circuit, so the capacitive reactance increases with decrease in frequency and if a graph is plotted between capacitive reactance and frequency, it is an hyperbolic curve as shown in figure above.

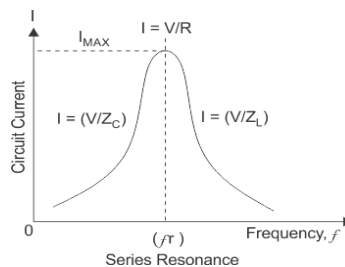
Variation of Impedance Vs Frequency



At **resonance in series RLC circuit**, two reactances become equal and cancel each other. So in resonant series RLC circuit, the opposition to the flow of current is due to resistance only. At resonance, the total impedance of series RLC circuit is equal to resistance i.e $Z = R$, impedance has only real part but no imaginary part and this impedance at resonant frequency is called dynamic impedance and this dynamic impedance is always less than impedance of series RLC circuit.

LECTURE11

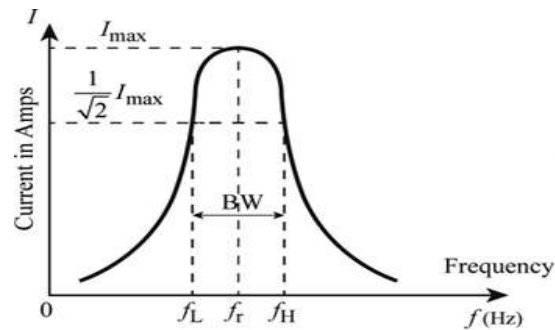
Resonant Current:



In series RLC circuit, the total voltage is the phasor sum of voltage across resistor, inductor and capacitor. At **resonance in series RLC circuit**, both inductive and capacitive reactances cancel each other and, so the voltage across inductor and capacitor is equal in magnitude and opposite in direction and thereby they cancel each other. So, in a series resonant circuit, voltage across resistor is equal to supply voltage i.e $V = V_r$.

Bandwidth of Series Resonant Circuit

The distance between these two points, i.e. ($f_H - f_L$) is called the **Bandwidth**, (BW) and is the range of frequencies over which at least half of the maximum power and current is provided.



From Figure , the maximum current occurs at the resonant frequency (f_r), the current is 0.707 of maximum current .

$$\left(\text{i.e. } \frac{1}{\sqrt{2}} I_{\max} \right)$$

Here,

f_L is the lower cut – off frequency

f_H is the higher cut – off frequency.

Band width is, $B.W = f_H - f_L$

Frequencies f_L and f_H are also called *half-power frequencies* because the power dissipation in the circuit at these frequencies is half of the power dissipation at the resonant frequency f_r . This may be seen as below:

Taking $I_{\max} = I_0$, we get

$$P_0 = \text{Power dissipation at } f_r = I_0^2 R$$

$$P_1 = \text{Power dissipation at } f_L = I_1^2 R = \frac{I_0^2 R}{2} = \frac{P_0}{2}$$

$$P_2 = \text{Power dissipation at } f_H = I_2^2 R = \frac{I_0^2 R}{2} = \frac{P_0}{2}$$

Quality Factor:

The *Quality factor*, Q , is a measure of the sharpness (or selectivity) of the resonance peak. It is the ratio of the resonant frequency to the "bandwidth".

$$Q = \frac{\omega_0 L}{R} = \frac{\omega_0}{\left(\frac{R}{L}\right)} = \frac{\omega_0}{(\omega_2 - \omega_1)}$$

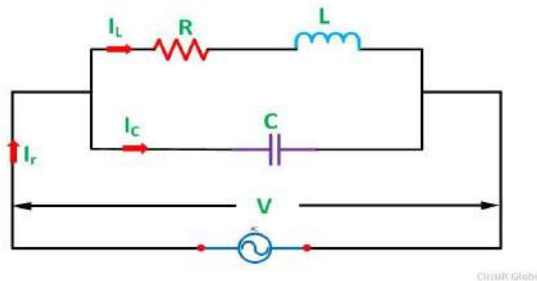
$$= \frac{\text{Resonant frequency}}{\text{Band Width}}$$

LECTURE12

Parallel Resonance:

Parallel Resonance means when the circuit current is in phase with the applied voltage of an AC circuit containing an Inductor and a Capacitor connected together in parallel.

Let us consider the figure shown below:

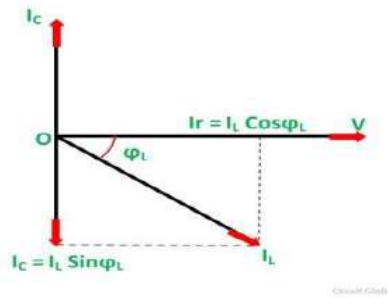


An Inductor of L Henry having some resistance of R ohms is connected in parallel with a capacitor of capacitance C farads. A supply voltage of V volts is connected across these elements. The circuit current I_r will only be in phase with the supply voltage when the following condition given below in the equation is satisfied.

$$I_C = I_L \sin \phi_L$$

The **phasor diagram** of the given circuit is shown below:

Phasor Diagram:



At some frequency, f_r called resonance frequency.

$$I_C = I_L \sin \phi_L$$

Where,

$$I_L = \frac{V}{Z_L}$$

Substituting I_C , I_L , $\sin \phi_L$, we get

$$\sin \phi_L = \frac{X_L}{Z_L} \text{ and } I_C = \frac{V}{X_C}$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$$

$$\text{or } X_L X_C = Z_L^2$$

$$\text{or } \frac{\omega L}{\omega C} = Z_L^2 = (R^2 + X_L^2)$$

$$\text{or } \frac{L}{C} = R^2 + (2\pi f_r L)^2$$

$$\text{or } 2\pi f_r L = \sqrt{\frac{L}{C} - R^2}$$

$$\text{or } f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Effect of Parallel Resonance

At parallel resonance line current $I_r = I_L \cos\phi$ or

$$\frac{V}{Z_r} = \frac{V}{Z_1} \times \frac{R}{Z_1} \quad \text{or} \quad \frac{1}{Z_r} = \frac{R}{Z_L^2} \quad \text{or}$$

$$\frac{1}{Z_r} = \frac{R}{L/C} = \frac{CR}{L} \quad \left(\text{as } Z_L^2 = \frac{L}{C} \right)$$

Therefore, the circuit impedance will be given as

$$Z_r = \frac{L}{CR}$$

The following **conclusions** are made about Parallel **Resonance**.

- The circuit impedance is purely resistive because there is no frequency term present in it. If the value of Inductance, Capacitance and Resistance is in Henry, Farads and Ohm than the value of circuit impedance Z_r will be in Ohms.
- The value of Z_r will be very high because the ratio L/C is very large at parallel resonance.
- The value of circuit current, $I_r = V/Z_r$ is very small because the value of Z_r is very high.
- The current flowing through the capacitor and the coil is much greater than the line current because the impedance of each branch is quite lower than that of circuit impedance Z_r .

Since the parallel resonant circuit can draw a very small current and power from the mains, therefore, it is also called as **Rejector Circuit**.

LECTURE13

Problem: The bandwidth of a series resonant circuit is 500Hz.If the resonant frequency is 6000Hz,What is the value of Q ? If $R=10\text{ohm}$,what is the value of inductive reactance at resonance. Calculate the circuit inductance and capacitance.

Solution:

$$BW(\text{Bandwidth})= \text{resonant frequency}/Q \text{ Factor}= 6000/500=12$$

$$\text{Also } Q= X_L/R$$

$$X_L = QR = 12 \times 10 = 120\Omega$$

$$X_L = 2\pi frL$$

$$\text{So } L = X_L / 2\pi fr = 120 / (2 \times 6000 \times \pi) = 3.18 \text{ milli henry}$$

We know at resonance, $|X_L| = |X_C|$

$$\text{So } X_C = 1 / (2\pi frC)$$

$$C = 1 / (2\pi \times 6000 \times 120) = 0.22 \mu\text{F}$$

Practical Resonance Circuits:

In the power system, at the resonant condition, the excessive voltage is built up across the inductive and capacitive component of the circuit such as circuit breaker, reactors, etc., may cause damage. Therefore, the series resonant condition is avoided in the power system. However, in some of the electronics devices such as antenna circuit of radio and TV receiver, tuning circuit the series resonant condition is used to increase the signal voltage and current at the desired frequency (fr).

Uses of series resonance circuit:

- As frequency selection circuit in radio and TV tuner circuits.
- As band pass filter circuit.

Uses of parallel resonance circuit:

- As a Band Stop Filter.
- As a tank circuit in Oscillators

MODULE IV: COUPLED CIRCUITS

LECTURE 14

Magnetic flux:

Magnetic flux (Φ_B) is the number of magnetic field lines passing through a surface (such as a loop of wire). The magnetic flux through a closed surface (such as a ball) is always zero. This implies that there cannot be magnetic charges in classical electromagnetism.

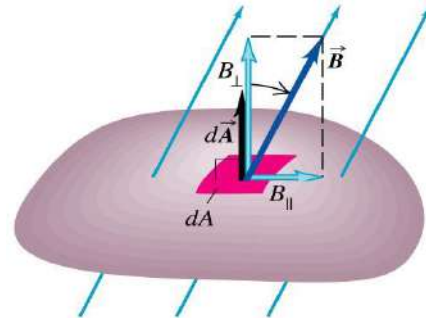
The SI unit of magnetic flux is the Weber (Wb) (in derived units: volt-seconds). The CGS unit is the Maxwell.

Magnetic flux is sometimes used by electrical engineers designing systems with electromagnets or designing dynamos. Physicists designing particle accelerators also calculate magnetic flux.

$$\Phi = \iint_S \mathbf{B} \cdot d\mathbf{A}$$

where

- Φ is the magnetic flux
- B is the magnetic field
- S is the surface area
- \cdot denotes dot product
- dA is the infinitesimal vector



Self-Inductance:

Self-inductance or in other words inductance of the coil is defined as the property of the coil due to which it opposes the change of current flowing through it. Inductance is attained by a coil due to the self-induced emf produced in the coil itself by changing the current flowing through it.

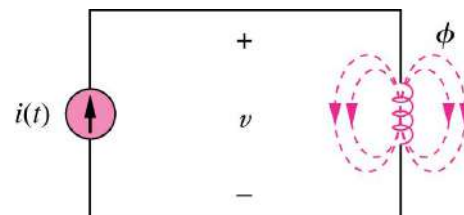
If the current in the coil is increasing, the self-induced emf produced in the coil will oppose the rise of current, that means the direction of the induced emf is opposite to the applied voltage.

You can determine the self-inductance of a coil by the following expression:

$$E = L \frac{di}{dt} \text{ or } N \frac{d\phi}{dt}$$

Where,

- L= inductance
- I=Current flowing through coil
- ϕ = Magnetic flux
- N= No of turns of coil



Mutual Inductance:

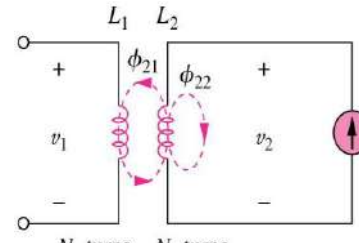
- Production of magnetic flux by a current
- The production of voltage by time varying magnetic field
- Current flowing in one coil established flux about that coil and about second coil nearby
- The time varying flux surrounding second coil produces a voltage across the terminals of the second coil
- This voltage is proportional to time rate of change of current in first coil
- Define coefficient of mutual inductance or simply mutual inductance

$$v_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d(\phi_{11} + \phi_{21})}{dt}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d(\phi_{12} + \phi_{22})}{dt}$$

$$v_2 = M_{21} \frac{di_1}{dt} \quad v_1 = M_{12} \frac{di_2}{dt}$$

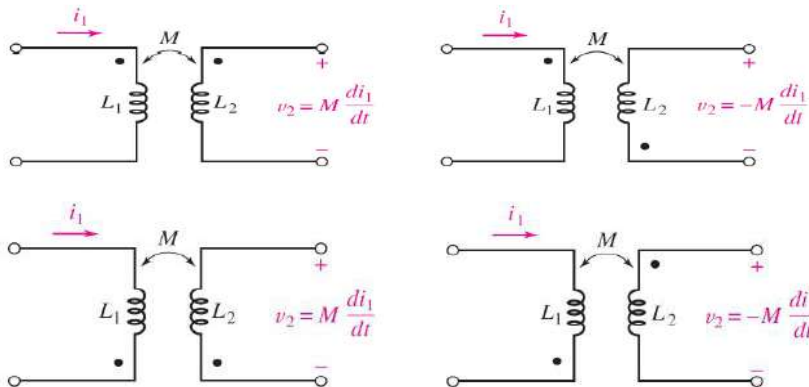
$$M_{12} = M_{21} = M$$



LECTURE15

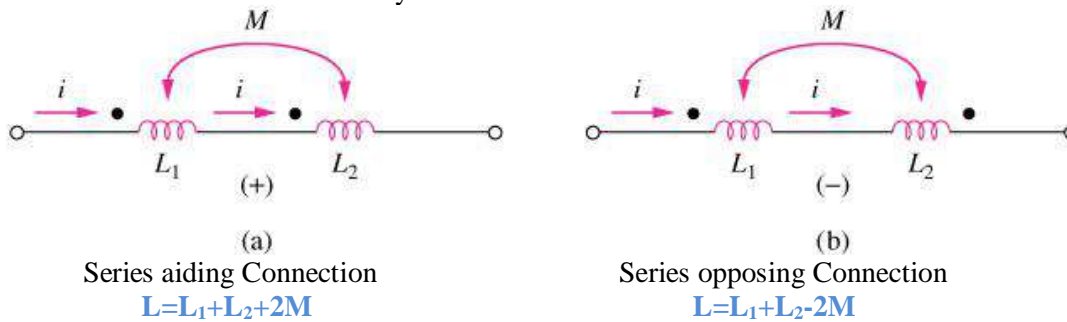
Dot Convention:

- A current entering the dotted terminal of one coil produces an open circuit voltage with positive voltage reference at the dotted terminal of the second coil
- A current entering the un dotted terminal of one coil provides a voltage that is positively sensed at the un dotted terminal of the second coil



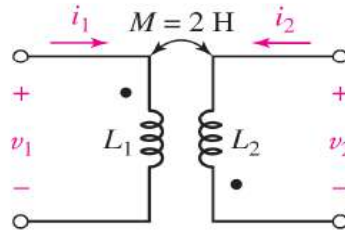
Coils in series:

The total inductance of two coupled coils in series depend on the placement of the dotted ends of the coils. The mutual inductances may add or subtract.



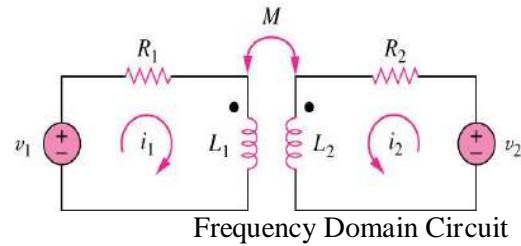
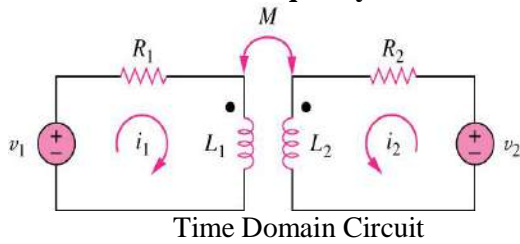
Example: (a) determine v_1 if $i_2 = 5 \sin 45t$ A and $i_1 = 0$;
 (b) Determine v_2 if $i_1 = -8e^{-t}$ A and $i_2 = 0$ di₂

The current i_2 (entering undotted terminal) results in a positive reference for the voltage induced across the left coil is the undotted terminal



Ans: $V_1(t) = \frac{di_2t}{dt} = -450 \cos 45t$
 $V_2(t) = \frac{di_1t}{dt} = -16 e^{-t}$ V

Time Domain and Frequency Domain Analysis:



Time Domain

$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Frequency Domain

$$V_1 = (R_1 + j\omega L_1)I_1 + j\omega M I_2$$

$$V_2 = j\omega M I_1 + (R_2 + j\omega L_2)I_2$$

LECTURE16

Energy in Coupled Circuit:

The total energy w stored in a mutually coupled inductor is:

- **Positive sign** is selected if both currents ENTER or LEAVE the dotted terminals.
- Otherwise we use Negative sign.

$$w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

Coupling Coefficient:

The Coupling Coefficient k is a measure of the magnetic coupling between two coils $0 \leq k \leq 1$

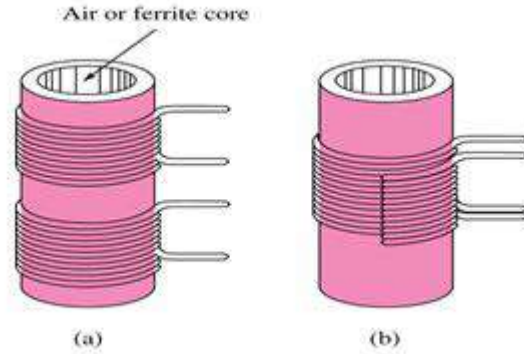
$$0 \leq k \leq 1$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$K = 1$ perfect coupling

$K < 0.5$ Loosely Coupling

$K > 0.5$ Tightly coupling



(a) Loosely Coupled Circuit (b) Tightly Coupled

Example 1:

Determine the coupling coefficient. Calculate the stored energy in the coupled inductors at $t=1$ s if $v=60 \cos(4t+30)$ V.

Ans:

The coupling coefficient is

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

After making mesh analysis we can found out that,

$$I_2 = 3.254 \angle 160.60^\circ \text{ A}$$

$$I_1 = -1.2 I_2$$

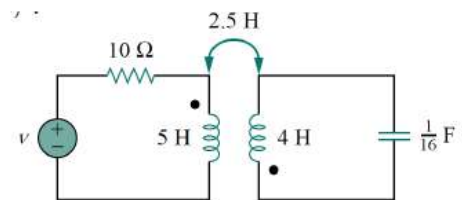
In the time domain

$$I_1 = 3.095 \cos(4t - 19.4^\circ)$$

$$I_2 = 3.254 \cos(4t - 199.4^\circ)$$

The total energy stored in the coupled inductors is

$$\begin{aligned}
 W &= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2 \\
 &= 20.73 \text{ J}
 \end{aligned}$$



MODULE V: GRAPH THEORY

LECTURE 17

Graph of a network is a simplest form of electric circuit representation. All the elements (resistor, inductor, capacitor) of an electric circuit can be replaced by lines with dots at both ends to form a graph for a given network.

Basic Terminologies associated with Graph:

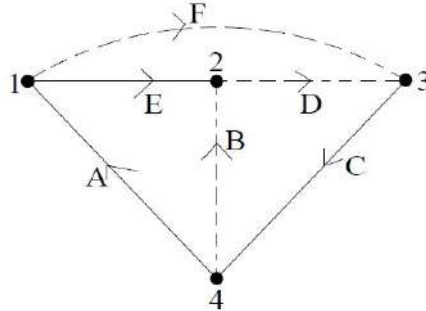


Fig 1

Branch: A branch is a line segment representing one network element/ combination of network elements connected between two points. Fig 1 shows A,B,C,D,E,F are branches.

Node: It is defined as an end point of a line segment that exists at the junction between two branches. Fig. 1 shows 1,2,3,4 are nodes.

Degree of a Node: It is the number of branches incident to it. Fig 1 shows degree of node 2 is 3.

Tree: It is an interconnected open set of branches of a graph which includes all the nodes and should never contain any loop within it. A tree is formed from a graph by deleting some branches of a graph. Fig 1 shows tree is formed by bold lines.

Twig: It is defined as the branches of a tree. Fig 1 shows A,C,E are twigs.

Tree link/ Chord: It is those branches of a graph that does not belong to that particular tree. Fig 1 shows B,D,F are links.

Directed or Oriented Graph: A graph is said to be directed when all the branches and nodes are numbered and branches are assigned with directions (arrows).

Relation between Twigs & Links

Let,

N = No of nodes; B = No of branches of a graph; L = No. of links, T = No. of twigs,

Then,

$$T = N - 1 ; \quad B = T + L$$

So, $L = B - N + 1$

Properties of a Tree

- i. Consists of all the nodes of the graph.
- ii. If a tree is obtained from a graph having N branches, then number of twigs will be N-1.
- iii. A tree should not have any closed loop.
- iv. For a particular graph, different trees can be obtained.

Formation of graph from electric circuit

The basic electrical circuit elements should be replaced as follows:

Resistor – By a shorted line between two nodes.

Inductor - By a shorted line between two nodes.

Capacitor - By a shorted line between two nodes.

Voltage source - By a shorted line between two nodes.

Current Source - By an open circuit.

Illustration:

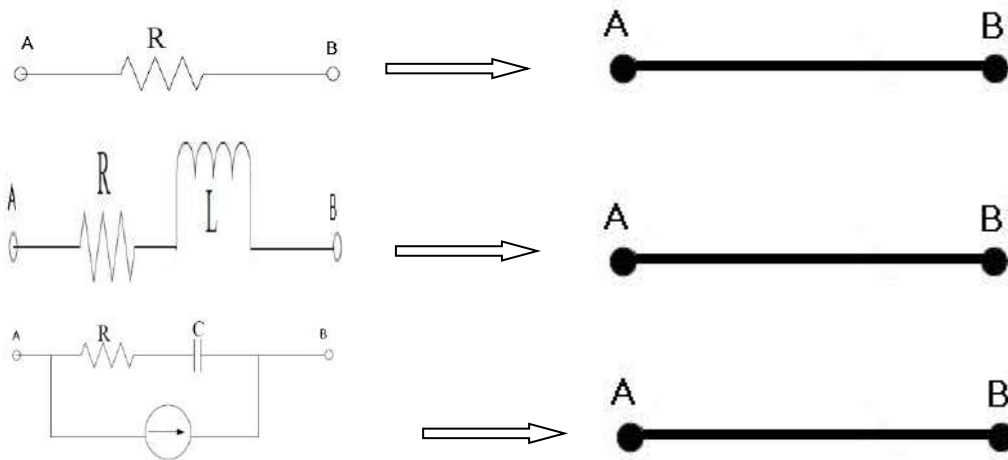


Fig. 2

LECTURE18

Incidence Matrix

This matrix gives relation between nodes & branches of a graph. It clearly indicates whether a branch is leaving a node or entering the node.

Formation of Incidence Matrix:

Let, N = Number of nodes of graph ; B = Number of branches of graph; I_{ij} = Incidence matrix of $N \times B$ dimension;

- i. If branch j is oriented away from node i , the matrix element $I_{ij} = 1$.
- ii. If branch j is oriented towards node i , the matrix element $I_{ij} = -1$.
- iii. If branch j is not incident at node i , the matrix element $I_{ij} = 0$.

The complete set of incidence matrix is called augmented incidence matrix.

Note that algebraic sum of the column entries of an incidence matrix is always zero.

Example I: Obtain incidence matrix of the graph shown in figure below:

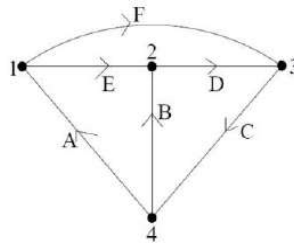


Fig. 3

Node	Branch	→	A	B	C	D	E	F
↓			$I_{ij} = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \\ 1 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$					

It is observed that column wise algebraic sum is zero. This is because a branch can leave a particular node and enters a particular node. It is a $N \times B$ matrix.

Reduced incidence Matrix:

It is possible to delete any one row from incidence matrix I_{ij} without losing any information about graph. The matrix obtained from incidence matrix by deleting one row is known as Reduced incidence matrix.

For a graph having N nodes & B branches, the order of the reduced incidence matrix is $(N - 1) \times B$.

Example II: Obtain reduced incidence matrix of the graph shown in figure below:

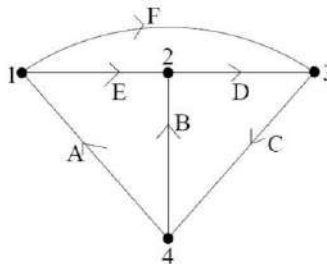


Fig. 4

Node	Branch	→	A	B	C	D	E	F
↓			$I_{ij} = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix}$					

Here node 4 (4th row) is deleted to obtain reduced incidence matrix. Note that as column wise algebraic sum is zero, hence it can easily be understood that what should be 4th row matrix elements.

LECTURE19

Fundamental Tie Set Matrix or Fundamental Loop Matrix:

A fundamental Tie set is formed by choosing a tree from a given graph. The fundamental loops should have only one link and other twigs. The number of fundamental loops is equal to number of links.

Formation of Fundamental Tie Set Matrix:

Step I: Choose a tree arbitrarily.

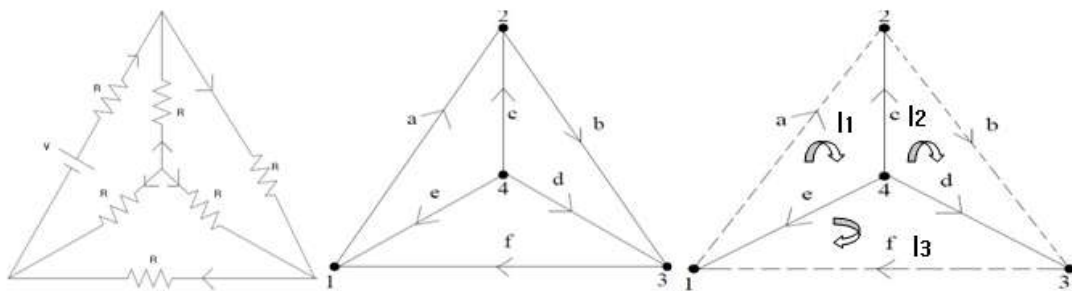
Step II: Form fundamental loops with one link & other twigs.

Step III: Assume direction of loop currents in the direction of links.

Step IV: Form fundamental Tie set Matrix T_{ij} as follows;

- i. If branch j is oriented in the direction of loop i , $T_{ij} = 1$.
- ii. If branch j is oriented in opposite direction of loop i , $T_{ij} = -1$.
- iii. If branch j is not part of loop i , $T_{ij} = 0$.

Example III: Draw graph for the circuit given below and obtain Tie set matrix.



Here, dotted lines show links and bold lines show twigs. The loop currents are assumed in the direction of links.

Loop currents	Branch	→	a	b	c	d	e	f
↓								

$$T_{ij} = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

Fundamental Cut Set Matrix:

A fundamental cut set of a graph with respect to a tree is formed by one twig & other links. The number of cut sets of a graph with respect to a tree is equal to number of twigs.

Formation of Fundamental Cut Set Matrix:

Step I: Choose a tree arbitrarily.

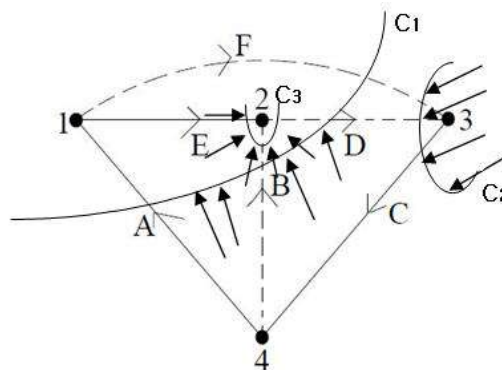
Step II: Form fundamental cut sets with one twig & other links.

Step III: Assume direction of cut sets in the direction of twigs.

Step IV: Form fundamental Cut set Matrix C_{ij} as follows;

- i. If branch j has same orientation of cut set i , $C_{ij} = 1$.
- ii. If branch j has opposite orientation of cut set i , $C_{ij} = -1$.
- iii. If branch j is not part of cut set i , $C_{ij} = 0$.

Example IV: Obtain cut set matrix of the graph shown in figure below:



Cut Set C_1 is formed with Twig A, Links B & D.

Cut Set C_2 is formed with Twig C, Links B & D.

Cut Set C_3 is formed with Twig E, Links F & D.

Cut set	Branch	→	A	B	C	D	E	F
↓								

$$I_{ij} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

MODULE VI: CIRCUIT TRANSIENTS

LECTURE 20

On completion of this unit you will be able to:

- ❖ Understand transient response of basic RC & RL circuits
- ❖ First order & Second Order Circuits
- ❖ Solve first order RL & RC circuits
- ❖ Understand the concepts of Transient & Steady State Response
- ❖ Solve RLC circuits in DC steady state conditions
- ❖ Relate the step response of a second order system to its natural frequency and damping ratio
- ❖ Understand the transient response of RC, RL, & RLC circuits under AC analysis

What is transient?

The time varying currents and voltages resulting from the sudden application of source, usually due to switching, are called transients. By writing circuit equations, we obtain integral differential equations.

Transient & Steady State Response

Complete response = Transient Response + Steady State Response

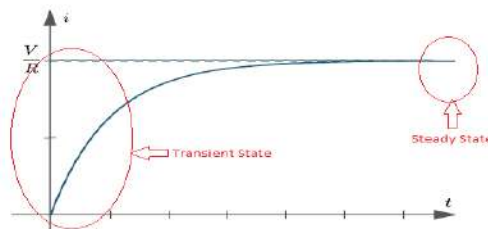
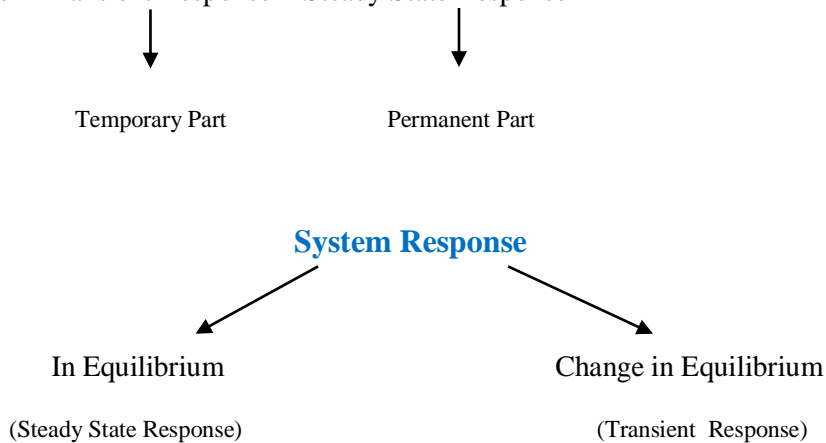
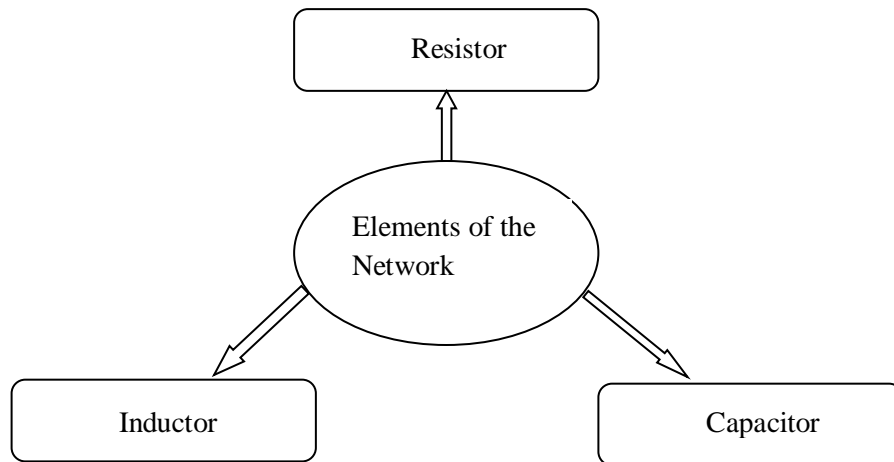


Fig1. Transient & Steady State Response

Transient response is the response of a system from rest or equilibrium to steady state.

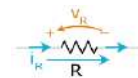
The values of voltage and current after the transient has died out are known as the steady state responses.

Basic Elements of the Network:



Resistor: A resistor is a two terminal electronic component. It produces a voltage across its terminals that is proportional to the current following through it in accordance with Ohm’s Law:

$$V = IR$$



Thus, in resistor, change in current is instantaneous as there is no storage of energy in it, in units of Ohm.

Inductor: An inductor is a passive component that can store energy in a magnetic field created by the current passing through it. An inductor’s ability to store magnetic energy is measured by its inductance, in



units of Henries. Current in an inductor cannot change instantaneously.

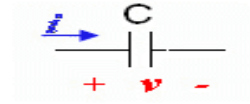
$$v(t) = L \frac{d}{dt} i(t)$$

$$i(t) = \frac{1}{L} \int_{t_0}^t v(\tau) dt + i(t_0)$$

Capacitor: The current through a capacitor can be changed instantly, but it takes time to change the voltage across a capacitor. Units is Farads.

$$i(t) = C \frac{d}{dt} v(t)$$

$$v(t) = \frac{1}{C} \int_{t_0}^t i(\tau) dt + v(t_0)$$



Passive Linear Circuit Elements

	Resistor	Capacitor	Inductor
Current Voltage Characteristic	$V = IR$	$i = C \frac{dv}{dt}$	$v = L \frac{di}{dt}$
Series Equivalent	$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$
Parallel Equivalent	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Complex Impedance	$z(s) = \frac{V(s)}{I(s)}$	$z(s) = \frac{1}{sC} = \frac{1}{j\omega c}$	$z(s) = sL = j\omega L$
Time Constant (With Resistor)		$\tau = RC$	$\tau = \frac{L}{R}$

Initial Conditions: We need initial conditions to evaluate the arbitrary constants in the general solution of differential equations. We find the change in selected variables in a circuit when one or more switches are moved from open to closed positions or vice versa. The representation of initial conditions of RLC circuit excited by a DC source is shown in table 1

- $t = 0^-$ indicated the time just before changing the position of the switch





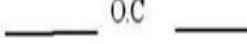

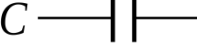

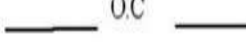
- $t = 0$ indicated the time when the position of switch is changed
- $t = 0^+$ indicated the time immediately after changing the position of switch

Initial condition for the resistor: For a resistor current and voltage are related by $v(t) = Ri(t)$. The current through a resistor will change instantaneously if the voltage changes instantaneously. Similarly the voltage will change instantaneously, if the current changes instantaneously.

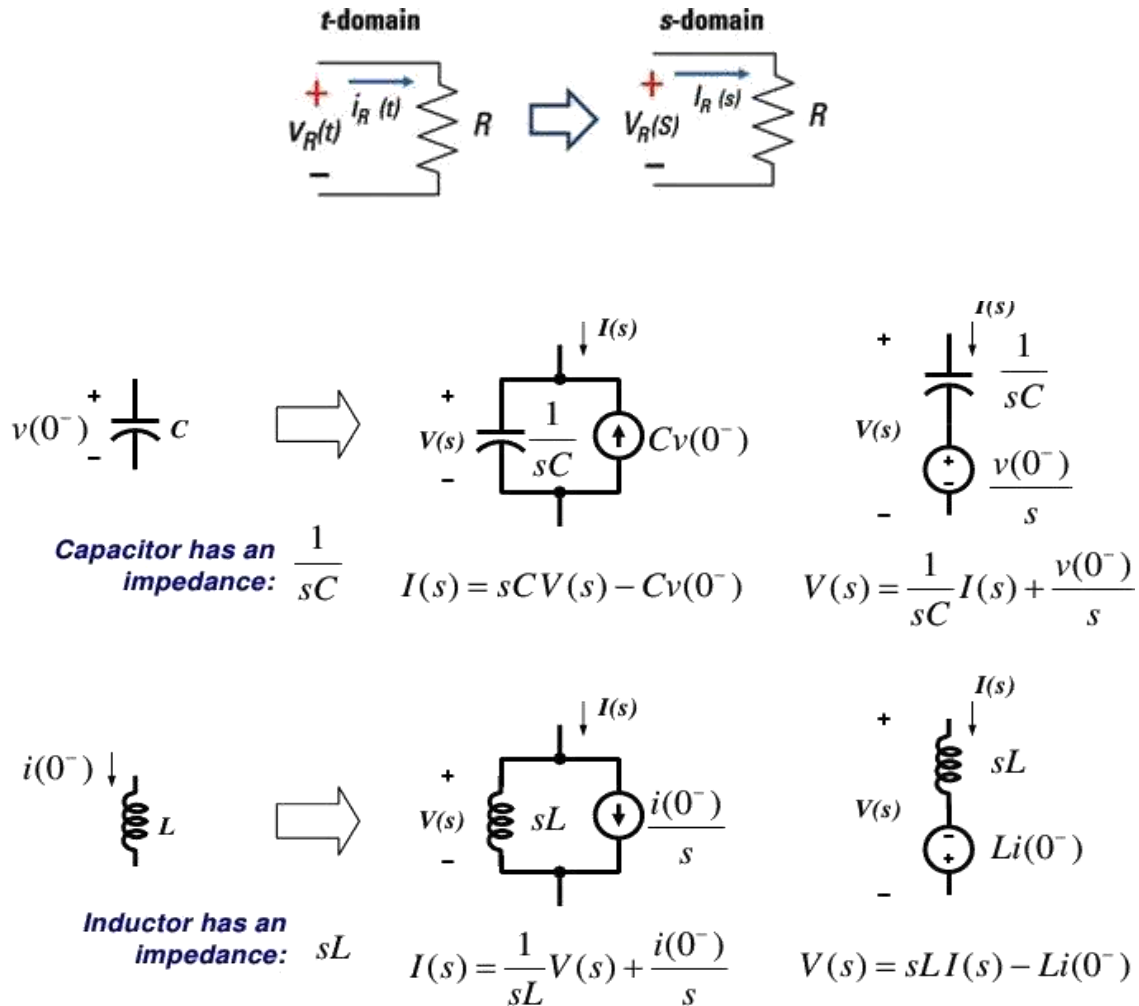
Initial condition for the inductor: Voltage across the inductor is proportional to the rate of change of current. It is impossible to change the current through an inductor by a finite amount in zero time. This requires an infinite voltage across the inductor.

Initial condition for the capacitor: Current through a capacitor is proportional to the rate of change of voltage. Due to this change the voltage across a capacitor by a amount in zero time. This requires an finite current through the capacitor .

Table: representation of initial and final conditions of an RLC circuit excited by a DC source

Circuit Elements	at initial conditions $t = 0^+$	at final conditions $t = \infty$
		
	 If there is no current flowing through an inductor at $t=0$, the inductor will act as open circuit at $t = 0^+$	
	 If there is no voltage across the capacitor at $t=0$, the capacitor will act as a short at $t = 0^+$	

Representation of RLC devices in time-domain and s-Domain:



Solution to First Order Differential Equation:

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Let the initial condition be $x(t = 0) = x(0)$, then we solve the differential equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete solution consists of two parts:

- ✚ the homogeneous solution (natural solution)
- ✚ the particular solution (forced solution)

The Natural Response:

Let the general Equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation $f(t)$ equal to zero,

$$\tau \frac{dx_n(t)}{dt} + x_n(t) = 0 \text{ or } \frac{dx_n(t)}{dt} = -\frac{x_n(t)}{\tau}, \frac{dx_n(t)}{x_n(t)} = -\frac{dt}{\tau}$$

$$\int \frac{dx_n(t)}{x_n(t)} = \int -\frac{dt}{\tau}, \quad x_n(t) = \alpha e^{-t/\tau}$$

$X_n(t)$ is called the natural response.

The Forced Response:

Let the general equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation $f(t)$ equal to F , a constant for $t \geq 0$

$$\tau \frac{dx_f(t)}{dt} + x_f(t) = K_s F$$

$$x_f(t) = K_s F \text{ for } t \geq 0$$

$X_f(t)$ is called the forced response

The Complete Response:

Let the general Equation and Solve for α ,

$$\text{for } t = 0$$

$$x(t=0) = x(0) = \alpha + x(\infty)$$

$$\alpha = x(0) - x(\infty)$$

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete response is:

- the natural response is $x = x_n(t) + x_f(t)$
- the forced response is $= \alpha e^{-t/\tau} + K_s F$

The Complete solution is:

$$= \alpha e^{-t/\tau} + x(\infty)$$

$$x(t) = [x(0) - x(\infty)]e^{-t/\tau} + x(\infty)$$

$$[x(0) - x(\infty)]e^{-t/\tau}$$

It is called transient response

$$x(\infty)$$

It is called steady state response

Complete response = natural response + forced response

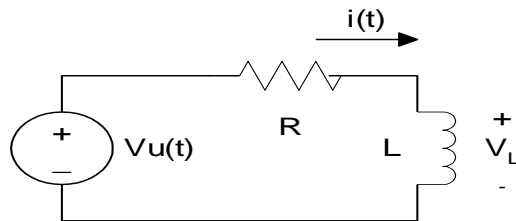


Due to the initial condition

Due to the external excitation

LECTURE21

DC Response of an R-L Circuit:



$$Ri + L \frac{di}{dt} = V$$

$$\frac{Ldi}{V - Ri} = dt$$

$$-\frac{L}{R} \ln(V - Ri) = t + k$$

Integrating both sides to have

$$i(0^+) = 0, \text{ thus } k = -\frac{L}{R} \ln V$$

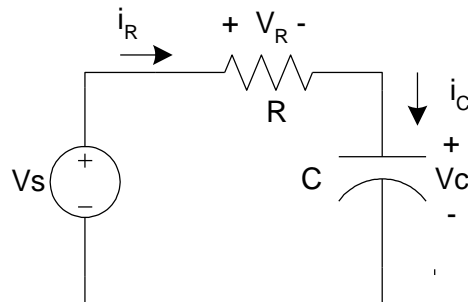
$$-\frac{L}{R} [\ln(V - Ri) - \ln V] = t$$

$$\frac{V - Ri}{V} = e^{-Rt/L} \quad \text{or}$$

$$i = \frac{V}{R} - \frac{V}{R} e^{-Rt/L}, \text{ for } t > 0$$

Where L/R is the time constant

DC Response of an R-C Circuit:



$$i_R = i_C$$

$$i_R = \frac{v_s - v_R}{R}, i_C = C \frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} + \frac{1}{RC} v_R = \frac{1}{RC} v_s$$

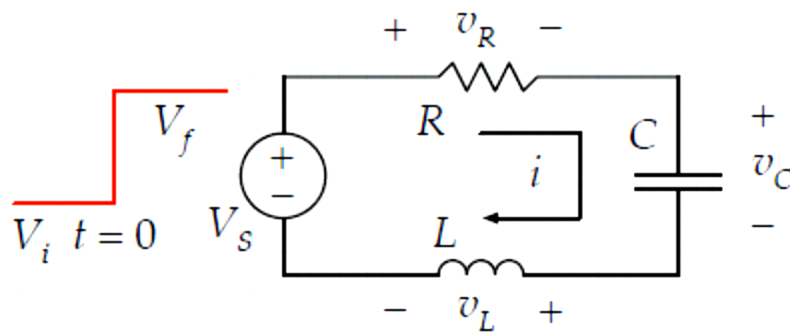
The solution will give natural response $i_R = i_C = Ke^{-t/RC}$

The corresponding voltage drops across the resistor and capacitor can be obtained as follows:

$$v_R = iR = -Ve^{-t/RC}$$

Where RC is a time constant

DC Response of an R-L-C Circuit:



$$t > 0: V_f = iR + v_C + L \frac{di}{dt}$$

$$i = C \frac{dv_C}{dt}$$

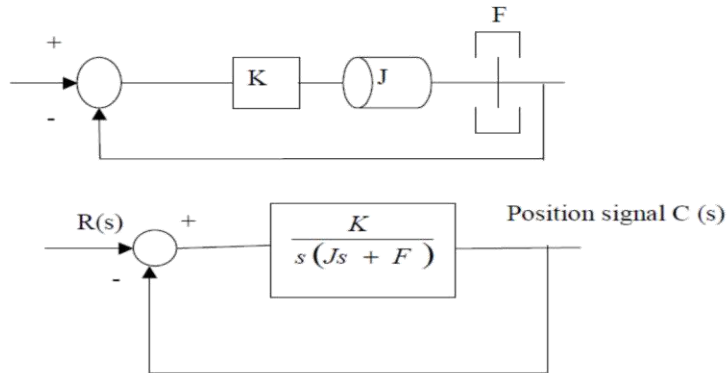
$$V_f = RC \frac{dv_C}{dt} + v_C + LC \frac{d^2v_C}{dt^2}$$

$$\frac{d^2v_C}{dt^2} + \frac{R}{L} \frac{dv_C}{dt} + \frac{v_C}{LC} = \frac{V_f}{LC}$$

Second-order differential equation.

2nd Order System:

Block Diagram



Transfer function:

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Fs + K}$$

$$= \frac{\frac{K}{J}}{\left[s + \frac{F}{2J} + \sqrt{\left(\frac{F}{2J}\right)^2 - \frac{K}{J}} \right] \left[s + \frac{F}{2J} - \sqrt{\left(\frac{F}{2J}\right)^2 - \frac{K}{J}} \right]}$$

Substitute in the transfer function:

$$\frac{K}{J} = \omega_n^2$$

$$\frac{F}{J} = 2 \zeta \omega_n = 2\sigma$$

$$\zeta = \frac{F}{2 \sqrt{JK}}$$

- ζ : damping ratio
- ω_n : undamped natural frequency
- σ : stability ratio

to obtain

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- **Underdamped** case: $0 < \zeta < 1$

$$F^2 - 4JK < 0 \quad \text{two complex conjugate poles}$$

- **Critically damped** case: $\zeta = 1$

$$F^2 - 4JK = 0 \quad \text{two equal real poles}$$

- **Overdamped** case: $\zeta > 1$

$$F^2 - 4JK > 0 \quad \text{two real poles}$$

MODULE VII: LAPLACE TRANSFORM

LECTURE22

The **Laplace Transform** is a powerful tool that is very useful in Electrical Engineering. The transform allows equations in the "time domain" to be transformed into an equivalent equation in the **Complex S Domain**.

The mathematical definition of the Laplace transform is as follows:

$$F(s) = \mathcal{L} \{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t) dt$$

Laplace Domain

The **Laplace domain**, or the "Complex s Domain" is the domain into which the Laplace transform transforms a time-domain equation. s is a complex variable, composed of real and imaginary parts:

$$s = \sigma + j\omega$$

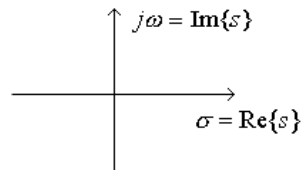


Fig 3.1 S-plane

The Laplace domain graphs the real part (σ) as the horizontal axis, and the imaginary part (ω) as the vertical axis. The real and imaginary parts of s can be considered as independent quantities.

LECTURE23

Properties of Laplace Transformer:

Laplace transform of derivative-

$$\mathcal{L} \{f'(t)\} = sF(s) - f(0)$$

Likewise, we can express higher-order derivatives in a similar manner:

$$\mathcal{L} \{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

For integrals, we get the following:

$$\mathcal{L} \left\{ \int_0^t f(t) dt \right\} = \frac{1}{s} F(s)$$

LECTURE24

Initial Value Theorem

The **Initial Value Theorem** of the laplace transform states as follows:

$$f(0) \Leftrightarrow \lim_{s \rightarrow \infty} sF(s)$$

N.B-This is useful for finding the initial conditions of a function needed when we perform the transform of a differentiation operation

LECTURE25

Final Value Theorem:

Similar to the Initial Value Theorem, the **Final Value Theorem** states that we can find the value of a function f , as t approaches infinity, in the laplace domain, as such

$$\lim_{t \rightarrow \infty} f(t) \Leftrightarrow \lim_{s \rightarrow 0} sF(s)$$

N.B-This is useful for finding the steady state response of a circuit. The final value theorem may only be applied to stable systems.

LECTURE26

Laplace transform of different function

$f(t) \quad (t \geq 0)$	$F(s)$
$\delta(t)$ (unit impulse)	1
$u(t)$ (unit step)	$\frac{1}{s}$
t (unit ramp)	$\frac{1}{s^2}$
$t^n \quad (n > -1)$	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$\sin bt$	$\frac{b}{s^2 + b^2}$
$\cos bt$	$\frac{s}{s^2 + b^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$

LECTURE27

Transfer Function:

A transfer function $G(s)$ or $H(s)$ is defined as the ratio of Laplace transform of the output $C(s)$ to Laplace transform of input $R(s)$

$$G(S) = \frac{C(S)}{R(S)}$$

Poles & Zeros :

The roots of $C(s)$ (numerator) for which the value of transfer function become zero is known as zeros.

The roots of $R(s)$ (denominator) for which $G(s)$ become infinite is known as poles.

Example

Find the pole-zero representation of the system with the transfer function:

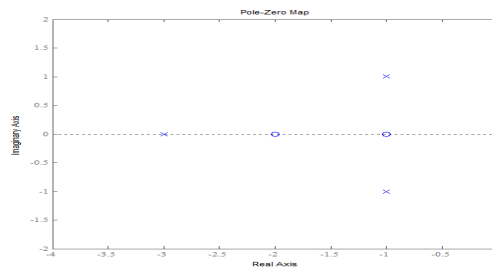
$$H(s) = \frac{6s^2 + 18s + 12}{2s^3 + 10s^2 + 16s + 12}$$

First rewrite in our standard form (note: the polynomials were factored with a computer).

$$H(s) = \frac{6}{2} \frac{s^2 + 3s + 2}{s^3 + 5s^2 + 8s + 6} = 3 \frac{(s+1)(s+2)}{(s+1-j)(s+1+j)(s+3)} \quad j = \sqrt{-1}$$

So the pole-zero representation consists of:

- zeros at $s=-1$ and $s=-2$, and
- polese at $s=-1+j$, $s=-1-j$ and $s=-3$.



LECTURE28

Properties of Laplace Transform:

Property 1. Constant Multiple

If a is a constant and $f(t)$ is a function of t , then

$$\mathcal{L}\{a \cdot f(t)\} = a \cdot \mathcal{L}\{f(t)\}$$

Property 2. Linearity Property

If a and b are constants while $f(t)$ and $g(t)$ are functions of t , then

$$\mathcal{L}\{a \cdot f(t) + b \cdot g(t)\} = a \cdot \mathcal{L}\{f(t)\} + b \cdot \mathcal{L}\{g(t)\}$$

Property 3. Change of Scale Property

If $\mathcal{L}\{f(t)\} = F(s)$ then $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

Property 4. Shifting Property (Shift Theorem)

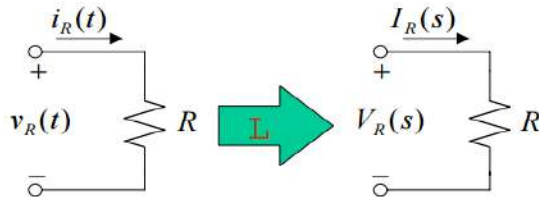
$$\mathcal{L}\{e^{at} f(t)\} = F(s - a)$$

LECTURE29

Circuit Elements in S domain

Time Domain

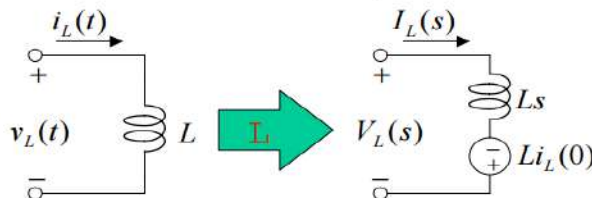
Resistor:
 $v_R(t) = Ri_R(t)$



s-Domain

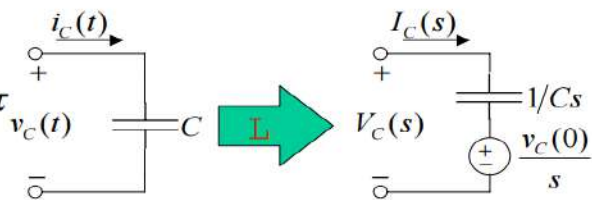
Resistor:
 $V_R(s) = RI_R(s)$

Inductor:
 $v_L(t) = L \frac{di_L(t)}{dt}$



Inductor:
 $V_L(s) = LsI_L(s) - Li_L(0)$

Capacitor:
 $v_C(t) = \frac{1}{C} \int_0^t i_C(\tau) d\tau + v_C(0)$



Capacitor:
 $V_C(s) = \frac{1}{Cs} I_C(s) + \frac{v_C(0)}{s}$

MODULE VIII: TWO PORT NETWORK
LECTURE 30

When a number of impedances are connected together to form a system that consist of set of interconnected circuits performing specific function, is called as a network.

One Port Network: A pair of terminals at which a signal may enter or leave a network is called a port n/w or one port n/w.

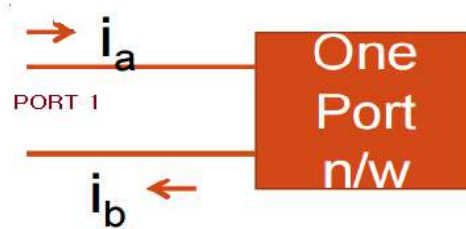


Fig 1. One port Network

Two Port Parameters It represented by a black box with four variables, □ two voltage (V_1, V_2) and two currents (I_1, I_2), which are available for measurements. Out of these four variables, which two variables □ may be considered ‘independent’ and which two ‘dependent’ is generally decided by the problem under consideration.



Fig 2. Two port Network

Z – PARAMETERS (Open Circuit Impedance Parameters):

The figure below shows a two port network. The voltages & currents of both ports are shown. The currents inward to the network are considered as positive.



Fig 3. Two port Network

For a two port network,

$$[V] = [Z][I] \dots \dots \dots (i)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \dots\dots\dots (ii)$$

Where, $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ = Impedance Matrix

Equation (ii) can be written as,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \dots\dots\dots (iii)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \dots\dots\dots (iv)$$

To find

Z_{11} & Z_{21} , output port should be open circuited, i.e.; $I_2 = 0$,

So, from equation (iii), $Z_{11} = \left[\frac{V_1}{I_1} \right]_{I_2=0}$; Known as Input driving point impedance

From equation (iv), $Z_{21} = \left[\frac{V_2}{I_1} \right]_{I_2=0}$; Known as forward transfer impedance

To find

Z_{12} & Z_{22} , input port should be open circuited, i.e.; $I_1 = 0$,

So, from equation (iii), $Z_{12} = \left[\frac{V_1}{I_2} \right]_{I_1=0}$; Known as reverse transfer impedance

From equation (iv), $Z_{22} = \left[\frac{V_2}{I_2} \right]_{I_1=0}$; Known as Output driving point impedance

Z_{11} , Z_{12} , Z_{21} , Z_{22} all are impedances and unit is ohm.

As Z parameters are obtained by open circuiting any of the two ports and expressed as ratio of voltage & current, these are known as **OPEN CIRCUIT IMPEDANCE PARAMETERS.**

Equivalent circuit model of Z parameter representation:

The following circuit diagram shows equivalent circuit model of Z parameters which satisfies equation (iii) & (iv).

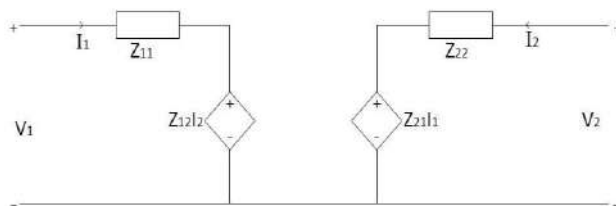


Fig 4. Equivalent circuit model of Z parameter representation

Example I: Find z parameters of the following network shown in fig 5.

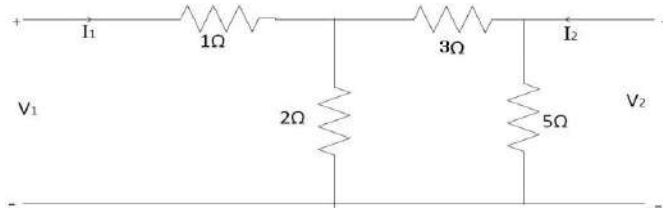


Fig 5.

Assuming first that the output is open circuited, $I_2 = 0$,

Looking back to the network from terminals ab,

$$R_{eq1} = [(3 + 5) \parallel 2] + 1 = 2.6 \Omega.$$

Let us assume I_1 to be the input current such that,

$$V_1 = I_1 R_{eq1} = 2.6 I_1 \text{ volt}$$

And, $Z_{11} = \frac{V_1}{I_1}$ when $I_2 = 0$

$$Z_{11} = 2.6 \Omega.$$

Let us assume the current through 5 Ω resistor be I_x

Then, $I_x = I_1 * \frac{2}{2+3+5} = \frac{I_1}{5}$ A

And, $V_2 = \frac{I_1}{5} * 5 = I_1$ Volt

This gives, $Z_{21} = \frac{V_2}{I_1}$ when $I_2 = 0$

$$Z_{21} = 1 \Omega.$$

Next assuming the input to be open, $I_1 = 0$. The equivalent resistance of the network looking back from the terminal gh is given by

$$R_{eq2} = [(3 + 2) \parallel 5] = 2.5 \Omega.$$

Let us assume I_2 to be the input current such that,

$$V_2 = I_2 R_{eq2} = 2.5 I_2 \text{ volt}$$

And, $Z_{22} = \frac{V_2}{I_2}$ when $I_1 = 0$

$$Z_{22} = 2.5 \Omega.$$

Let us assume the current through 2 Ω resistor be I_y

Then, $I_y = I_2 * \frac{5}{2+3+5} = \frac{I_2}{2}$ A

With terminal a-b open, $V_{a-b} = V_{c-d}$, i.e., the voltage across c-d terminal is V_1 too.

Hence, $V_1 = I_y * 2 = \frac{I_2}{2} * 2 = I_2$ Volt

This gives, $Z_{12} = \frac{V_1}{I_2}$ when $I_1 = 0$

$Z_{21} = 1 \Omega$.

LECTURE31

Example II: Find z parameters of the following network shown in fig 6.

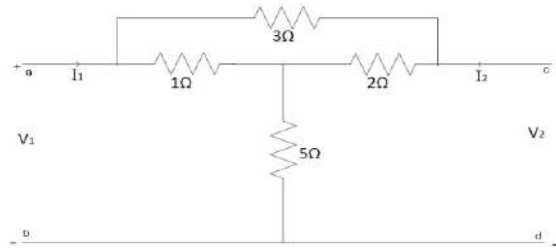


Fig 6.

Let us first assume the output terminal c-d are open circuited and V_1 is applied at a-b terminals (input port).

Looking back to the network from terminals ab,

$R_{eq1} = [(3 + 2) \parallel 1] + 5 = \frac{35}{6} \Omega$.

Let us assume I_1 to be the input current such that,

$V_1 = I_1 R_{eq1} = \frac{35}{6} I_1$ volt

And, $Z_{11} = \frac{V_1}{I_1}$ when $I_2 = 0$

$Z_{11} = \frac{35}{6} \Omega$.

Then, $I_3 = I_1 * \frac{1}{2+3+1} = \frac{I_1}{6}$ A

Thus drop V_2 across terminal c-d is given by

$V_2 = I_3 * 2 + I_1 * 5 = \frac{I_1}{6} * 2 + I_1 * 5$ Volt

This gives, $Z_{21} = \frac{V_2}{I_1}$ when $I_2 = 0$

$$Z_{21} = \frac{16}{3} \Omega.$$

Next let us assume the oinput terminal a-b are open circuited and V_2 is applied at c-d terminals (output port).

$$R_{eq2} = [(3 + 1) \parallel 2] + 5 = \frac{19}{3} \Omega.$$

Let us assume I_2 to be the input current such that,

$$V_2 = I_2 R_{eq2} = \frac{19}{3} I_2 \text{ volt}$$

And, $Z_{22} = \frac{V_2}{I_2}$ when $I_1 = 0$

$$Z_{22} = \frac{19}{3} \Omega.$$

Then, $I_3 = I_2 * \frac{2}{2+3+1} = \frac{I_2}{3}$ A

Hence, $V_1 = \frac{I_2}{3} * 1 + 5I_2 = \frac{16}{3} I_2$ Volt

This gives, $Z_{12} = \frac{V_1}{I_2}$ when $I_1 = 0$

$$Z_{21} = \frac{16}{3} \Omega.$$

LECTURE32

Y – PARAMETERS (Short Circuit Admittance Parameters):

The figure below shows a two port network. The voltages & currents of both ports are shown. The currents inward to the network are considered as positive.



Fig 7. Two port Network

For a two port network,

$$[I] = [Y][V] \dots \dots \dots (v)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \dots\dots\dots (vi)$$

Where, $[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ = Admittance Matrix

Equation (ii) can be written as,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \dots\dots\dots (vii)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \dots\dots\dots (viii)$$

To find

Y_{11} & Y_{21} , output port should be short circuited, i.e; $V_2 = 0$,

So, from equation (vii), $Y_{11} = \left[\frac{I_1}{V_1} \right]_{V_2=0}$; Known as Input driving point admittance

From equation (viii), $Y_{21} = \left[\frac{I_2}{V_1} \right]_{V_2=0}$; Known as forward transfer admittance

To find

Y_{12} & Y_{22} , input port should be short circuited, i.e; $V_1 = 0$,

So, from equation (vii), $Y_{12} = \left[\frac{I_1}{V_2} \right]_{V_1=0}$; Known as reverse transfer admittance

From equation (viii), $Y_{22} = \left[\frac{I_2}{V_2} \right]_{V_1=0}$; Known as Output driving point admittance

Y_{11} , Y_{12} , Y_{21} , Y_{22} all are admittances and unit is mho.

As Y parameters are obtained by short circuiting any of the two ports and expressed as ratio of current voltage, these are known as **SHORTCIRCUIT ADMITTANCE PARAMETERS.**

Equivalent circuit model of Y parameter representation:

The following circuit diagram shows equivalent circuit model of Y parameters which satisfies equation (vii) & (viii).

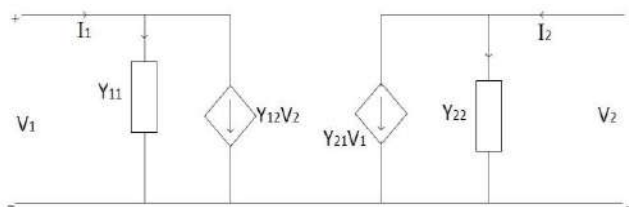


Fig 8. Equivalent circuit model of Y parameter representation

Example III: Find Y parameters of the following network shown in fig 9.

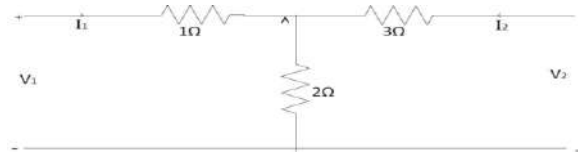


Fig 9.

Let voltage across 2 ohm resistance is V_3 .

From above figure,

$$I_1 = \frac{V_1 - V_3}{1}$$

$$I_1 = V_1 - V_3 \quad \dots\dots(i)$$

$$I_2 = \frac{V_2 - V_3}{3} \quad \dots\dots(ii)$$

$$\text{Applying KCL at Node A, } I_1 + I_2 = \frac{V_3}{2} \quad \dots\dots(iii)$$

Substituting Eqn. (i) and (ii) in Eqn. (iii),

$$V_1 - V_3 + \frac{V_2 - V_3}{3} = \frac{V_3}{2}$$

$$V_3 = \frac{6}{11}V_1 + \frac{2}{11}V_2 \quad \dots\dots(iv)$$

Substituting Eqn. (iv) in Eqn. (i),

$$I_1 = V_1 - \frac{6}{11}V_1 + \frac{2}{11}V_2$$

$$I_1 = \frac{5}{11}V_1 - \frac{2}{11}V_2 \quad \dots\dots(v)$$

Substituting Eqn. (iv) in Eqn. (ii),

$$I_2 = \frac{V_2}{3} - \frac{1}{3}\left(\frac{6}{11}V_1 + \frac{2}{11}V_2\right)$$

$$I_2 = -\frac{2}{11}V_1 + \frac{3}{11}V_2 \quad \dots\dots(vi)$$

Comparing Eqn. (v) and (vi) with Y- parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

Example IV: Obtain Y parameters of the following network shown in fig 10.

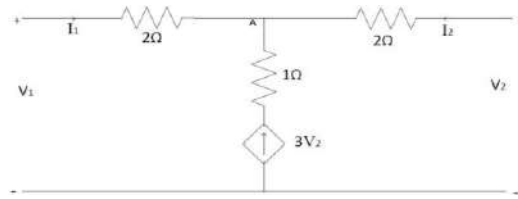


Fig 10.

Let, V_3 be voltage at node A.

From figure, we can write,

$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_1 = \frac{1}{2}V_1 - \frac{1}{2}V_3 \quad \dots\dots(i)$$

$$I_2 = \frac{V_2 - V_3}{2} \quad \dots\dots(ii)$$

$$\text{Applying KCL at Node A, } I_1 + I_2 + 3V_2 = 0 \quad \dots\dots(iii)$$

Substituting Eqn. (i) and (ii) in Eqn. (iii),

$$\frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{2} + 3V_2 = 0$$

$$V_3 = \frac{1}{2}V_1 + \frac{7}{2}V_2 \quad \dots\dots(iv)$$

Substituting Eqn. (iv) in Eqn. (i),

$$I_1 = \frac{1}{2}V_1 - \frac{1}{2}\left(\frac{1}{2}V_1 + \frac{7}{2}V_2\right)$$

$$I_1 = \frac{1}{4}V_1 - \frac{7}{4}V_2 \quad \dots\dots(v)$$

Substituting Eqn. (iv) in Eqn. (ii),

$$I_2 = \frac{V_2}{2} - \frac{1}{2}\left(\frac{1}{2}V_1 + \frac{7}{2}V_2\right)$$

$$I_2 = -\frac{1}{4}V_1 - \frac{5}{4}V_2 \quad \dots\dots(vi)$$

Comparing Eqn. (v) and (vi) with Y- parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{7}{4} \\ -\frac{1}{4} & -\frac{5}{4} \end{bmatrix}$$

LECTURE33

h – PARAMETERS (Hybrid Parameters):

h – parameter representations are widely used in electronic circuit modelling, especially in transistor modelling. To find h parameters, both open circuit & short circuit conditions are utilized at input & output ports respectively, hence it is termed as hybrid parameters.



Fig 11: Two Port Network

For a two port network,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \dots\dots\dots (ix)$$

Equation (ii) can be written as,

$$V_1 = h_{11}I_1+h_{12}V_2 \dots\dots\dots (x)$$

$$I_2 = h_{21}I_1+h_{22}V_2 \dots\dots\dots (xi)$$

To find

h_{11} & h_{21} , output port should be short circuited, i.e; $V_2 = 0$,

So, from equation (x), $h_{11} = \left[\frac{V_1}{I_1} \right]_{V_2=0}$; Unit : ohm

From equation (xi), $h_{21} = \left[\frac{I_2}{I_1} \right]_{V_2=0}$; Unit : None

To find

h_{12} & h_{22} , input port should be open circuited, i.e; $I_1 = 0$,

So, from equation (v), $h_{12} = \left[\frac{V_1}{V_2} \right]_{I_1=0}$; Unit :None

From equation (vi), $Z_{22} = \left[\frac{I_2}{V_2} \right]_{I_1=0}$; Unit : mho

Equivalent circuit model of h parameter representation:

The following circuit diagram shows equivalent circuit model of Y parameters which satisfies equation (x) & (xi).

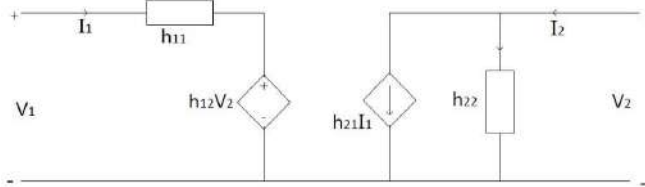


Fig 12. Equivalent circuit model of h parameter representation

LECTURE34

ABCD PARAMETERS (Transmission line or Chin Parameters):

ABCD parameters are widely used in transmission line parameter calculations, hence these are also known as transmission line parameters.



Fig 13: Two Port Network

It is observed from above figure that, I_1 is positive as it is inward to the network, whereas I_2 is negative as it is leaving the network.

For a two port network,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \dots\dots\dots (xii)$$

Equation (ii) can be written as,

$$V_1 = AV_2 - B I_2 \dots\dots\dots (xiii)$$

$$I_1 = CV_2 - D I_2 \dots\dots\dots (xiv)$$

To find

A & C, output port should be open circuited, i.e; $I_2 = 0$,

So, from equation (xiii), $A = \left[\frac{V_1}{V_2} \right]_{I_2=0}$; Unit : None

From equation (xiv), $C = \left[I_1 / V_2 \right]_{I_2=0}$; Unit : mho

To find

B & D, output port should be short circuited, i.e; $V_2 = 0$,

So, from equation (xiii), $B = \left[V_1 / -I_2 \right]_{V_2=0}$; Unit : ohm

From equation (xiv), $D = \left[I_1 / -I_2 \right]_{V_2=0}$; Unit : None

Example III: Find ABCD parameters of the following network shown in fig 14.

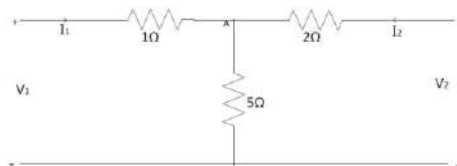


Fig 14.

Applying KVL at left mesh, $V_1 = 6I_1 + 5I_2$ (i)

Applying KVL at right mesh, $V_2 = 5I_1 + 7I_2$ (ii)

Hence, $5I_1 = V_2 - 7I_2$

$$I_1 = \frac{1}{5}V_2 - \frac{7}{5}I_2 \quad \text{.....(iii)}$$

Substituting Eqn. (iii) in Eqn. (i),

$$\begin{aligned} V_1 &= 6\left(\frac{1}{5}V_2 - \frac{7}{5}I_2\right) + 5I_2 \\ &= \frac{6}{5}V_2 - \frac{17}{5}I_2 \quad \text{.....(iv)} \end{aligned}$$

Substituting Eqn. (iv) in Eqn. (i),

$$\begin{aligned} I_1 &= \frac{1}{2}V_1 - \frac{1}{2}\left(\frac{1}{5}V_2 + \frac{7}{5}I_2\right) \\ I_1 &= \frac{1}{4}V_1 - \frac{7}{4}I_2 \quad \text{.....(v)} \end{aligned}$$

Comparing Eqn. (iii) and (iv) with transmission parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{17}{5} \\ \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$