## GURUNANAK INSTITUTE OF TECHNOLOGY

157/F, Nilgunj Road, Panihati
Kolkata -700114
Website: www.gnit.ac.in
Email: info.gnit@jisgroup.org
Approved by A.I.C.T.E., New Delhi
Affiliated to MAKAUT, West Bengal

# GNIT 

ONLINE COURSE WARE
PAPER NAME: UNIT OPERATION OF CHEMICAL ENGINEERING I

PAPER CODE: FT 403A
CREDIT: 3

NAME OF THE COORDINATOR:

NAME OF THE TEAM MEMBER:

Dr. Kakali Banyopadhyay (GNIT)

Prof. Uttam Raychaudhuri (GNIT)

## Module 1

## Lecture 1

Topics Covered: Basic concepts of Fluid Mechanics (Module I): Importance of this subject in application of Food Technology, Introduction of Dimensions \& Units, Dimensional Analysis, Calculation of Dimensional formulas

The engineering branches which describing the behavior of fluids and where fluid means liquids, gases and vapors is called fluid mechanics.

Fluid mechanics can be divided into two important branches to the study of unit operations:1. fluid statics; where fluid is in equilibrium state with no shear and 2 . fluid dynamics where portion of fluid are in motion relative to other parts.

## Nature of a Fluid

Fluid is a substance having the following characteristics :
a) fluids, if allowed, can flow.
b) It does not have a definite shape of its own but it conforms to the shape of the containing vessel.
c) It keeps on changing its shape so long external force acts on it.
d) In case of gases under normal temperature and pressure the effects of intermolecular attractions are very small because of large distance between molecules which is sufficiently strong in liquid state.
e) When density of fluid in little affected by the moderate changes in temperature and pressure, fluid is said to be incompressible and if the density is sensible towards the changes in variables, fluid is said to be compressible.
f) Ideal fluid offers no resistance to change in shape, it is frictionless and incompressible.

## Some Properties of Fluids

1) Mass Density (Specific mass $\rho$ ) : Mass density of fluid means mass per unit volume. In SI unit, it is expressed as $\mathrm{kg} / \mathrm{m}^{3}$. Specific mass of pure water at 4 C is $1000 \mathrm{~kg} / \mathrm{m}^{3}$ or $1 \mathrm{gm} / \mathrm{c} . \mathrm{c}$. Mass density varies directly with fluid pressure and inversely with fluid temperature.
2) Weight Density (Specific weight w): Weight density of fluid means the weight of unit volume. In SI unit, it is expressed as $\mathrm{N} / \mathrm{m}^{3}$. Specific weight of pure water at $4^{\circ} \mathrm{C}$ is $9810 \mathrm{~N} / \mathrm{m}^{3}$ or 1000 kgf/m3.

Relationship between mass density and weight density : $\mathrm{W}=\rho \mathrm{g}[\mathrm{g}=$ gravitational acceleration. Specific weight varies with the value of g , temperature, pressure of fluid etc.
3) Specific Gravity : Specific gravity of fluid means the ratio of mass or weight density of fluid to the mass or weight density of standard fluid.

Water at $4^{\circ} \mathrm{C}$ is considered as standard liquid, while air or $\mathrm{H}_{2}$ at NTP is considered as standard gas.
4) Specific Volume : Specific volume is the volume of unit weight of fluid. Specific volume is the reciprocal of specific weight.

## Viscosity

Definition : Viscosity is the property of fluid which offers resistance to motion of translation of one layer relative to other or in other word, viscosity means friction of fluids. In general it can be said that viscosity is thickness. Thus water is considered thin with lower viscosity and honey as thick with higher viscosity. The resistance is called viscous force which acts tangential to the surface along which the adjacent layers come in contact. The viscous force acting per unit area is called shear stress. All real fluids have some resistance to stress but a fluid which has no resistance to shear stress is called an ideal fluid or inviscid fluid. The study of viscosity is also known as rheology.

## Newton's Law of Viscosity

"The shear stress on a layer of a fluid is directly proportional to the rate of shear strain (or velocity gradient) in the direction perpendicular to the motion"

Velocity gradient means the change in fluid velocity per unit distance perpendicular to the motion.

If two layers are separated by distance " y ", have relative velocity of " V ", then the velocity gradient $: \tau \infty$ $\mathrm{V} / \mathrm{y}$; or, $\tau=\mu \mathrm{V} / \mathrm{y} \quad[\mu=$ coefficient of viscosity $]$

The viscous force between two adjacent layers having contact area "A" would be, $\mathrm{F}=\tau$. A

Or, $\quad \mathrm{F}=\mu . \mathrm{V} / \mathrm{y} . \mathrm{A}$
(Newton)
Or, $F=1 / g_{c} . \mu . V / y . A \quad$ (kg.f or gmf).

## Viscosity coefficient

When discussing about viscosity, the number that one most often used is the viscosity coefficient. Depending on the nature of applied stress and nature of fluid there are several viscosity coefficient:
a. Dynamics or absolute viscosity
b. Kinematic viscosity
c. Volume or bulk viscosity
d. Shear viscosity
e. Extensional viscosity.

## Absolute or Dynamic Viscosity ( $\mu$ )

The coefficient of dynamic viscosity can be expressed as:
$\mu=$ F $/$ A.V/y $=F / A / V / y=$ Shear stress $/$ Shear strain $=\tau / V / y$
S.I. unit of $\mu=\mathrm{N} /\left(\mathrm{m}^{2} \mathrm{~m} / \mathrm{s} .1 / \mathrm{m}\right)=\mathrm{N}$-sec. $/ \mathrm{m}^{2}=$ Pa.s $=\mathrm{Kg} / \mathrm{m} / \mathrm{s}$.
C.G.S unit of $\mu=$ dyne-sec $/ \mathrm{cm}^{2}$ or "poise" [1 poise $=1$ dyne-sec. $\left./ \mathrm{cm}^{2}=0.1 \mathrm{~N}-\mathrm{sec} / \mathrm{m}^{2}\right]$

Gravitational unit of $\mu=\tau \mathrm{g}_{\mathrm{c}} / \mathrm{V} / \mathrm{y}$ as kgf-sec/ $\mathrm{m}^{2}$
The CGS physical unit of dynamic viscosity is Poise (P), named after Jean Louis Marie Poiseullie. It is commonly used in ASTM standards as centipoise (cP). Water at $20^{\circ} \mathrm{C}$ has a viscosity of 1.0020 cP .

The relation between poise and pascal second is,
$10 \mathrm{P}=1 \mathrm{Kg} / \mathrm{m} / \mathrm{s}=1 \mathrm{~Pa} . \mathrm{s}$
$1 \mathrm{cP}=0.001 \mathrm{~Pa} . \mathrm{s}=1 \mathrm{mPa} . \mathrm{s}$.

## Kinematic Viscosity (v)

The ratio of absolute or dynamic viscosity to the mass density $\rho$ of fluid is called as its kinematic viscosity (v). The kinematic viscosity is sometime referred to as diffusivity of momentum, because it is comparable to and has the same unit $\left(\mathrm{m}^{2} / \mathrm{sec}\right)$ as diffusivity of heat and diffusivity of mass. It is therefore used in dimensionless numbers which compare the ratio of the diffusivities.

So, $v=\mu / \rho$.
S.I. unit of $v=\mathrm{N}-\mathrm{sec} / \mathrm{m}^{2} / \mathrm{kg} / \mathrm{m}^{3}=\mathrm{N}-\mathrm{m}-\mathrm{sec} / \mathrm{kg} .=\left(\mathrm{kg}-\mathrm{m} / \mathrm{sec}^{2}\right) / \mathrm{kg} . \mathrm{m} . \mathrm{sec}=\mathrm{m}^{2} / \mathrm{sec}$.
C.G.S. unit of $v=\mathrm{cm}^{2} / \mathrm{sec}$ or "stokes" [1 stoke $\left.=1 \mathrm{~cm}^{2} / \mathrm{sec}\right]$

Gravitational unit of $v=\mathrm{kgf}-\mathrm{sec} / \mathrm{m}^{2} / \mathrm{kg} / \mathrm{m}^{3}=\mathrm{m}-\mathrm{sec}$.

## Classification of Fluids:

Fluids are generally classified according to their viscosity. The different types of fluids are :

1. Ideal Fluid : these are the fluids having zero viscosity. Actually there is hardly any ideal fluid. Air, water etc have very low viscosity and thereby treated as ideal fluid.
2. Real Fluid : these are the fluids having viscosity.
3. Newtonian Fluid : these are the fluids which obey the Newton's law of viscosity. In other words it can be said that the coefficient of viscosity $(\mu)$ does not change with velocity gradient ( $\tau$ and $V / y$ ).
4. Non-Newtonian Fluid : these fluids does not obey Newton's law of viscosity. In case of these fluids, viscosity changes with velocity gradient. There are three types of non-Newtonian fluids which are:
a. Bingham fluids or Bingham plastics:There are fluids which have a linear shear stress/ shear strain relationship which require a finite yield stress before they begin to flow. That is shear stress, shear strain curve does not pass through the origin. Examples are clay suspensions, drilling mud, toothpaste, mayonnaise, chocolate and mustard.
b. Pseudoplastic fluids: They have a viscosity which decreases with an increasing velocity gradient. They are also known as shear rate thinning fluid. Examples are polymer solutions, muds and most slurries.
c. Dilatant fluids: They have a viscosity that increases with an increasing velocity gradient. They are also called shear rate thickening fluids. Examples are starch suspension in water, suspension paper pulp.


There are also fluids whose strain rate is a function of time. Fluids that require a gradually increasing shear stress to maintain a constant strain rate are known as Rheopectic. An opposite case of this, is a fluid that thins out with time and requires a decreasing stress to maintain a constant strain rate, called Thixotropic.

## Classification:

| Time- | Rheopectic | Apparent viscosity increases with duration of stress | Some lubricants |
| :---: | :---: | :---: | :---: |
| viscosity | Thixotropic | Apparent viscosity decreases with duration of stress | Some Clays, Some Drilling Mud, synovial fluid, Honey under certain conditions |
| Shear-stressdependent viscosity | Dilatant | Apparent viscosity increases with increased stress | Suspensions of corn starch or sand in water |
|  | Pseudoplastic | Apparent viscosity decreases with increased stress | Paper pulp in water, latex paint, blood plasma, syrup, molasses |
| Generalized Newtonian fluids |  | Stress depends on normal and shear strain rates and also the pressure applied on it | Blood, Custard |

## Lecture 2

Topics Covered: Definition of different terms related to fluid mechanics: Steady \& Unsteady state concept. Conversion of equations, proof of its dimensional homogeneity

## Fluid Pressure :

The pressure exerted by a fluid at all points of the container per unit area is called the pressure intensity. If " P " pressure acts on area " A ", then pressure intensity " p " is given by : $\mathrm{p}=\mathrm{P} / \mathrm{A}$

## Pressure Head :

Considering a liquid of specific weight " $w$ " placed inside the cylinder vessel having base area " $A$ ", the hydrostatic pressure is given by : $\mathrm{P}=$ weight of liquid / area of cylinder base

Or, $\mathrm{P}=$ Weight/Volume $\times$ volume of liquid $/ \mathrm{A}=\mathrm{w} . \mathrm{A} . \mathrm{h} / \mathrm{A}=\mathrm{h} . \rho . \mathrm{g}$
[ $\rho=$ specific mass liquid, $\mathrm{A}=$ area of cylinder base, $\mathrm{h}=$ column height of liquid, $\mathrm{w}=$ weight/volume].

Therefore, intensity of pressure at any point in a liquid increases with its depth.
Then, $\mathrm{h}=\mathrm{P} / \mathrm{w}=\mathrm{P} / \mathrm{\rho g}$.

## Units of Pressure :

|  | $\frac{\text { pascal }}{(\mathrm{Pa})}$ | $\frac{\mathrm{bar}}{(\mathrm{bar})}$ | technical atmosphere <br> (at) | $\frac{\text { atmosphere }}{(\mathrm{atm})}$ | $\stackrel{\text { torr }}{\text { (Torr) }}$ | $\frac{\text { pound-force }}{\text { per }}$ $\frac{\text { square inch }}{(\mathrm{psi})}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 Pa | $\equiv 1 \mathrm{~N} / \mathrm{m}^{2}$ | $10^{-5}$ | $1.0197 \times 10^{-5}$ | $9.8692 \times 10^{-6}$ | $7.5006 \times 10^{-3}$ | $145.04 \times 10^{-6}$ |
| 1 bar | 100,000 | $\equiv 10^{6} \underline{\mathrm{dyn} / \mathrm{cm}^{2}}$ | 1.0197 | 0.98692 | 750.06 | 14.5037744 |
| 1 at | 98,066.5 | 0.980665 | $\equiv 1 \mathrm{kgf} / \mathrm{cm}^{2}$ | 0.96784 | 735.56 | 14.223 |
| 1 atm | 101,325 | 1.01325 | 1.0332 | $\equiv 1 \mathrm{~atm}$ | 760 | 14.696 |
| 1 torr | 133.322 | $1.3332 \times 10^{-3}$ | $1.3595 \times 10^{-3}$ | $1.3158 \times 10^{-3}$ | $\begin{aligned} & \equiv 1 \mathrm{Torr} ; \\ & \approx 1 \mathrm{mmHg} \end{aligned}$ | $19.337 \times 10^{-3}$ |
| 1 psi | 6,894.76 | $68.948 \times 10^{-3}$ | $70.307 \times 10^{-3}$ | $68.046 \times 10^{-3}$ | 51.715 | $\equiv 1 \underline{\mathrm{lbf}} / \mathrm{in}^{2}$ |

Example reading: $1 \mathrm{~Pa}=1 \mathrm{~N} / \mathrm{m}^{2}=10^{-5} \mathrm{bar}=10.197 \times 10^{-6} \mathrm{at}=9.8692 \times 10^{-6} \mathrm{~atm}$, etc.

1. Engineering Units : $\quad \mathrm{kg} / \mathrm{m}^{2}$ or $\mathrm{kg} / \mathrm{cm}^{2}$
2. S.I. Units $\quad: \quad \mathrm{N} / \mathrm{m}^{2}$ or $\operatorname{Pascal}(\mathrm{Pa}) \quad: \quad 1 \mathrm{~Pa}=1 \mathrm{~N} / \mathrm{m}^{2}$
3. FPS Units $\quad: \quad \mathrm{lb} / \mathrm{in}^{2}(\mathrm{psi})$
4. Bar : $1.0917 \mathrm{~kg} / \mathrm{cm}^{2}$

5 Atmospheric $\quad: \quad 1 \mathrm{~atm}=1.0332 \mathrm{~kg} / \mathrm{cm}^{2}$
6. Head Units : Expressed in terms of head or height in $\mathrm{m} / \mathrm{cm}$
$1 \mathrm{psi}=0.0068046 \mathrm{~atm} .=0.0068948 \mathrm{bar}$
$1 \mathrm{bar} \approx 1 \mathrm{~atm} . \approx 1 \mathrm{~kg} / \mathrm{cm}^{2}$
Pressure Measuring Instruments:

| [A] Tube Gauges | [B] Mechanical Gauges |
| :--- | :--- |
| 1) Piezometer Tube | 1)Burdon's Tube Pressure Gauge |
| 2) "U"-Tube Manometer | 2)Diaphgram Pressure Gauge |
|  | 3)Dead Weight Pressure Gauge |

A. 1) Piezometer Tube: Piezometer tube is perhaps the simplest of the pressure measuring devices and consists of a vertical tube. In its application one end is connected to the pressure to be measured while the other end is open to the atmosphere as shown.

2) The "U"-Tube Manometer

The "U"-Tube can be used to measure the pressure of both liquids and gases. The "U" tube is connected as in the figure below and filled with a fluid called the manometric fluid. The fluid whose pressure is being measured should have a mass density less than that of the manometric fluid and the two fluids must be immiscible.

"U"-Tube manometer

Pressure in a continuous static fluid is the same at any horizontal level so,

$$
\text { pressure at } \begin{aligned}
\mathrm{B} & =\text { pressure at } \mathrm{C} \\
p_{B} & =p_{C}
\end{aligned}
$$

For the left hand side
pressure at $B=$ pressure at $A+$ pressure due to hei ght $h_{1}$ of fluid being meas ured

$$
\begin{equation*}
p_{B}=p_{A}+\rho g h_{1} \tag{1}
\end{equation*}
$$

For the right hand side

$$
\begin{align*}
& \text { pressure at } \mathrm{C}=\text { pressure at } \mathrm{D}+\text { pressure due to hei ght } \mathrm{h}_{2} \text { of manome tric fluid } \\
& \qquad p_{\mathrm{C}}=p_{\text {Amospheric }}+\rho_{\operatorname{man}} g h_{2} \tag{2}
\end{align*}
$$

As we are measuring gauge pressure we can subtract $p_{\text {atmospheric }}$ by subtracting eqn (2) from eqn (1) we have:

$$
\begin{gathered}
p_{B}=p_{C} \\
p_{A}=\rho_{\operatorname{man}} g h_{2}-\rho g h_{1}
\end{gathered}
$$

If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the manometric fluid. In this case the term $\rho \mathrm{gh}_{1}$ can be neglected, and the gauge pressure becomes

$$
p_{A}=p_{\operatorname{man}} g h_{2}
$$

B. 1) Burdon's Tube Pressure Gauge:

The Burdon's Tube Pressure Gauge is most suitable to measure the pressure of fluid, above or below the Atmosphere Pressure. In its simplest form, consists of an elliptical tube bent into an arc of a circle.

This bent-up tube is connected to the fluid, the pressurised fluid flows into the tube, as a result of the increased Pressure, tends to straighten itself.

The tube tends to become circular as it is encased in circular cover. With the help of simple Pinion and Sector arrangement, the elastic deformation of the Burdon's tube rotates the Pointer.This Pointer moves over a Calibrated Scale Which directly shows the Pressure.
2) Diaphragm Pressure Gauge:-

This Pressure Gauge is also used to found out the Pressure of a fluid, above or below atmosphere pressure. Instead of Burdon's tube a corrugated diaphragm is used, hence it is called Diaphragm Pressure Gauge.

When the Gauge is connected to the Fluid, causes some deformation of the diaphragm due to fluid pressure. With the help of some pinion arrangement, the elastic deformation of the diaphragm rotates the pointer. This Pointer moves over the calibrated scale which directly shows the pressure.

A Diaphragm Pressure Gauge is generally, used to measure relatively Low pressure.
3) Dead Weight Pressure Gauge:-

This type of Pressure Gauge is generally used for the Calibration of the other Pressure Gauge in a Laboratory. In its simplest form, consist of a Piston and a Cylinder of known area and connected to a Fluid by a Tube.

A pressure gauge, to be calibrated, is fitted on the other end of the tube. By changing the weight, on the piston, the pressure on the Fluid is calculated and marked on the pointer.

By taking adequate precaution, a small error due to frictional resistance to the motion of the piston, can be avoided.

## Classification of Fluid Flow :

Types of Fluid Flows -
1.Uniform and Non-uniform Flow: It is based on the fluid velocity. When the velocity of liquid particles is same in magnitude and direction at every section of the pipe the flow is called uniform otherwise it is non-uniform
2. Steady and Unsteady Flow - The classification is based on the property of the fluid. In steady flow, the fluid properties like volume, pressure, density etc. do not change with time.
3. Compressible and Incompressible Flow: This classification is based on the volume change. A flow, in which the volume or density of flowing fluid may change with time is called as compressible flow, where as fluids having a fixed volume or density during the flow is called incompressible.
4. Laminar or Turbulent Flow: The classification is based on the nature of flow.
(a) Laminar or Streamline flow: Here each liquid particle has certain fixed, definite path and paths of individual particles do not cross each other is said to be laminar flow. This is happened at relatively low velocity when fluid particles move smoothly, parallel to each other.
(b)Turbulent flow: Here liquid particles do not have any fixed path and different layers of fluid are mixed with each other. Almost all natural flows are turbulent in nature. At relatively high velocities, the
individual particles of the liquid, instead of flowing in an orderly manner, move in an erratic manner so that there is complete mixing.

## Reynold's Number :

The nature of flow not only depends on the velocity of the fluid but on the parameter Dvp/ $\mu$, where D , pipe diameter; v , average fluid velocity in the pipe; $\rho$ and $\mu$ are density and viscosity of the fluid. This dimensionless parameter is known as Reynold's number.
$N_{R e}=F_{i} / F_{v} \quad\left[F_{i}=\right.$ inertia force; $\quad F_{v}=$ viscous force $]$
$\mathrm{N}_{\mathrm{Re}}=\operatorname{Dv} \rho / \mu=\operatorname{Dv} / v \quad \nu=\mu / \rho=$ kinematic viscosity.
At lower flow velocities, $F_{v}$ is greater than $F_{i}$ (even for low velocity fluids) and it makes the flow laminar or viscous most of the time. But at higher flow velocities, $\mathrm{F}_{\mathrm{i}}$ is greater than $\mathrm{F}_{\mathrm{v}}$, and it makes the flow turbulent or less viscous.
i) When $\mathrm{N}_{\mathrm{Re}}<2100$, then the flow is laminar
ii) When $\mathrm{N}_{\mathrm{Re}}>4000$, then the flow is turbulent.
iii) When $2100<\mathrm{N}_{\mathrm{Re}}<4000$, the flow is in transition state.
$\mathrm{N}_{\mathrm{Re}}=2000$ is the lower critical value and $\mathrm{N}_{\mathrm{Re}}=2800$ is the upper critical value .

Reynold's Experiment : Reynold's experiment for determining the nature of fluid flow (laminar or turbulent) is described by fig: . The experimental apparatus consists of a tank A, filled with water of constant head and a small tank B containing dye solution (usually $\mathrm{KMnO}_{4}$ solution). A horizontal glass tube is fitted with tank A so that water can flow through this tube whose velocity is adjusted with the valve V . Dye is introduced into the tube at the bell mouth entrance. Now the valve is regulate so that there will be a steady flow of water through the tube. A jet of dye is now allowed to enter through the center of the glass tube in any of the three ways discussed in the following:
a) At low water velocities the dye is flowing in the form of a straight and stable filament This indicates that at low flow velocities there is no intermixing of water layers and dye particles or liquid flow in parallel layers. This flow is considered as laminar or streamline flow.
b) After that the water velocity is slowly increased and then the dye thread starts to be irregular The velocity at which laminar flow changes to turbulent flow is called critical velocity. The velocity at which, the dye thread starts to diffuse is called "lower critical velocity" and when the whole dye threads diffused,
it is called "upper critical velocity". Experimentally it has been found that threr will be no abrupt change fro laminar to turbulent region ruther it goes through a transition period.
c) When the water flow velocity is increased above the upper critical velocity, the fluctuation in the dye filament increases and that results in diffusion of dye in the entire cross section of the tube. This indicates complete mixing of liquid particles and is called "turbulent flow regime".

## Lecture 3

Topics Covered: Proof of their dimensional homogeneity, Basic equations of Fluid Flow (Laminar/Turbulent), Proof of Hagen -Poiseille's Equation, Fanning Friction Factor

## Hagen - Poiseulle's Equation :

In fluid dynamics, the Hagen-Poiseuille equation is a physical law that gives the pressure drop in fluid flowing through a long cylindrical pipe. The assumptions of the equation are that the flow is laminar or viscous and incompressible and the flow is through a constant circular cross-section that is significantly longer than its diameter.The viscous fluid flows with a pressure, which overcomes fluid friction or viscous force. Hagen - Poiseulle's law gives the concept of loss of fluid pressure in overcoming the fluid friction.

The law gives the pressure energy lost in overcoming fluid friction as : $\Delta \mathrm{P}=\mathrm{P}_{1}-\mathrm{P}_{2}=32 \mu \mathrm{vL} / \mathrm{D}^{2}$ (S.I. Units) $=32 \mu \mathrm{LL} / \mathrm{g}_{\mathrm{c}} \mathrm{D}^{2}$ (gravitational units) $\qquad$ ..(1), where $\Delta \mathrm{P}=$ the pressure drop, $\mathrm{L}=$ distance between two sections in the pipeline carrying fluids, $\mu=$ Coefficient of viscosity of fluid, $\mathrm{v}=$ average velocity of fluid, $\mathrm{D}=$ diameter of the pipe.

The above figure shows the pipe of uniform diameter carrying some liquid. The velocity at distance y from the centre line is given by parabolic law :
$\mathrm{V}=\mathrm{wH}_{\mathrm{L}} / 4 \mu \mathrm{~L}\left(\mathrm{r}^{2}-\mathrm{y}^{2}\right) \quad \ldots \ldots \ldots \ldots \ldots .(2)$, where $\mathrm{w}=$ specific weight of liquid, $\mathrm{H}_{\mathrm{L}}=$ head lost in friction, $\mu=$ coefficient of viscosity, $L=$ length of pipe, $r=$ radius of the pipe.

The maximum velocity is obtained at the centerline of the pipe which is far apart from the wall of the pipe, so the frictional force causes by the wall of the pipe is minimum here.

Therefore, by putting y $=0$ in (2) Vmax can be obtained.
$\mathrm{V}_{\text {max }}=\mathrm{wH}_{\mathrm{L}} \mathrm{r}^{2} / 4 \mu \mathrm{~L}$

Therefore, $\mathrm{V} / \mathrm{V}_{\max }=1-\mathrm{r}^{2} / \mathrm{y}^{2}$

The average velocity of laminar flow for parabolic variation is given by :
$\mathrm{V}_{\text {avg }}=\mathrm{V}_{\text {max }} / 2=\mathrm{wH}_{\mathrm{L}} \mathrm{r}^{2} / 8 \mu \mathrm{~L}$ $\qquad$
For turbulent flow
$\mathrm{V}_{\text {avg }}=0.83 \mathrm{~V}_{\text {max }}$.
This gives Hagen - Poiseuille's law as :
$\mathrm{H}_{\mathrm{L}}=\mathrm{P}_{1}-\mathrm{P}_{2} / \mathrm{w}=8 \mu \mathrm{VL} / \mathrm{wr}^{2}=32 \mu \mathrm{VL} / \mathrm{wD}^{2}[\mathrm{D}=$ diameter of the pipe $]$.

## FLUID FRICTION

Liquid, during flow in a pipe, experiences some resistance or more precisely the frictional resistance to flow, that results decrease in velocity and total energy. This frictional resistance to flow depends on 1) type of flow and 2) the roughness in inside of the pipe.

This frictional resistance can be explained for laminar and turbulent flow separately as follows:

## A] Friction losses in Laminar flow through circular pipe \& Flow Through Smooth Pipes (derivation of Hagen - Poiseuille's Equation):

Frictional force in laminar flow depends on the following factors:

1) It is directly proportional to the volume of the flow.
2) It is directly proportional to the contact surface area.
3) It is greatly affected by the variation of flowing fluid temperature.
4) It is independent of nature of contact surface.
5) It is independent of pressure.

Consider a Newtonian fluid in steady state flowing through a horizontal circular pipe of radius R and length L . The pressure drop across the pipe is $\Delta \mathrm{P}$. Let us consider a small concentric portion of the pipe with radius $r$ and length $\Delta \mathrm{L}$. For flow at steady state the pressure must be balanced by the viscous force acting at the boundary of the cylindrical element. So, the force caused by the pressure drop of $\Delta \mathrm{P}$ is

$$
\mathrm{F}_{\mathrm{p}}=(-\Delta \mathrm{P}) \pi \mathrm{r}^{2}
$$

The viscous force acting in the opposite direction of fluid flow is

$$
\mathrm{F}_{\mathrm{v}}=\tau(2 \pi \mathrm{r} \Delta \mathrm{~L})
$$

The condition steady state is: $\mathrm{F}_{\mathrm{p}}+\mathrm{F}_{\mathrm{v}}=0$
Therefore, $\tau=(-\Delta \mathrm{P} / 2 \Delta \mathrm{~L}) \cdot \mathrm{r}$

So, the shear stress at pipe wall is obtained by putting $\mathrm{r}=\mathrm{R}$ at the above equation,

$$
\tau_{\mathrm{w}}=(-\Delta \mathrm{P} / 2 \Delta \mathrm{~L}) \cdot \mathrm{R}
$$

So, the shear stress is zero at the center of the tube and it is maximum at the wall.

$$
\tau / \tau_{\mathrm{w}}=\mathrm{r} / \mathrm{R}
$$

Now, according to Newton's law of viscosity of a liquid flowing through a circular pipe of radius R , average velocity of $\mathrm{v}, \quad \quad \tau=-\mu \mathrm{dv} / \mathrm{dr}=(-\Delta \mathrm{P} / 2 \Delta \mathrm{~L}) . \mathrm{r}$

$$
\begin{aligned}
& \quad \text { or } \quad \mathrm{v}=1 / 4 \mu(-\Delta \mathrm{P} / \Delta \mathrm{L})\left(\mathrm{R}^{2}-\mathrm{r}^{2}\right) \\
& =(-\Delta \mathrm{P} / \Delta \mathrm{L}) \mathrm{R}^{2} / 4 \mu\left[1-(\mathrm{r} / \mathrm{R})^{2}\right]
\end{aligned}
$$

The above equation describes the variation of the fluid velocity v with radius r of the pipe. The velocity of the fluid is maximum at the centerline of the pipe.

$$
\mathrm{v}_{\max }=-(\Delta \mathrm{P} / \Delta \mathrm{L}) \mathrm{R}^{2} / 4 \mu
$$

Therefore, $\mathrm{v} / \mathrm{v}_{\text {max }}=1-(\mathrm{r} / \mathrm{R})^{2}$
The average velocity at any given cross section is:

$$
\begin{aligned}
& =2 / \mathrm{R}^{2} \cdot(-\Delta \mathrm{P} / \Delta \mathrm{L}) \cdot \mathrm{R}^{2} / 4 \mu \cdot \operatorname{Int}_{0}{ }^{\mathrm{R}}\left[1-(\mathrm{r} / \mathrm{R})^{2}\right] \mathrm{r} \mathrm{dr} \\
& =(-\Delta \mathrm{P} / \Delta \mathrm{L}) \cdot 1 / 2 \mu \cdot\left[1 / 4 \mathrm{R}^{2}\right] \\
& =\mathrm{v}_{\text {max }} / 2
\end{aligned}
$$

Volumetric flow rate, Q is expressed as:

$$
\mathrm{Q}=\pi \mathrm{R}^{2} \mathrm{~V}_{\text {avg }}=(-\Delta \mathrm{P} / \Delta \mathrm{L}) \cdot \pi \mathrm{R}^{4} / 8 \mu
$$

$$
\text { or } \quad-\Delta \mathrm{P} / \Delta \mathrm{L}=8 \mu \mathrm{~V}_{\text {avg }} / \mathrm{R}^{2}=32 \mu \mathrm{~V}_{\text {avg }} / \mathrm{D}^{2} \quad[\mathrm{D}=\text { diameter of the pipe }]
$$

This is Hagen - Poiseuille's equation
The total drag force acting on the fluid, resulting from the shear force transmitted from to the wetted surface area of the wall can be expressed as:

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{d}}= \\
& =\left(2 \pi \mathrm{wetted} \text { surface area of the wall } \times \text { Shear stress } \cdot \tau_{\mathrm{w}}\right. \\
& \quad=(2 \pi \mathrm{R} \Delta \mathrm{~L}) \cdot(-\Delta \mathrm{P} / 2 \Delta \mathrm{~L}) \cdot \mathrm{R} \\
& \left.=\pi \mathrm{F}_{\mathrm{d}}=\text { total drag force }\right] \\
& =8 \mathrm{P}) \\
& =8 \pi \mu(\Delta \mathrm{~L}) \cdot \mathrm{V}_{\text {avg }}
\end{aligned}
$$

This shows that for a specific fluid drag force is directly proportional to the average fluid velocity.

The Fanning friction factor (f) is defined as:

$$
\begin{aligned}
\mathrm{f}=\mathrm{F}_{\mathrm{d}} & (\text { wetted surface area). (kinetic energy per unit volume) } \\
& =\left(8 \pi \mu \cdot \Delta \mathrm{~L} . \mathrm{V}_{\text {avg }}\right) /(\pi \mathrm{D} \Delta \mathrm{~L}) \cdot\left(\rho \mathrm{V}_{\text {avg }}^{2} / 2\right) \\
& =16 \mu / \mathrm{DV}_{\text {avg }} \rho \\
& =16 \mathrm{~N}_{\mathrm{Re}}
\end{aligned}
$$

The Fanning friction factor, f describes the skin friction which is a tangential friction associated with a fluid flowing through a pipe. In case of fluid flowing through spherical or other bodies, a part of total friction is contributed by skin friction and the other part is by form friction. The later is associated with the pressure difference between the fluid in front and that in behind the object.
$\mathrm{f}=16 \mathrm{~N}_{\mathrm{Re}}$ can also be proved by following way:
Loss of head due to viscosity is given by Hagen - Poiseuille's law as: $\mathrm{H}_{\mathrm{L}}=32 \mu \mathrm{VL} / \mathrm{wD}_{2}$ .......(1)

Loss of head due to friction or skin friction loss is given by Darcy - Weisbach's equation as : $\quad H_{L}=$ $4 \mathrm{fLV}_{2} / 2 \mathrm{gD}$, putting $\mathrm{V}=\mathrm{Q} / \pi / 4 \mathrm{D}_{2}$. So $\mathrm{f}=$ Darcy's coefficient or friction coefficient or friction factor $=0.005(1+1 / 12 \mathrm{D})$ [for smooth pipes];
$=0.01(1+1 / 12 \mathrm{D})[$ rough pipes $]$

In laminar flow the resistance is mainly due to viscosity.
Therefore, $4 \mathrm{fLV} 2 / 2 \mathrm{gD}=32 \mu \mathrm{VL} / \mathrm{wD}_{2}$
So, $\mathrm{f}=16 \mu \mathrm{~g} / \mathrm{wDV}=16 \mu / \mathrm{VD} \times \mathrm{g} / \mathrm{w}=16 \mu / \mathrm{VD} \times 1 / \rho=16 / \mathrm{DV} \rho / \mu=16 / \mathrm{N}_{\mathrm{Re}}$
f is also called fanning friction factor.

## B] Frictional losses in Turbulent Flow \& Flow Through Rough Pipes :

Frictional force in turbulent flow depends on the conditions of flow like fluid velocity, contact surface area as well as the physical properties of the system like density of flowing fluid, fluid temperature etc.

The frictional pressure drop in a pipe can be calculated from Fanning equation:

$$
\begin{aligned}
-\Delta \mathrm{P}_{\mathrm{f}} & =\left(2 \mathrm{f} \rho \mathrm{v}^{2} \mathrm{~L}\right) / \mathrm{D} \ldots \ldots \ldots \ldots \ldots \ldots, \mathrm{~Pa} \\
-\Delta \mathrm{P}_{\mathrm{f}} / \rho & =2 \mathrm{fv}^{2}(\mathrm{~L} / \mathrm{D}) \ldots \ldots \ldots \ldots \ldots \ldots, \mathrm{J} / \mathrm{Kg} \\
\mathrm{~h}_{\mathrm{f}} & =-\Delta \mathrm{P}_{\mathrm{f}} / \rho \mathrm{fg}=2 \mathrm{f}^{2} / \mathrm{g} \cdot(\mathrm{~L} / \mathrm{D}) \ldots \ldots \ldots, \mathrm{m} \quad\left[\mathrm{~h}_{\mathrm{f}}=\text { frictional loss }\right]
\end{aligned}
$$

The friction factor f , depends on the Reynold's number of the flow and the relative roughness of the pipe $(k / D)$ as expressed in following figure

Where $\mathrm{k} / \mathrm{D}=$ relative roughness $=$ Average height of pipe wall roughness/pipe diameter.
Therefore, $\quad \mathrm{f}=\left(\mathrm{N}_{\mathrm{Re}}, \mathrm{k} / \mathrm{D}\right)$
Friction factor for turbulent flow in smooth pipe is given by $\mathrm{f}=0.079 / \mathrm{N}_{\mathrm{Re}} 0.25$
and $\quad 1 / \sqrt{ } \mathrm{f}=4 \log \left(\mathrm{~N}_{\mathrm{Re}} \sqrt{ } \mathrm{f}\right)-0.4 \quad$ [Nikuradse equation]

Friction factor for turbulent flow in rough pipes depends on both the $\mathrm{N}_{\mathrm{Re}}$ and $\mathrm{k} / \mathrm{D}$. The empirical formulae for friction factor in this case is given by:

1) $1 / \sqrt{ } \mathrm{f}=-4 \log \left[(\mathrm{k} / 3.7 \mathrm{D})+\left(6.81 / \mathrm{N}_{\mathrm{Re}}\right)^{0.9}\right]$
2) $1 / \sqrt{ } \mathrm{f}=3.48-4 \log \left[(2 \mathrm{k} / \mathrm{D})+\left(9.35 / \mathrm{N}_{\mathrm{Re}} \sqrt{ } \mathrm{f}\right)\right]$

## Friction Factor Chart :

It is a $\log -\log$ plot of friction factor " f " against $\mathrm{N}_{\mathrm{Re}}$ over a wide range of $\mathrm{N}_{\mathrm{Re}}$ for a flow, both in smooth as well as rough pipes. The chart is also called as Stanton diagram.
i) For laminar flow region $\left(N_{R e}<2100\right): f=64 / N_{R e}$

$$
\text { Or, } \log \mathrm{f}=\log 64-\log \mathrm{N}_{\mathrm{Re}}
$$

Therefore, $\log -\log$ plot of f vs. $\mathrm{N}_{\mathrm{Re}}$ will be a straight line with slope -1
ii) For turbulent flow region $\left(N_{R e}>4100\right)$, $f$ is also a function of $k / D$ of pipe.

## C] Other Frictional losses :

## (i) Frictional loss due to sudden expansion of the pipe diameter:

The frictional loss due to sudden expansion is given by:

$$
\mathrm{h}_{\mathrm{fe}}=\left[1-\mathrm{A}_{1} / \mathrm{A}_{2}\right]^{2} \cdot \mathrm{v}_{1}{ }^{2} / 2 \mathrm{~g}
$$

where $A_{1}=$ initial pipe area, $A_{2}=$ enlarged pipe area, $\mathrm{v}_{1}=$ initial flow velocity, $\mathrm{v}_{2}=$ subsequent decrease in flow velocity due to enlargement.

## (ii) Frictional loss due to sudden contraction of the pipe diameter:

The eqation can be expressed as:

$$
\mathrm{h}_{\mathrm{fc}}=0.4\left[1-\mathrm{A}_{2} / \mathrm{A}_{1}\right] \cdot \mathrm{V}_{2}^{2} / 2 \mathrm{~g}
$$

where $A_{1}=$ initial pipe area, $A_{2}=$ decreased pipe area, $\mathrm{v}_{1}=$ initial flow velocity, $\mathrm{v}_{2}=$ subsequent increase in flow velocity due to enlargement.

## Lecture 4

Topics Covered: Introduction of Bernoulli's Equation, Derivation and Limitations

## Bernoulli's Equation (mechanical energy balance for stedy flow):

In fluid dynamics, Bernoulli's principle states that for an incompressible fluid, an increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential
energy. Bernoulli's principle is named after the Dutch-Swiss mathematician Daniel Bernoulli. Bernoulli's. In fact, there are different forms of the Bernoulli equation for different types of flow. The simple form of Bernoulli's principle is valid for incompressible flows (e.g. most liquid flows) and also for compressible flows (e.g. gases) moving at low Mach numbers ( $\mathrm{Ma}=$ flow velocity/sonic velocity).

Bernoulli's principle can be derived from the principle of conservation of energy. It is basically energy balance of flowing liquid, which states that : "for a perfect incompressible liquid flowing in a continuous stream, the total energy of liquid particles remain the same, while the particle moves from one point to another".

Fluid particles are subject only to pressure and their own weight. If a fluid is flowing horizontally and along a section of a streamline, where the speed increases it can only be because the fluid on that section has moved from a region of higher pressure to a region of lower pressure; and if its speed decreases, it can only be because it has moved from a region of lower pressure to a region of higher pressure.
Consequently, within a fluid flowing horizontally, the highest speed occurs where the pressure is lowest, and the lowest speed occurs where the pressure is highest.

The following assumptions must be taken for the equation to apply:

- The fluid must be incompressible - even though pressure varies, the density must remain constant.
- The streamline must not enter a boundary layer. (Bernoulli's equation is not applicable where there are viscous forces, such as in a boundary layer.)

If we consider a perfectly incompressible liquid flowing through a non-uniform pipe shown in two sections $A-A$ and $B-B$ of the pipe is to be considered. Let $z_{1}, p_{1}, A_{1}, V_{1}$ and $z_{2}, p_{2}, A_{2}, V_{2}$ be the elevation above datum line, static pressure, flow area and flow volume at the section $\mathrm{A}-\mathrm{A}$ and $\mathrm{B}-\mathrm{B}$ respectively.

Now, the liquid between the sections $\mathrm{A}-\mathrm{A}$ and $\mathrm{B}-\mathrm{B}$ gets displaced between $\mathrm{A}^{\prime}-\mathrm{A}^{\prime}$ and $\mathrm{B}^{\prime}-\mathrm{B}^{\prime}$ through very small length is equivalent to the movement of the liquid between $A-A$ and $A^{\prime}-A^{\prime}$ to $B-$ $B$ and $B^{\prime}-B^{\prime}$, the remaining liquid between $A^{\prime}-A^{\prime}$ and $B-B$ being unaffected.

Let $w$ be the specific weigh of liquid.
Therefore, weight of liquid between $\mathrm{A}-\mathrm{A}$ and $\mathrm{A}^{\prime}-\mathrm{A}^{\prime}$ would be :
$\mathrm{W}=$ weight $/$ volume $\times$ volume of liquid
$=$ Weight $/$ Volume $\times$ flow area $\times$ displacement
$=\mathrm{w} \times \mathrm{A}_{1} \times \mathrm{dl}_{1}($ at section $\mathrm{A}-\mathrm{A})$
$=\mathrm{w} \times \mathrm{A}_{2} \times \mathrm{dl}_{2}$ (at section $\mathrm{B}-\mathrm{B}$ )
Therefore, $\mathrm{W} / \mathrm{w}=\mathrm{A}_{1} \mathrm{dl}_{1}=\mathrm{A}_{2} \mathrm{dl}_{2}$
Work done by the pressure at $\mathrm{A}-\mathrm{A}$ in moving liquid to $\mathrm{A}^{\prime}-\mathrm{A}^{\prime}$ is
$\mathrm{W}_{1}=$ force $\times$ distance traveled between $\mathrm{A}-\mathrm{A}$ and $\mathrm{A}^{\prime}-\mathrm{A}^{\prime}$.
$\mathrm{W}_{1}=$ pressure $\times$ area $\times$ distance traveled

$$
=\mathrm{p}_{1} \times \mathrm{A}_{1} \times \mathrm{dl}_{1}
$$

Similarly, $\mathrm{W}_{2}=-\mathrm{p}_{2} \mathrm{~A}_{2} \mathrm{dl}_{2}$ [because, $\mathrm{p}_{2}$ is opposite in direction of $\mathrm{p}_{1}$ ]
Therefore, total work done $=\mathrm{W}_{1}+\mathrm{W}_{2}=\mathrm{p}_{1} \mathrm{~A}_{1} \mathrm{dl}_{1}-\mathrm{p}_{2} \mathrm{~A}_{2} \mathrm{dl}_{2}$

$$
\begin{aligned}
& =\mathrm{p}_{1} \mathrm{~A}_{1} \mathrm{dl}_{1}-\mathrm{p}_{2} \mathrm{~A}_{1} \mathrm{dl}_{1}\left[\text { because } \mathrm{A}_{1} \mathrm{dl}_{1}=\mathrm{A}_{2} \mathrm{dl}_{2}\right] \\
& =\mathrm{A}_{1} \mathrm{dl}_{1}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) \\
& =\mathrm{W} / \mathrm{w}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) \quad\left[\text { since, } \mathrm{A}_{1} \mathrm{dl}_{1}=\mathrm{W} / \mathrm{w}\right]
\end{aligned}
$$

Loss of potential energy ( PE ) $=\mathrm{W}\left(\mathrm{z}_{1}-\mathrm{z}_{2}\right)$
Gain in kinetic energy $(\mathrm{KE})=\mathrm{W}\left(\mathrm{v}^{2} / 2 \mathrm{~g}-\mathrm{v}^{2}{ }_{1} / 2 \mathrm{~g}\right)$

$$
=\mathrm{W} / 2 \mathrm{~g}\left(\mathrm{v}^{2}{ }_{2}-\mathrm{v}^{2}{ }_{1}\right)
$$

Now,
Loss of PE + work done by pressure = gain KE
Therefore, $\mathrm{W}\left(\mathrm{z}_{1}-\mathrm{z}_{2}\right)+\mathrm{W} / \mathrm{w}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right)=\mathrm{W} / 2 \mathrm{~g}\left(\mathrm{v}^{2}{ }_{2}-\mathrm{v}^{2}{ }_{1}\right)$
Or, $\left(\mathrm{z}_{1}-\mathrm{z}_{2}\right)+1 / \mathrm{w}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right)=1 / 2 \mathrm{~g}\left(\mathrm{v}^{2}{ }_{2}-\mathrm{v}^{2}{ }_{1}\right)$
Or, $\mathrm{Z}_{1}+\mathrm{p}_{1} / \mathrm{w}+\mathrm{v}^{2}{ }_{1} / 2 \mathrm{~g}=\mathrm{z}_{2}+\mathrm{p}_{2} / \mathrm{w}+\mathrm{v}^{2}{ }_{2} / 2 \mathrm{~g}$
Or, $\mathrm{z}+\mathrm{p} / \mathrm{w}+\mathrm{v}^{2} / 2 \mathrm{~g}=$ constant.

Or, potential head + pressure head + velocity head $=$ constant.

The simplified form of Bernoulli's equation can be summarized in the following memorable word equation:
static pressure + dynamic pressure $=$ total pressure

Every point in a steadily flowing fluid, regardless of the fluid speed at that point, has its own unique static pressure $p$ and dynamic pressure $q$. Their sum $p+q$ is defined to be the total pressure $p_{0}$. The significance of Bernoulli's principle can now be summarized as total pressure is constant along a streamline.

Lecture 5
Topics Covered: Recap of Bernoulli's Equations problems, Explanation of Fluid Friction for laminar \& Turbulent Flow

## Interpretation of Bernoulli's Equation :

Bernoulli's Equation can be interpreted with the help of three energy heads:

## (i) Potential Energy (PE) head:

It is the energy of fluid particle due to its position or height Z with respect to certain datum line. PE of the fluid particle of mass m at height Z above the datum line is given by -
$\mathrm{PE}=\mathrm{m} . \mathrm{g} \cdot \mathrm{z} ; \mathrm{PE} /$ Unit mass $=\mathrm{g} . \mathrm{z} ; \mathrm{PE} /$ unit weight $=\mathrm{m} . \mathrm{g} \cdot \mathrm{z} / \mathrm{m} \cdot \mathrm{g}=\mathrm{z}=$ potential head.

## (ii) Kinetic Energy head:

It is the energy of fluid particle due to its motion. The KE of fluid particle of mass m moving with velocity V is given by -
$K E=1 / 2 \mathrm{~m} \cdot \mathrm{v}^{2} ; \quad \mathrm{KE} /$ unit mass $=\mathrm{v}^{2} / 2 ; \quad \mathrm{KE} /$ Unit weight $=\mathrm{v}^{2} / 2 \mathrm{~g}$.

## (iii) Pressure Energy [Flow Energy (FE)] head :

It is the energy of fluid particle due to pressure acting on it. Work done in moving a fluid particle of mass density $\rho$ at static pressure p appears as the flow energy is given by -
$\mathrm{FE}=$ Work done $=$ force $\times$ displacement
$=$ force $/$ area $\times$ area $\times$ displacement
$=$ pressure $\times$ volume.
FE $/$ Unit mass $=$ pressure $\times$ volume $/$ mass $=$ pressure $/($ mass $/$ volume $)=$ Pressure $/$ mass density $=$ P/ $\rho$

FE $/$ Unit Weight $=$ pressure head $=($ pressure $\times$ volume $) /$ weight $=$ Pressure $/($ weight/volume $)$
$=\mathrm{P} / \mathrm{w}$

Therefore considering PE, KE, FE, the total energy between the sections A -A and A' -A'
The total energy / mass :
$\mathrm{E} /$ mass $=\mathrm{gz}_{1}+\mathrm{v}_{1} / 2+\mathrm{p}_{1} / \rho$
The total energy / weight :
$\mathrm{E} /$ weight $=\mathrm{z}_{1}+\mathrm{v}^{2}{ }_{1} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}$
As the unit of energy / unit weight is: $\mathrm{N}-\mathrm{m} / \mathrm{N}=\mathrm{m}$
Therefore the terms also represent the respective energy heads.
Total head $=\mathrm{z}_{1}+\mathrm{v}^{2} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}=$ potential head + volume head + pressure head.

## Limitations of Bernoulli's Equation :

i) Bernoulli's equation is derived, after assuming that the velocity of every liquid particle at every point of a pipe is uniform. But in actual cases the velocity of liquid particle is zero at the pipe wall and maximum at centre of the pipe. Hence KE in Bernoulli's equation must be calculated by using average velocity. Therefore KE head $=\alpha v_{2} / 2 \mathrm{~g}, \alpha=$ correction factor. For fluid flow through circular pipes. When $\alpha=2$, the flow is laminar and when $\alpha=1$, the flow is turbulent.
ii) In Bernoulli's equation, it has been assumed that there is no loss of fluid energy. But in actual cases certain energy is lost in overcoming fluid friction in laminar flow and in case of turbulent flow, some KE is converted to heat. Thus fluid energy decreases in the direction of flow.

Therefore, energy head at (1) - (1) = energy head at $(2)-(2)+h_{f}$
Or, $\mathrm{z}_{1}+\mathrm{v}^{2} / 2 \mathrm{~g}+\mathrm{P}_{1} / \mathrm{w}=\left(\mathrm{z}_{2}+{ }^{\mathrm{v} 2} / 2 \mathrm{~g}+\mathrm{P}_{2} / \mathrm{w}\right)+\mathrm{h}_{\mathrm{f}}$
Where $\mathrm{h}_{\mathrm{f}}=$ energy head lost during flow
iii) While deriving Bernoulli's equation, it is assumed that no external force other than gravity force is acting on the liquid. But, in actual practice, pump is used to increase the mechanical energy of fluid so as to maintain the flow.

Let $\mathrm{w}_{\mathrm{p}}=$ in the work done by the pump / unit mass.
Therefore actual work delivered by the pump to the liquid is $\eta \omega_{p} \quad$ [ $\eta=$ pump efficiency]
$\eta<1$ if the pump is used to lift liquid from section (1) - (1) [lower elevation] to (2) - (2) [higher elevation]

Therefore, modified Bernoulli's equation can be written as

$$
\mathrm{gz}_{1}+\mathrm{v}^{2}{ }_{1} / 2 \mathrm{~g}+\mathrm{p}_{1} / \rho+\eta \mathrm{w}_{\mathrm{p}}=\mathrm{gz}_{2}+\mathrm{v}^{2} / 2 \mathrm{~g}+\mathrm{p}_{2} / \rho+\mathrm{h}_{\mathrm{f}} .
$$

## Lecture 6

Topics Covered: Flow through Packed Bed, Concept of Fluidization, Discussion of Fluidization \& its application in Food Technology, Explanation of Fluid Bed drier in experimental Set-up

## Equivalent Particle Diameter : Sphericity of particle

The equivalent particle diameter of a non-spherical particle is defined as the diameter of a sphere having the same volume as the particle.

Therefore, sphericity $\Phi_{\mathrm{s}}=$ Surface area of this sphere/actual surface area of the particle.
As for sphere, $S_{P}=\pi D_{P}{ }^{2} ; \quad \mathrm{v}_{\mathrm{P}}=1 / 6 \pi \mathrm{D}_{\mathrm{P}}{ }^{3}$
$S_{P} / V_{P}=6 / \Phi_{S} D_{P}$ $\qquad$
Ergun has correlated experimental data and showed that, $\mathrm{K}_{1}=150 / 36$ and $\mathrm{K}_{2}=1.75 / 6$ and $\mathrm{S}_{\mathrm{P}} / \mathrm{V}_{\mathrm{P}}=$ $6 / \Phi_{S} D_{P}$

Therefore, equation (6) becomes :
$\left[\Delta \mathrm{P} . \mathrm{g}_{\mathrm{c}} / \mathrm{L}\right] .\left[\Phi_{\mathrm{S}} \mathrm{D}_{\mathrm{P}} / \rho \cdot \mathrm{V}_{0}{ }^{2}\right] \cdot\left[\varepsilon^{3} /(1-\varepsilon)\right]=150(1-\varepsilon) / \Phi_{\mathrm{S}} \mathrm{D}_{\mathrm{P}} \mathrm{V}_{0} .(\rho / \mu)+1.75$.
This equation (8) is called Ergun Equation.

A friction factor for a packed bed is defined by the LHS of equation (8)
Therefore, $\mathrm{f}_{\mathrm{P}}=\left[\Delta \mathrm{P} \cdot \mathrm{g}_{\mathrm{c}} / \mathrm{L}\right] \cdot\left[\Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}} / \rho \cdot \mathrm{V}_{0}{ }^{2}\right] \cdot\left[\varepsilon^{3} /(1-\varepsilon)\right]$

$$
\begin{equation*}
\text { Or, } \mathrm{f}_{\mathrm{P}}=150(1-\varepsilon) / \Phi_{\mathrm{S}} \mathrm{~N}_{\mathrm{Re}-\mathrm{P}}+1.75 \tag{9}
\end{equation*}
$$

At low $\mathrm{N}_{\mathrm{Re}}$, the quantity 1.75 is negligible in comparison to the $1^{\text {st }}$ term.
Therefore, $\Delta \mathrm{P} . \mathrm{g}_{\mathrm{c}} / \mathrm{L} . \Phi_{\mathrm{S}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2} \varepsilon^{3} / \mathrm{V}_{0} \mu(1-\varepsilon)^{2}=150$
This equation (10) is called Kozeny-Carman equation. It is valid for laminar flow region, where $\mathrm{N}_{\mathrm{Re}}<1.0$
For $\mathrm{N}_{\mathrm{Re}}>10^{3}$, the $1^{\text {st }}$ term of equation (9) becomes negligible compared to the $2^{\text {nd }}$ term.
Therefore, $\Delta \mathrm{P} . \mathrm{g}_{\mathrm{c}} / \rho \mathrm{L} \mathrm{V}_{0}{ }^{2} . \Phi_{\mathrm{s} \cdot} \mathrm{D}_{\mathrm{P}} \varepsilon^{3} /(1-\varepsilon)=1.75$
Equation (11) is called Blake-Plummer equation.
For $\mathrm{N}_{\mathrm{Re}}$ between 1 to $10^{3}$, equation (8) is used.

## Fluidization :

When a fluid is passed upwards through the bed of solids, the pressure drop will be the same as for downward flow at low rates, the particles do not move, but when the frictional drag on the particles becomes equal to the apparent weight (actual weight - buoyancy) of the particles. In this way as the velocity of the fluid increased, the individual particles separate them from each other and become freely supported in the fluid and the bed is called "fluidized". The pressure drop is given by "Ergun's equation". If the fluid velocity is steadily increased, the pressure drop and the drag on the individual particles increase and eventually the particles start to move and become suspended in the fluid. The term fluidization and fluidized bed are used to describe the condition of fully suspended particles since the suspension behaves like a dense fluid. If the bed is tilted, the top surface remains horizontal and large objects will either float or sink in the bed depending on their density relative to the suspension. By this unit operation the fine granular solids are transformed into a fluid-like state through contact with a fluid.

## Lecture 7

Topics Covered: Calculation of minimum fluidization velocity, Conditions of fluidization

## Condition of Fluidization :

Condition of fluidization can be understood by taking one specific example of an upward flow of fluid through a vertical bed of fine particles. like a vertical tube partly filled with fine granular material such as catalytic cracking catalyst as shown in the following figure.

The tube is opened at the top and has a porous plate at the bottom to support the bed of catalyst. Air as a fluid is admitted below the distribution plate at a lower flow rate and passed upward through bed without
causing any particle motion. If the particles are quite small, flow in the channels between the particles will be laminar and the pressure drop across the bed will be proportional to the superficial velocity $\mathrm{V}_{0}$. As the velocity increases gradually, the $\Delta \mathrm{P}$ increases, but the particles do not move and the bed height remains the same. At a certain velocity, the drop of pressure $(\Delta \mathrm{P})$ across the bed counterbalances the force of gravity on the particles to move. This is point A but when the velocity increases further, the particles become separated up to a possible extent to move about in the bed and true fluidization begins (pt. B). Once the bed is fluidized, the $\Delta \mathrm{P}$ across the bed stays constant, but the bed height increases with increasing flow.

If the flow rate to the fluidized bed is gradually reduced, the $\Delta \mathrm{P}$ remains constant and the bed height decreases (CB). However, the final bed height may be greater than the initial value for the fixed bed. The $\Delta \mathrm{P}$ counterbalances weight of the bed at point B and this point rather than point A , should be considered to give the minimum fluidization velocity $\left(\mathrm{V}_{\mathrm{OM}}\right)$.

## Lecture 8

Topics Covered: Derivation of Ergun equation, Kozeny-Carmann equation, Blake-Plummer Equation.

## Calculation of Minimum Fluidization Velocity :

The equation for the minimum fluidization velocity can be obtained by setting the pressure drop across the bed equal to the weight per unit cross-sectional area, allowing for the buoyant force of the displaced fluid.

Therefore,

$$
\begin{equation*}
\Delta P=g / g_{c}(1-\varepsilon)\left(\rho_{P}-\rho\right) L \tag{1}
\end{equation*}
$$

Where, $\Delta \mathrm{P}=$ pressure drop across the bed, $\varepsilon=$ porosity of the fluidized bed, $\rho_{\mathrm{P}}=$ density of the particle, $\rho=$ density of fluid, L=length of bed.

At starting point of fluidization $\varepsilon$ is the minimum porosity $\varepsilon_{\mathrm{m}}$

Therefore,

$$
\begin{equation*}
\Delta \mathrm{P} / \mathrm{L}=\mathrm{g} / \mathrm{g}_{\mathrm{c}}\left(1-\varepsilon_{\mathrm{m}}\right)\left(\rho_{\mathrm{P}}-\rho\right) \tag{2}
\end{equation*}
$$

The pressure drop in flow through packed bed is given by Ergun's equation:
$(\Delta \mathrm{P} / \mathrm{L}) \cdot \mathrm{g}_{\mathrm{c}} \cdot\left(\Phi_{\mathrm{s}} \cdot \mathrm{D}_{\mathrm{P}}\right) /\left(\rho \mathrm{V}^{2}{ }_{0}\right) \cdot \varepsilon^{3} /(1-\varepsilon)=\mathrm{f}=150(1-\varepsilon) /\left[\left(\Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}} \mathrm{V}_{0}\right)(\rho / \mu)\right]+1.75 \ldots \ldots$.
Rearranging the above equation:
$(\Delta \mathrm{P} / \mathrm{L}) \cdot \mathrm{g}_{\mathrm{c}}=\left[150(1-\varepsilon)^{2} \mu \mathrm{~V}_{0}\right] /\left(\Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2} \varepsilon^{3}\right)+1.75 \mathrm{~V}_{0}{ }^{2} \rho /\left(\Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}}\right) .(1-\varepsilon) / \varepsilon^{3}$.
Applying the above equation for incipient fluidization for minimum fluidization velocity $\mathrm{V}_{\mathrm{mf}}$ and
$\varepsilon=\varepsilon_{\mathrm{m}}:$
$\Delta \mathrm{P} / \mathrm{L} \cdot \mathrm{g}_{\mathrm{c}}=150\left(1-\varepsilon_{\mathrm{m}}\right)^{2} \mu \mathrm{~V}_{\mathrm{mf}} / \Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2} \varepsilon_{\mathrm{m}}{ }^{3}+1.75 \mathrm{~V}_{0}{ }^{2} \rho / \Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}} \cdot\left(1-\varepsilon_{\mathrm{m}}\right) / \varepsilon_{\mathrm{m}}^{3}$

Comparing equation (2) and (5) :
$\left(150 \mu \mathrm{~V}_{\mathrm{mf}}\right) /\left(\Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2}\right) \times .\left(1-\varepsilon_{\mathrm{m}}\right) / \varepsilon^{3}{ }_{\mathrm{m}}+\left(1.75 \mathrm{~V}_{\mathrm{mf}}{ }^{2} \rho\right) / \Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}} .1 / \varepsilon_{\mathrm{m}}^{3}=\mathrm{g}\left(\rho_{\mathrm{P}}-\rho\right)$

For the very small particles, only the laminar flow term of Ergun equation is significant. $\left(1.75 \mathrm{~V}_{\mathrm{mf}}{ }^{2}\right) \rho /$ $\Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}} .1 / \varepsilon_{\mathrm{m}}^{3}$ term can be neglected.

Therefore, with $\mathrm{N}_{\mathrm{Re}}<1$, the equation for minimum fluidization velocity is:

$$
150\left(1-\varepsilon_{\mathrm{m}}\right) \mu \mathrm{V}_{\mathrm{mf}} / \Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2} \varepsilon_{\mathrm{m}}{ }^{3}=\mathrm{g}\left(\rho_{\mathrm{P}}-\rho\right)
$$

Therefore, $\mathrm{V}_{\mathrm{mf}}=\mathrm{g}\left(\rho_{\mathrm{P}}-\rho\right) / 150 \mu . \Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2} \cdot \varepsilon^{3}{ }_{\mathrm{m}} /\left(1-\varepsilon_{\mathrm{m}}\right)$

This is called Kozeny-Carmon equation.

This equation is valid up to particle size of $300 \mu \mathrm{~m}$.

Therefore, particles larger than 1 mm , laminar flow term becomes negligible $\left(\mathrm{N}_{\mathrm{Re}}\right.$ above 1000$)$

Therefore, the term $\left(150 \mu \mathrm{~V}_{\mathrm{mf}}\right) /\left(\Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2}\right) \times .\left(1-\varepsilon_{\mathrm{m}}\right) / \varepsilon_{\mathrm{m}}^{3}$ can be neglected and equation (6) becomes:

$$
1.75 \mathrm{~V}_{\mathrm{mf}}^{2} \rho / \Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}} .1 / \varepsilon_{\mathrm{m}}^{3}=\mathrm{g}\left(\rho_{\mathrm{P}}-\rho\right)
$$

So, $\mathrm{V}_{\mathrm{mf}}=\left[g\left(\rho_{\mathrm{P}}-\rho\right) \Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}} \varepsilon_{\mathrm{m}}^{3} / 1.75 \rho\right]^{1 / 2}$

This is called Blake Plummer equation.

The terminal velocity for individual particles falling in still air shows that, for low Reynold's number, terminal velocity $\left(\mathrm{U}_{\mathrm{t}}\right)$ is obtained by Stoke's law:

$$
\mathrm{U}_{\mathrm{t}}=\mathrm{gD}_{\mathrm{P}}^{2}\left(\rho_{\mathrm{P}}-\rho\right) / 18 \mu
$$

and $\mathrm{V}_{\mathrm{mf}}$ is obtained from equation (7), $\mathrm{V}_{\mathrm{mf}}=\mathrm{g}\left(\rho_{\mathrm{P}}-\rho\right) / 150 \mu . \Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2} \varepsilon_{\mathrm{m}}{ }^{3} /\left(1-\varepsilon_{\mathrm{m}}\right)$

$$
\begin{equation*}
\mathrm{U}_{\mathrm{t}} / \mathrm{V}_{\mathrm{mf}}=8.33\left(1-\varepsilon_{\mathrm{m}}\right) / \Phi_{\mathrm{s}}{ }^{2} \cdot \varepsilon_{\mathrm{m}}^{3} \tag{9}
\end{equation*}
$$

Therefore, $\mathrm{U}_{\mathrm{t}} / \mathrm{V}_{\mathrm{mf}}$ depends mainly on void fraction at minimum fluidization.
For large particles, the terminal velocity is given by Newton's law :

$$
\mathrm{U}_{\mathrm{t}}=1.75 \sqrt{ }\left[\mathrm{~g} \mathrm{D} \mathrm{D}_{\mathrm{P}}\left(\rho_{\mathrm{P}}-\rho\right) / \rho\right]
$$

For spheres, with $\left.N_{R e}>10^{3}, \quad U_{t} / V_{m f}=1.75\left[g D_{P}\left(\rho_{P}-\rho\right) / \rho\right]^{1 / 2} \cdot[1.75 \rho) / g D_{P}\left(\rho_{P}-\rho\right) \varepsilon_{m}^{3}\right]^{1 / 2}$

$$
\begin{equation*}
\mathrm{U}_{\mathrm{t}} / \mathrm{V}_{\mathrm{mf}}=2.32 \varepsilon_{\mathrm{m}}^{3 / 2} \tag{10}
\end{equation*}
$$

$\qquad$

For spheres, with $, \varepsilon_{\mathrm{m}}=0.45, \quad, \quad \mathrm{U}_{\mathrm{t}} / \mathrm{V}_{\mathrm{mf}}=50.22$ (for low $\mathrm{N}_{\mathrm{Re}}$ )

And,
$\mathrm{V}_{\mathrm{mf}}=7.68\left(\right.$ for $\left.\mathrm{N}_{\mathrm{Re}}>10^{3}\right)$

## Types of Fluidization :

There are two types of fluidization :

1) Particulate Fluidization: For this type of fluidization the expansion of bed is uniform and Ergun's equation applied to fixed bed is expected to hold approximately for slightly expanded bed. So, for laminar flow of particles between the bed, neglecting the term $\left(1.75 \mathrm{~V}_{\mathrm{mf}}{ }^{2} \rho\right) / \Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}} .1 / \varepsilon_{\mathrm{m}}^{3}$.

$$
\begin{align*}
& \quad 150\left(1-\varepsilon_{\mathrm{m}}\right) \mu \mathrm{V}_{\mathrm{mf}} / \Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2} \varepsilon_{\mathrm{m}}{ }^{3}=\mathrm{g}\left(\rho_{\mathrm{P}}-\rho\right) \\
& \text { Putting } \varepsilon_{\mathrm{m}}=\varepsilon \text { and } \mathrm{V}_{\mathrm{mf}}=\mathrm{V}_{0}, \quad \varepsilon^{3} /(1-\varepsilon)=150 \mu \mathrm{~V}_{0} /\left[\Phi_{\mathrm{s}}{ }^{2} \mathrm{D}_{\mathrm{P}}{ }^{2} \mathrm{~g}\left(\rho_{\mathrm{P}}-\rho\right)\right] \tag{12}
\end{align*}
$$

The expanded bed height can be obtained by using the following equation:

$$
\begin{equation*}
\mathrm{L}=\mathrm{L}_{\mathrm{m}}\left(1-\varepsilon_{\mathrm{m}}\right) /(1-\varepsilon) \tag{13}
\end{equation*}
$$

Particulate fluidization can be explained by taking the example of fluidizing sand, the particles move further apart with water and their motion become more vigorous as the velocity is increased, but the average bed density at a given velocity is the same in all sections of the bed. This is called Particulate fluidization and is characterized by a large but uniform expansion of the bed at high velocities.
2) Bubbling Fluidization : for this type of fluidization, the expansion of bed comes mainly from the space occupied by gas bubbles. Beds of solids, fluidized with air usually exhibit aggregative or bubbling fluidization. At superficial velocity, much greater than $\mathrm{V}_{\mathrm{mf}}$ most of the gas passes through the bed as bubbles or voids which are almost free of solids, and only small fraction of gas flows in the channels
between the particles. The particles move erratically and are supported by the fluid, but in space between bubbles, the void fraction is same as at incipient fluidization.

## Module II

## Lecture 9

Topics Covered: Introduction of Module II: Flow Measurements and Machineries, Introduction of Different flow measuring devices

Flow Meters:The flow rate of fluids in a process equipment is one important variable which can be measured by inferential or direct methods. Inferential methods determine the rate of flow by measuring related variables such as the differential pressure or the pressure head. The above methods are used to determine the instantaneous value of flow rate of fluid.

In case of variable head flow meters, such as the orifice meter, venturi meter, pitot tube, the fluid velocity is either accelerated or retarded by changing the flow area at measuring section and the resulting change in flow area is measured.

For variable area meter such as rotameter, there is a constant pressure drop. The flow is indicated by the area of the annular opening between the float and the tapered tbe through which the fluid is flowing.

## Flow Measuring Instruments:

1) Orifice Meter : The cross-sectional area of flow passage is suddenly reduced at the flow restriction with the help of orifice plate in Orifice meter. That orifice plate develops differential pressure across the restriction and this differential pressure varies with the rate of flow or discharge of fluid through the restriction. Thus, any changes in fluid flow rate through the restriction could be measured in terms of change in differential pressure across the restriction or constriction. Thus for this reason, orifice meter is also called variable head flow meter.
2. Venturimeter: This venturimeter is also another type of variable head flow meter. Here, venturitube gradually reduces the flow area and a differential pressure area is created across it. The below mentioned figure shows a venturitube having a specially shaped length of pipe resembling two funnels joined at their ends. It has a convergent inlet cone (angled between $19^{\circ}$ and $23^{\circ}$ ), after that a uniform diameter throat (having length equal to its diameter) and a divergent outlet cone(angled between $5^{\circ}$ and $15^{\circ}$ ). The high pressure tap is located on the convergent section and low pressure tap is located at the middle of the
throat. A manometer is connected in between the taps. The venturitube is made up of cast iron or steel, but usually the throat is made up of bronze.
3. Rotameter (Variable Area Flowmeter): A Rotameter is a device that measures the flow rate of liquid or gas in a closed tube. It belongs to a class of meters called variable area meters, which measure flow rate by allowing the cross-sectional area the fluid travels through to vary, causing some measurable effect.

## Lecture 10

Topics Covered: Flow through pipes \& open channels, Flow measurement through Orifice Meter, Venacontracta

Orifice Meter : The cross-sectional area of flow passage is suddenly reduced at the flow restriction with the help of orifice plate in Orifice meter. That orifice plate develops differential pressure across the restriction and this differential pressure varies with the rate of flow or discharge of fluid through the restriction. Thus, any changes in fluid flow rate through the restriction could be measured in terms of change in differential pressure across the restriction or constriction. Thus for this reason, orifice meter is also called variable head flow meter.


The above figure shows orifice meter, consisting of orifice plate inserted into a pipeline by means of flanges. The orifice plate is a circular plate with a hole which restricts the fluid flow through the pipeline. Orifice plates are made up of steel, stainless steel, monel, phosphor bronze or any metal which can resist corrosive action of fluid. Orifice plate is installed concentrically with the pipe. Usually, the orifice plate has sharp square edge but it may be changed in such a way that it can be used for viscous liquids. The upstream pressure tap is kept sufficiently away from the orifice while the downstream pressure-tap is located at 'vena-contracta' where flow area becomes minimum. The manometer measures the differential pressure across the orifice-plate which can be calibrated in terms of fluid discharge through the orifice.

The orifice meter consists of a flat orifice plate with a circular hole drilled in it. There is a pressure tap upstream from the orifice plate and another just downstream. There are three recognized methods of placing the taps. And the coefficient of the meter will depend upon the position of taps.

| Type of tap | Distance of upstream tap from face of orifice | Distance of downstream tap from downstream face |
| :---: | :---: | :---: |
| Flange | 1 inch | 1 inch |
| Vena contracta | 1 pipe diameter (actual inside) | 0.3 to 0.8 pipe diameter, depending on b |
| Pipe | 2.5 times nominal pipe diameter | 8 times nominal pipe diameter |

## Flow equation of Orifice meter:



Consider section (1) - (1) at the upstream tap and section (2) - (2) at the vena contracta. Let $\mathrm{A}_{1}$, $\mathrm{V}_{1}, \mathrm{p}_{1}$ and $\mathrm{A}_{2}, \mathrm{~V}_{2}, \mathrm{p}_{2}$ be the flow area, flow velocity and static pressure at the respective sections. So, according to continuity equation:
$\mathrm{Q}=\mathrm{A}_{1} \mathrm{~V}_{1}=\mathrm{A}_{2} \mathrm{~V}_{2} \quad[\mathrm{As} \mathrm{Q}=$ discharge rate or volumetric flow rate of fluid $]$

As, $\mathrm{A}_{2}<\mathrm{A}_{1}$ and $\mathrm{V}_{2}>\mathrm{V}_{1}$

So, at the restriction, the flow area decreases and the flow velocity increases.
Therefore, applying Bernoulli's equation between section (1) - (1) and (2) - (2):
$\mathrm{Z}_{1}+\mathrm{V}^{2} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}=\mathrm{Z}_{2}+\mathrm{V}^{2} / 2 \mathrm{~g}+\mathrm{p}_{2} / \mathrm{w}$
As $Z_{1}=Z_{2}, \quad V^{2} / 2 g+p_{1} / w=V^{2} / 2 g+p_{2} / w \ldots \ldots(1)[\mathrm{w}=$ specific weight of fluid $]$
As, $\mathrm{V}_{2}>\mathrm{V}_{1}, \quad \mathrm{p}_{2}<\mathrm{p}_{1}$

So at the restriction, velocity head increases at the cost of decrease in pressure head.
From equation (1) $1 / 2 \mathrm{~g}\left(\mathrm{~V}^{2}{ }_{2}-\mathrm{V}^{2}{ }_{1}\right)=\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}$
$\mathrm{V}^{2}{ }_{2}-\mathrm{V}^{2}{ }_{1}=2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}$
Again from continuity equation, $\mathrm{V}_{1}=\left(\mathrm{A}_{2} / \mathrm{A}_{1}\right) \mathrm{V}_{2}$ $\qquad$
Putting the value of $\mathrm{V}_{1}$ in equation (2), we have: $\mathrm{V}^{2}{ }_{2}-\left(\mathrm{A}_{2} / \mathrm{A}_{1}\right) 2 \mathrm{~V}^{2}{ }_{2}=2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}$

$$
\begin{array}{r}
\operatorname{Or}, \mathrm{V}_{2}^{2}\left[1-\left(\mathrm{A}_{2} / \mathrm{A}_{1}\right)^{2}\right]=2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w} \\
\operatorname{Or}, \mathrm{~V}_{2}=\sqrt{ }\left[2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right)\right] /\left[\mathrm{w}\left(1-\mathrm{A}_{2}{ }_{2} / \mathrm{A}^{2}{ }_{1}\right)\right] \ldots \ldots . .(4) \tag{4}
\end{array}
$$

Applying continuity equation between orifice plated and vena contracta:
$A_{0} V_{0}=A_{2} V_{2}$, Or, $V_{0}=A_{2} / A_{0} . V_{2}, \quad$ Or, $V_{0}=C_{c} . V_{2}$ $\qquad$
$\mathrm{V}_{0}=$ fluid velocity at orifice, $\mathrm{A}_{0}=$ orifice area,
$C_{c}=$ coefficient of contraction $=$ area of Vena contracta/orifice area $=A_{2} / A_{0}$.

Therefore, $\mathrm{A}_{2}=\mathrm{C}_{\mathrm{c}} . \mathrm{A}_{0}$

Using these values of $\mathrm{V}_{0}$ and $\mathrm{A}_{2}$ in equation (4) and (5), it can be obtained that -

$$
\begin{equation*}
\mathrm{Q}=\mathrm{A}_{0} \mathrm{~V}_{0}=\mathrm{A}_{0} \mathrm{C}_{\mathrm{c}} \mathrm{~V}_{2}=\mathrm{C}_{\mathrm{c}} \mathrm{~A}_{0} \sqrt{ }\left[2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}\left(1-\mathrm{C}_{\mathrm{c}} \mathrm{~A}^{2} / \mathrm{A}^{2}{ }_{1}\right)\right] \tag{6}
\end{equation*}
$$

From the equation (6) the theoretical discharge $\mathrm{Q}_{\mathrm{th}}$ is obtained but not the actual discharge because it has been derived without considering any frictional loss.

Therefore, $\mathrm{Q}_{\mathrm{th}}=\left(\mathrm{A}_{2} / \mathrm{A}_{0}\right) \cdot \mathrm{A}_{0} \sqrt{ }\left[2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) \times \mathrm{A}^{2}{ }_{1}\right] /\left[\mathrm{w}\left(\mathrm{A}^{2}{ }_{1}-\mathrm{A}^{2}{ }_{2}\right)\right]$

$$
\begin{aligned}
= & A_{1} \cdot A_{2} \sqrt{ } 2 g\left(p_{1}-p_{2}\right) /\left[\rho \cdot g \cdot\left(A_{1}^{2}-A_{2}^{2}\right)\right] \\
& =A_{1} \cdot A_{2} \sqrt{ } 2 g \cdot h /\left(A_{1}^{2}-A_{2}^{2}\right)
\end{aligned}
$$

The actual discharge is obtained by using coefficient of discharge $\mathrm{C}_{\mathrm{d}}$ :

$$
\mathrm{Q}_{\mathrm{a}}=\mathrm{C}_{\mathrm{d}} \cdot \mathrm{Q}_{\mathrm{th}}=\mathrm{C}_{\mathrm{d}} \cdot \mathrm{C}_{\mathrm{c}} \mathrm{~A}_{0} \sqrt{ }\left[2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}\left(1-\mathrm{C}_{\mathrm{c}} \mathrm{~A}^{2} / \mathrm{A}^{2}{ }_{1}\right)\right]
$$

Calibration of Orifice Meter: It means the establishment of correlation between fluid flow rate $Q$ and corresponding manometer reading $H$. For problem solving: $Q=C A_{1} A_{2} \sqrt{ } 2 g h /\left(A_{1}^{2}-A_{2}^{2}\right), C$ $=$ orifice constant.

Actual discharge is given by - $\mathrm{Q}_{\mathrm{a}}=\mathrm{C}_{\mathrm{d}} \cdot \mathrm{Q}_{\mathrm{th}}=\mathrm{C}_{\mathrm{d}} \cdot \mathrm{C}_{\mathrm{c}} \cdot \mathrm{A}_{0} \sqrt{ } 2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}\left(1-\mathrm{C}^{2}{ }_{\mathrm{c}} \mathrm{A}^{2}{ }_{2} / \mathrm{A}_{1}{ }_{1}\right)$

$$
\begin{align*}
& =C_{d} \cdot C_{c} / \sqrt{ }\left(1-C_{c}^{2} A_{2}^{2} / A_{1}^{2}\right) \cdot A_{0} \cdot \sqrt{ } 2 g\left(p_{1}-p_{2}\right) / \mathrm{w} \\
& =C_{d} \cdot C \cdot A_{0} \sqrt{ } 2 g h \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \cdot(7) \tag{7}
\end{align*}
$$

[Here, $\mathrm{C}=\mathrm{C}_{\mathrm{c}} / \sqrt{ }\left(1-\mathrm{C}_{\mathrm{c}}{ }^{2} \mathrm{~A}^{2}{ }_{0} / \mathrm{A}^{2}{ }_{1}\right)=$ constant of given orifice
$=C_{c} / \sqrt{ } 1-C^{2}{ }_{c} \beta^{4}, \quad$ where $\beta=$ diameter ratio $=$ orifice diameter $/$ pipe diameter $=d / D$
Therefore, $\mathrm{A}_{0}{ }^{2} / \mathrm{A}^{2}{ }_{1}=\left(\pi / 4 . \mathrm{d}^{2}\right)^{2} /\left(\pi / 4 . D^{2}\right)^{2}=\mathrm{d}^{4} / \mathrm{D}^{4}=\beta^{4}$

Therefore the differential pressure head $h=p_{1}-p_{2} / w=\left(w_{m}-w\right) H / w \quad$ in SI unit

$$
\begin{aligned}
& =\left(\rho_{\mathrm{m}}-\rho\right) \mathrm{g} / \rho \mathrm{g} \cdot \mathrm{H} \\
& =\left(\rho_{\mathrm{m}} / \rho-1\right) \cdot \mathrm{H}
\end{aligned}
$$

$\mathrm{w}_{\mathrm{m}}=$ Specific weight of manometric liquid, $\rho_{\mathrm{m}}=$ specific mass of manometric liquid,
$\mathrm{H}=$ manometer reading.

Applications:

Orifice plates are most commonly used for continuous measurement of fluid in pipes. They are also used in some small river systems to measure flow at locations where the river passes through a culvert or drain. Only a small number of rivers are appropriate for the use of the technology since the plate must remain completely immersed i.e the approach pipe must be full, and the river must be substantially free of debris.

In the natural environment large orifice plates are used to control onward flow in flood relief dams. in these structures a low dam is placed across a river and in normal operation the water flows through the orifice plate unimpeded as the orifice is substantially larger than the normal flow cross section. However, in floods, the flow rate rises and floods out the orifice plate which can then only pass a flow determined by the physical dimensions of the orifice. Flow is then held back behind the low dam in a temporary reservoir which is slowly discharged through the orifice when the flood subsides.

## Lecture 11

Topics Covered: Coefficient of discharge of orifice meter, Introduction of Venturimeter, Derivation of working formula, Cd

Venturimeter: This venturimeter is also another type of variable head flow meter. Here, venturitube gradually reduces the flow area and a differential pressure area is created across it. The below mentioned figure shows a venturitube having a specially shaped length of pipe resembling two funnels joined at their ends. It has a convergent inlet cone (angled between $19^{\circ}$ and $23^{\circ}$ ), after that a uniform diameter throat (having length equal to its diameter) and a divergent outlet cone(angled between $5^{\circ}$ and $15^{\circ}$ ). The high pressure tap is located on the convergent section and low pressure tap is located at the middle of the throat. A manometer is connected in between the taps. The venturitube is made up of cast iron or steel, but usually the throat is made up of bronze.


As fluid enters the convergent cone, it smoothly converges with corresponding increase in flow velocity and decrease in static pressure. In throat section there is no change in flow area, which provides the point of minimum pressure measurement. In divergent section, flow smoothly diverges to pipe diameter with increase in pressure head at the cost of decrease in velocity head. This is called as pressure recovery which is higher for venturimeter than for orifice meter. This is because an orifice meter sudden contraction of flow area at the orifice plate results in loss of energy in eddies formation. The differential pressure across the venturitube is measured by manometer which is correlated with the fluid flow rate.

Flow Equation for venturimeter : Consider section (1) - (1) at the upstream tap and section (2) - (2) at the tap on throat. Let $\mathrm{A}_{1}, \mathrm{~V}_{1}, \mathrm{p}_{1}$ and $\mathrm{A}_{2}, \mathrm{~V}_{2}, \mathrm{p}_{2}$ be the flow area, flow velocity and fluid pressure at the respective sections.

Applying Bernoulli's equation between these two sections, we get :
$\mathrm{Z}_{1}+\mathrm{V}_{1}{ }^{2} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}=\mathrm{Z}_{2}+\mathrm{V}_{2}^{2} / 2 \mathrm{~g}+\mathrm{p}_{2} / \mathrm{w}$
As $\mathrm{Z}_{1}=\mathrm{Z}_{2}, \quad \mathrm{~V}^{2} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}=\mathrm{V}^{2} / 2 \mathrm{~g}+\mathrm{p}_{2} / \mathrm{w}$
Or, $\mathrm{V}^{2}{ }_{2}-\mathrm{V}^{2}{ }_{1}=2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}$ $\qquad$

Applying continuity equation between sections (1) - (1) and (2) - (2) we get :
$\mathrm{A}_{1} \mathrm{~V}_{1}=\mathrm{A}_{2} \mathrm{~V}_{2}$
$\mathrm{V}_{1}=\left(\mathrm{A}_{2} / \mathrm{A}_{1}\right) \mathrm{V}_{2}$
Putting this value of $\mathrm{V}_{1}$ in equation (1) :
$\mathrm{V}_{2}{ }^{2}-\left(\mathrm{A}_{2} / \mathrm{A}_{1}\right)^{2} \cdot \mathrm{~V}_{2}{ }^{2}=2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}$
$\left(\mathrm{A}_{2} / \mathrm{A}_{1}\right)^{2}=$ throat area $/$ pipe area $=\pi / 4 \mathrm{~d}^{2} / \pi / 4 \mathrm{D}^{2}=(\mathrm{d} / \mathrm{D})^{4}=\beta^{4}$
$\beta=$ throat diameter/pipe diameter $=\mathrm{d} / \mathrm{D}$
$\mathrm{V}_{2}^{2}\left(1-\beta^{4}\right)=2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}$
Therefore, $\mathrm{V}_{2}=\sqrt{ } 2 \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}\left(1-\beta^{4}\right)$ $\qquad$

Using this value of $\mathrm{V}_{2}$ in continuity equation, the fluid flow rate can be expressed as :
$\mathrm{Q}=\mathrm{A}_{2} \mathrm{~V}_{2}=\mathrm{A}_{2} \sqrt{ } \mathrm{~g}\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}\left(1-\beta^{4}\right)$

Equation (4) gives theoretical discharge rate because fluid friction is not considered here.
Therefore, $C_{d}=$ coefficient of discharge $=Q_{a} / Q_{t h} \quad\left[Q_{a}=\right.$ actual discharge rate $]$
Therefore, $\mathrm{Q}_{\mathrm{a}}=\mathrm{C}_{\mathrm{d}} \cdot \mathrm{Q}_{\mathrm{th}}$

$$
\begin{align*}
\mathrm{Q}_{\mathrm{a}} & =C_{d} \cdot A_{2} \sqrt{ } 2 g\left(p_{1}-p_{2}\right) / w\left(1-\beta^{4}\right) \\
& =C_{d} \cdot A_{2} \sqrt{ } 2 g \cdot \sqrt{ }\left(p_{1}-p_{2}\right) / w \cdot 1 / \sqrt{ }\left(1-\beta^{4}\right) \\
& =C_{d} \cdot A_{2} / \sqrt{ }\left(1-\beta^{4}\right) \cdot \sqrt{ } 2 g h \\
& =C_{d} \cdot C \cdot A_{2} \cdot \sqrt{ } 2 g h \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \ldots \ldots \ldots \tag{4}
\end{align*}
$$

Here, $\mathrm{C}=$ venture constant $=1 / \sqrt{ }\left(1-\beta^{4}\right)$;
$\mathrm{h}=$ differential pressure $=\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right) / \mathrm{w}=\left(\rho_{\mathrm{m}}-\rho\right) / \rho . \mathrm{H}=\left(\mathrm{w}_{\mathrm{m}}-\mathrm{w}\right) \cdot \mathrm{H} / \mathrm{w}$
Here, $\rho_{\mathrm{m}}=$ mass density of manometric liquid; $\mathrm{w}_{\mathrm{m}}=$ weight density of manometric liquid, $H$ $=$ manometer reading.

Calibration for venturimeter: $\mathrm{Q}_{\mathrm{th}}=\mathrm{CA}_{1} \mathrm{~A}_{2} \sqrt{ } 2 \mathrm{gh} / \sqrt{ }\left(\mathrm{A}^{2}{ }_{1}-\mathrm{A}^{2}{ }_{2}\right)[\mathrm{C}=$ constant for venturimeter]

From equation (4) it is evident that fluid discharge rate is directly proportional to the differential pressure head, so it is called a variable pressure head. The comparative values of $\mathrm{C}_{\mathrm{d}}$, coefficient of discharge for the two variable head meters are:

| Head Meters | Values of $\mathrm{C}_{\mathrm{d}}$ |
| :--- | :--- |
| Orifice meter | $0.7-1.0$ |
| Venturi meter | $0.3-0.7$ |

## Applications:

A venturi can be used to measure the volumetric flow rate $Q$.
A venturi can also be used to mix a liquid with a gas. If a pump forces the liquid through a tube connected to a system consisting of a venturi to increase the water speed (the diameter decreases), a short piece of tube with a small hole in it, and last a venturi that decreases speed (so the pipe gets wider again), the gas will be sucked in through the small hole because of changes in pressure. At the end of the system, a mixture of liquid and gas will appear. See aspirator and pressure head for a discussion of this type of siphon.

## Lecture 12

Topics Covered: Venacontracta, Introduction of Variable area meter, Derivation of flow equation for Rotameter
$\underline{\text { Rotameter (Variable Area Flowmeter): A Rotameter is a device that measures the flow rate of liquid }}$ or gas in a closed tube. It belongs to a class of meters called variable area meters, which measure flow rate by allowing the cross-sectional area the fluid travels through to vary, causing some measurable effect.

In case of variable head flowmeters (like orifice and venturimeter) the restriction size remains constant, due to which the differential pressure head across it varies with change in fluid flow rate. Where as in rotameter, a variable area meter the area available for the flow of liquid at the restriction is allowed to vary with the fluid flow rate so that the differential pressure across it remains constant. Thus any change in fluid flow rate can be measured in terms of change in flow area, hence the name is given as variable-area meter.


The above figure shows the construction of rotameter, consists of a tapered glass tube mounted vertically with bigger end on the upper side. The glass tubes are generally used for measuring the flow rates of low temperature and pressure fluids, but for fluids at high temperature and pressure metal tubes are used. Float is installed in the tube after the meter is mounted in the flow line. Floats are usually made of corrosion resistant metals like aluminium, bronze, monel, nickel, stainless steel etc. Float material decides the flow range of the rotameter. The meter must be installed vertically.

## Flow Equation:

If we consider (1) - (1) and (2) - (2) as shown in figure, at the float position, flow area suddenly decreases from $A_{1}$ to $A_{2}$ with subsequent decrease in pressure from $p_{1}$ to $p_{2}$. This gives rise to the differential pressure $\left(p_{1}-p_{2}\right)$ across the float. At given flow rate, float rises in the tube until its weight is balanced by the upward forces acting on the float.

Let, $\quad D_{f}=$ float diameter at the rim
$D_{t}=$ tube diameter at the float material
$w_{f}=$ weight density of float material
w = weight density of fluid
$\mathrm{V}_{\mathrm{f}}=$ volume of float
The force balance equation of float for dynamic equilibrium can be written as :
Net upward force $=$ net downward force.
Therefore, buoyant force + differential pressure force or drag force $=$ weight of float

$$
w V_{f}+\left(p_{1}-p_{2}\right) X \pi D_{f}^{2} / 4 .=w_{f} \cdot V_{f}
$$

Or, $\left(p_{1}-p_{2}\right) \cdot \pi D_{f}^{2} / 4 .=V_{f}\left(\mathrm{w}_{\mathrm{f}}-\mathrm{w}\right)$
Or, $\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right)=4 \mathrm{~V}_{\mathrm{f}}\left(\mathrm{w}_{\mathrm{f}}-\mathrm{w}\right) / \pi \mathrm{D}_{\mathrm{f}}{ }^{2}$
Now fluid is flowing through the annular section between float rim and metering tube. So the flow can be considered similar to the flow throurh orifice of diameter $\left(D_{t}-D_{f}\right)$ having the differential pressure $\left(\mathrm{p}_{1}-\mathrm{p}_{2}\right)$ across it.

Therefore the flow equation of orificemeter can be used to express the flow rate of fluid through rotameter as :

$$
\begin{align*}
Q & =C_{d} \cdot C \cdot A_{0} \cdot \sqrt{ } 2 g h . \\
& =C_{d} \cdot C \cdot \pi\left(D_{t}^{2}-D_{f}^{2}\right) / 4 \sqrt{ } 2 g\left(p_{1}-p_{2}\right) / w \tag{2}
\end{align*}
$$

Using the value of $\left(p_{1}-p_{2}\right)$ from the equation (1), we get,

$$
\begin{aligned}
\mathrm{Q} & =\mathrm{C}_{\mathrm{d}} \cdot \mathrm{C} \cdot \pi\left(\mathrm{D}_{\mathrm{t}}{ }^{2}-\mathrm{D}_{\mathrm{f}}{ }^{2}\right) / 4 \cdot \sqrt{ }\left(2 \mathrm{~g} / \mathrm{w} \cdot\left\{4 \mathrm{~V}_{\mathrm{f}}\left(\mathrm{w}_{\mathrm{f}}-\mathrm{w}\right) / \pi \mathrm{D}_{\mathrm{f}}{ }^{2}\right\}\right. \\
\mathrm{Q} & =\mathrm{C}_{\mathrm{d}} \cdot C \cdot\left(\mathrm{D}_{\mathrm{t}}{ }^{2}-\mathrm{D}_{\mathrm{f}}{ }^{2}\right) / \mathrm{D}_{\mathrm{f}} \sqrt{ } \pi \mathrm{~g} / 2 \sqrt{ } \mathrm{~V}_{\mathrm{f}}\left(\mathrm{w}_{\mathrm{f}}-\mathrm{w}\right) / \mathrm{w} \\
& =\mathrm{K} \cdot \sqrt{ } \mathrm{~V}_{\mathrm{f}} \cdot\left(\mathrm{w}_{\mathrm{f}}-\mathrm{w}\right) / \mathrm{w} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots(3)
\end{aligned}
$$

Here $K=C_{d} \cdot C \cdot \sqrt{ } \mathrm{~g} / 2 \cdot\left(\mathrm{D}_{\mathrm{t}}{ }^{2}-\mathrm{D}_{\mathrm{f}}{ }^{2}\right) / \mathrm{D}_{\mathrm{f}}=$ constant for a given rotameter.
Thus from the above equations it is evident that the factor $\left(D_{t}{ }^{2}-D_{f}{ }^{2}\right) / D_{f}$, representing the float position is a function of fluid flow rate.

Lecture 13
Topics Covered: Comparison of Orifice, Rota \& Venturimeter, Pitot Tube, Transportation of Fluids, Definition of Pump \& its classification

Pitot Tube : The pitot tube is a devise to measure the local velocity along a streamline. The principle of the devise is described from the following figure.


The opening of the impact tube a is perpendicular to the flow direction. The opening of the static tube b is parallel to the direction of flow. The two tubes are connected to the legs of a manometer or equivalent devise for measuring small pressure differences. The static tube measures the static pressure $\mathrm{p}_{0}$, since there is no velocity component perpendicular to its opening. The impact opening includes a stagnation point $B$ at which the streamline $A B$ terminates.

The measurement of the rate of flow of a fluid by a pitot tube depends on the measurement of the difference between the impact and static heads of the fluid. Orifice meter and venture meter measure the average liquid velocity where as the pitot tube measures the local velocity of liquid.

The basic pitot tube consists of a tube pointing directly into the fluid flow. As this tube contains fluid, a pressure can be measured; the moving fluid is brought to rest (stagnates) as there is no outlet to allow flow to continue. This pressure is the stagnation pressure of the fluid, also known as the total pressure or (particularly in aviation) the pitot pressure.

The measured stagnation pressure cannot of itself be used to determine the fluid velocity (airspeed in aviation). However, Bernoulli's equation states:

## Stagnation Pressure $=$ Static Pressure + Dynamic Pressure

Which can also be written

$$
p_{t}=p_{s}+\left(\frac{\rho * V^{2}}{2}\right)
$$

Solving that for velocity we get:

$$
V=\sqrt{\frac{2 *\left(p_{t}-p_{s}\right)}{\rho}}
$$

Where $V$ is fluid velocity
$p_{t}$ is stagnation or total pressure
$p_{s}$ is static pressure
and $\rho$ is fluid density

## Lecture 14

Topics Covered: Introduction of Centrifugal Pump \& its performance, calculation of total head, Cavitation, priming, NPSH, Characteristics Curves.

## Transportation of Fluids:

For transportation of liquids, pumps are used while gases are transported with the help of fans, blowers or compressors. In a pump, fan or compressor, mechanical work is transformed into fluid energy. The properties of fluids may different, it may be highly viscous, corrosive, or may contain suspended solid particles. So, the correct type of fluid-moving machinery is very much important for fluid transport.

PUMPS: Broadly, pump is defined as a machine for converting mechanical energy into pressure energy which is used to lift liquid from lower to a higher elevation. According to the working principle pumps can classified as:

| Classification of pump |  |  |
| :--- | :--- | :--- |
| 1. Centrifugal | 2. Rotary | 3. Reciprocating |
| a) Centrifugal | a) Cam | a) Piston |
| b) Propeller | b) Screw | b) Plunger |
| c) Mixed flow | c) Gear | c) Diaphragm |
| d) Turbine | d) Vane |  |
|  | e) Lobe |  |

Centrifugal Pumps : In this type of pump, the pressure is developed by centrifugal force on the liquid passing through it. The centrifugal pump consists of an impeller with blades or vanes, mounted on stationary casing. The shaft passes through the stuffing box and it is mounted on bearings.

The operation of the pump depends on increasing the velocity head of a fluid by the action of centrifugal force, then passing the fluid through increasing cross-sectional area, after that part of velocity head is converted into pressure head. To achieve sufficient centrifugal force the pump must be operated at higher speed and have large diameters. The mechanical energy is provided externally to the shaft which in turn rotates the impeller and the rotating impeller decreases pressure in the suction end and liquid flows
into the impeller. The impeller transfers its energy to the liquid, so that the liquid is thrown out through the delivery outlet by the acquired kinetic energy.

## Basic costruction of a Centrifugal Pump

1. Impellers : a) open, b) fully enabled, c) semi-enclosed.
2. Casing : a) Volute type, b) vertex type, c) volute type with guide blades.
3. Shaft and bearings :
4. Stuffing box :

## Performance of centrifugal pump (Hydraulic characteristics) :

i) Specific Speed : It is defined as the speed of an imaginary pump, identical with the given pump, which will discharge 1 lit. of liquid which is raised through a head of 1 m . The specific speed of a centrifugal pump is calculated by the following formula:
$N_{s}=N \sqrt{ } \mathrm{Q} / \mathrm{H}^{3 / 4}\left[\mathrm{~N}=\right.$ rotation speed of impeller in r.p.m.; $\mathrm{Q}=$ pump discharge in $\mathrm{m}^{3} / \mathrm{sec} ; \mathrm{H}=$ lift of the pump in m.]
ii) Total Head: The mechanical energy supplied to the impeller is converted into potential and kinetic energy which imparts pressure to liquid at the delivery end. The head is expressed in meters of liquid, the pump can deliver. It is independent of the nature of liquid being pumped and counts on a given speed of rotation and capacity.

The total head obtained from the pump is determined by the Bernoulli's equation, gives the difference between static pressure and velocity heads in addition to the friction losses for the suction ( $\mathrm{h}_{\mathrm{SL}}$ ) and discharge ( $\mathrm{h}_{\mathrm{DL}}$ ) sides of the pump. The above figure is a typical suction lift for a liquid of weight density w from the tank 1 at lower height at pressure $\mathrm{p}_{1}$ to the tank 2 at higher height at pressure $\mathrm{p}_{2}$.

Let $Z_{1}$ and $Z_{2}$ be the elevations of points $A$ and $B$ respectively above datum line and $V_{1}, V_{2}$ and $\mathrm{p}_{1}$ and $\mathrm{p}_{2}$ be the velocity and pressure at points A and B respectively. $\mathrm{W}_{\mathrm{P}}=$ working of pump with efficiency $\eta$.

From Bernoulli's equation, we have -
$\mathrm{Z}_{1}+\mathrm{V}_{1}{ }^{2} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}+\eta \mathrm{W}_{\mathrm{p}}=\mathrm{Z}_{2}+\mathrm{V}_{2}{ }^{2}+\mathrm{p}_{2} / \mathrm{w}+\mathrm{h}_{\mathrm{f}}$.
[where hf $=$ total friction loss $=h_{f \mathrm{fD}}-\mathrm{h}_{\mathrm{fS}}$
Therefore, $\eta \mathrm{W}_{\mathrm{p}}=\left(\mathrm{Z}_{2}-\mathrm{Z}_{1}\right)+1 / 2 \mathrm{~g}\left(\mathrm{~V}_{2}{ }^{2}-\mathrm{V}_{1}{ }^{2}\right)+1 / \mathrm{w}\left(\mathrm{p}_{2}-\mathrm{p}_{1}\right)+\mathrm{h}_{\mathrm{f}}$

Therefore, total head $\mathrm{H}=\eta \mathrm{Wp}=$ difference in (static head + velocity head + pressure head) + total friction loss.

So, $\quad \eta \mathrm{W}_{\mathrm{p}}=\left(\mathrm{Z}_{2}+\mathrm{V}_{2}{ }^{2} / 2 \mathrm{~g}+\mathrm{p}_{2} / \mathrm{w}+\mathrm{h}_{\mathrm{fD}}\right)-\left(\mathrm{Z}_{1}+\mathrm{V}_{1}{ }^{2} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}+\mathrm{h}_{\mathrm{fS}}\right)$ $=\mathrm{h}_{\mathrm{D}}-\mathrm{h}_{\mathrm{s}} .=$ discharge head - suction head

Therefore, power required $P=\eta W_{p} / 75 \quad h p$
Special conditions:
[A] If both tanks are at atmospheric pressure $\left(\mathrm{p}_{1}=\mathrm{p}_{2}=\mathrm{p}_{\mathrm{atm}}\right)$

$$
\mathrm{H}=\left(\mathrm{Z}_{2}-\mathrm{Z}_{1}\right)+1 / 2 \mathrm{~g}\left(\mathrm{~V}_{2}^{2}-\mathrm{V}_{1}^{2}\right)+\mathrm{h}_{\mathrm{f}}
$$

[B] If tank $A$ is at datum line $Z_{1}=0$ and velocity at point 1 is negligible $V_{1} \rightarrow 0$

$$
\mathrm{H}=\mathrm{Z}_{2}+\mathrm{V}_{2}{ }^{2} / 2 \mathrm{~g}+\mathrm{h}_{\mathrm{f}}
$$

To get centrifugal pump in properly working, all the air in system must be replaced by the liquid to be pumped before starting the motor otherwise one problem may arise, known as cavitation.

Cavitation: While working with centrifugal pump the concept of vapour pressure of the liquid is very much important. The vapour pressure of a liquid may be defined as the pressure at which the liquid will transform into vapour at the given temperature. Vapour pressure is a function of temperature. When centrifugal pump is running care must be taken so that the pressure at any point does not fall below the vapour pressure of liquid to be pumped. If the pressur reached below that level, then some of the liquid will be converted into vapour or if the liquid contain gases, they may come out of the solution forming gas pockets that damage the impeller. This phenomenon is known as cavitation. The sufficient suction head is not developed, when cavitation occurs and no liquid is pumped. Cavitation leads to mechanical damage to the pump as bubble collapse. This can be minimized by -i) keeping the velocity of the liquid in the suction pipe as less as possible, ii) avoiding the sharp bends in the suction pipe to reduce the loss of head.

Cavitation will not take place if the total head at the suction is sufficiently greater than the vapour pressure of the liquid at pumping temperature.

Net Positive Suction Head (NPSH) : NPSH of the pump is the amount by which the absolute pressure available at suction point is in excess or greater than the vapour pressure of the liquid at the pumping temperature. During selection of the pump the concept of NPSH is very important.

## Equation for NPSH :

NPSH $=\left(\right.$ Absolute pressure head available at suction point $\left.1^{\prime}\right)-($ Vapour pressure head $)$

$$
\begin{equation*}
=\left(\mathrm{V}_{1},^{2} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}\right)-\mathrm{p}_{\mathrm{v}} / \mathrm{w} \quad\left(\mathrm{p}_{\mathrm{v}}=\text { vapour pressure of liquid to be pumped }\right) . \tag{1}
\end{equation*}
$$

Applying Bernoulli's equation between points 1 and $1^{\prime}$
$\mathrm{Z}_{1}+\mathrm{V}_{1}{ }^{2} / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}=\mathrm{Z}_{1},+\mathrm{V}^{2}{ }_{1}, / 2 \mathrm{~g}+\mathrm{p}_{1} / \mathrm{w}+\mathrm{h}_{\mathrm{LS}}$ $\qquad$

Since, $Z_{1}=0$ and $V_{1}=0$, equation (2) becomes,
$\mathrm{V}_{1}{ }^{2} / 2 \mathrm{~g}+\mathrm{p}_{\mathrm{l}} / \mathrm{w}=\mathrm{p}_{1} / \mathrm{w}-\mathrm{h}_{\mathrm{LS}}-\mathrm{Z}_{\mathrm{l}}$,
From equation (1) and (3), we have :-

$$
\text { NPSH }=\mathrm{p}_{1} / \mathrm{w}-\mathrm{p}_{\mathrm{v}} / \mathrm{w}-\mathrm{h}_{\mathrm{LS}}-\mathrm{Z}_{\mathrm{l}^{\prime}}=\left(\mathrm{p}_{1}-\mathrm{p}_{\mathrm{v}}\right) / \mathrm{w}-\mathrm{h}_{\mathrm{LS}}-\mathrm{Z}_{\mathrm{l}^{\prime}}
$$

If pump centerline remains below the point 1 , then NPSH $=\left(p_{1}-p_{v}\right) / w-h_{L S}+Z_{1}$,
When, NPSH $=0$, then suction pressure $=$ vapour pressure

Therefore, cavitation occurs.
So, to avoid cavitation NPSH should be greater than 0 and usually 2 or more.

## Lecture 15

Topics Covered: Introduction of Reciprocating Pump; Difference between reciprocating and centrifugal pump, Introduction of Rotary Pump, Pumping devices used for transportation of gases, Blower and compressors, Reciprocating and Centrifugal compressors.

Priming : The pressure developed by the impeller in a centrifugal pump is proportional to the density of fluid in the impeller. When impeller is running with air, it will produce negligible pressure at the suction end which cannot suck liquid source since density of air is very less than that of liquid. This phenomenon is known as air-binding of the pump. This can be avoided by removing air from the suction line and pump casing which is called priming. It can be done by first filling the suction pipe and casing by liquid to be pumped. Then the pump is started with delivery valve closed, so that the rotating impeller
pushes the water in the delivery pipe. Finally the delivery valve is opened and the air is displaced by the liquid.

Characteristics Curve: Before using centrifugal pump it is very much important to determine the behaviour of the pump under different conditions. The essential characteristics curves are as follows:


Discharge Vs. head curve : The $\mathrm{H}-\mathrm{Q}$ curves are parabolic in nature, that means for a given rotational speed, the head decreases with increase in discharge.

Discharge Vs Power Curves: It is almost a straight line which means that for a given rotational speed power increases with increases with increase in discharge.

Discharge Vs Efficiency Curves: These curves are parabolic in nature. Efficiency of the pump reaches maximum at certain discharge and then decreases. The point of maximum efficiency is called "duty point" which represents the optimum operating conditions.

## Positive Displacement Reciprocating Pump, Piston type:

This type of pump consists of a cylinder fitted with a reciprocating pump. The crank is connected with piston by a connecting rod. The piston is reciprocated inside the cylinder with the help of crank which is rotated by steam engine. Positive displacement reciprocating pump may be single acting or double acting type.

For single acting type piston moves towards the right in its suction stroke and thus creating vacuum inside the cylinder, which opens the suction valve A and close the discharge valve B. As a result of which liquid enters into the cylinder. Then the piston moves towards the left in its delivery stroke, liquid gets compressed, thus creating the pressure inside the cylinder and the discharge valve $B$ opens with simultaneous closing of suction valve $A$.

For double acting pump liquid is liquid enters into the cylinder and then discharged from the pump during forward and backward stroke respectively. In backward stroke, liquid enters into the cylinder through suction valve A with simultaneous discharge through discharge valve C. In forward stroke liquid enters through suction valve D and simultaneously discharged through the valve B.

A reciprocating pump is also known as positive displacement pump because it discharges a define amount of liquid due to the movement of piston or plunger.

## Differences between Centrifugal and Reciprocating Pump

| Centrifugal Pump | Reciprocating Pump |
| :--- | :--- |
| 1) Simple, light weight with less number of parts. | 1) Complex, heavy construction with more number <br> of parts. |
| 2) It delivers the fluid continuously | 2) It has pulsating discharge. |
| 3) Requires less floor area. | 3) It requires more floor area. |
| 4) It can sustain less wear and tear with low | 4) It can sustain more wear and tear with high |
| maintenance cost. | maintenance cost. |
| 5) Requires priming before starting | 5) It does not require priming before starting. |
| 6) It has low efficiency. | 6) It has high efficiency. |

Positive Displacement Rotary Pump : Rotary pumps deliver a fixed amount of fluid per revolution. These pumps have the element that rotates inside the casing and provides reduction in pressure at the inlet side, that forces liquid inside the pump which gets trapped between rotating elements and casing, finally gets discharged. These pumps can handle liquids of any viscosity and they operate in moderate pressure ranges. The rate of discharge from rotary pump depends upon its size and speed of rotation. Rotary pumps are self priming, so can be used for lifting liquids from a level below the pump. Example: a) the screw pump, b) the gear pump.

## Centrifugal vs. Ratary Pump:

- Centrifugal pumps work best with low viscosity fluids (like water) that do not contain entrained air. A centrifugal pump has to be primed before it can pump liquid.
- Rotary pumps work best with viscous (thick) fluids because the viscous fluid fills the clearance areas as well as the pumping cavities, and the less clearance you have in a rotary pump the better it works.
- This means that rotary are more efficient than centrifugal pumps when the fluid is viscous, but less efficient with low viscosity fluids
- They also have the advantage of being self priming because they can pump gases as well as liquid.


## Pumping Devises for Gases:

For pumping gaseous substance, fans, blowers and compressors are generally used.

1) The lobe or roots type blower :
2) The Centrifugal Blower : The operating principle of the centrifugal blower is same as that of centrifugal pump. The impeller size is larger than that in pump which is necessary in order to generate high head with low density fluid like gas. The impeller is surrounded by diffuser or guide blades that convert the kinetic energy of gas leaving the impeller into pressure of 40 to 100 psi and operate at speeds $3500-30000$ r.p.m. These blowers are used for supplying air to furnaces, cooling and drying purposes and for ventilation purpose.
3) The Air Compressors :

The centrifugal compressors consist of impellers mounted on a single shaft which rotates at high speed inside the massive casing. As gas flows into the eye of the first impeller, it gets accelerated radially and flows into diffuser, which converts kinetic energy into pressure energy. This compressed gas then enters the next impeller and finally high pressure gas is obtained. These compressors are widely used in petroleum refineries.

Centrifugal compressor, (sometimes referred to as radial compressors) are a special class of radialflow work-absorbing turbomachinery that includes pumps, fans, blowers and compressors.

In contrast, modern centrifugal compressors are higher in speed and analysis must deal with compressible flow.

For purposes of definition, centrifugal compressors often have density increases greater than 5 percent. Also, they often experience relative fluid velocities above Mach 0.3 when the working fluid is air or nitrogen. In contrast, fans or blowers are often considered to have density increases of less than 5 percent and peak relative fluid velocities below Mach 0.3-0.5

| Reciprocating Compressors | Centrifugal Compressors |
| :--- | :--- |
| 1) It cannot be directly coupled with the drive unit | 1) It can be directly coupled with the drive unit |
| 2) It is a slow-speed machine | 2) It is high-speed machine. |
| 3) It can develop pressure up to $1 \mathrm{~N} / \mathrm{m} 2$ | 3) It can develop pressure up to $100 \mathrm{~N} / \mathrm{m} 2$ |
| 4) The compression process is isothermal because | 4) The compression process is adiabatic because of |
| coolers are used. | high speed operation. |

## Lecture 16

Topics Covered: Discussion of piping and pipe fittings, flow controlling valves, Gate valve, Globe Valve, Ball Valve etc.

Piping and Pipe Fitting: Within industry, piping is a system of pipes used to convey fluids (liquids and gases) from one location to another. The engineering discipline of piping design studies the efficient transport of fluid. Industrial process piping can be manufactured from wood, fiberglass, glass, steel, aluminum, plastic, copper, and concrete. The in-line components, known as fittings, valves, and other devices, typically sense and control the pressure, flow rate and temperature of the transmitted fluid, and usually are included in the field of Piping Design.

The difference between pipe and tube:

1. A pipe is a vessel - a tube is structural.
2. A pipe is measured ID - a tube is measured OD.
3. Pipes are measured ID or inside diameter because they are vessels. Tubes are measured OD or outside diameter because they are structural.
4. Pipes have a consistent ID regardless of wall thickness. In other words, a $1 / 2^{\prime \prime}$ high pressure pipe may need a 2 " thick wall, but the ID will still only be $1 / 2^{\prime \prime}$ even though the OD is $4.5^{\prime \prime}$.
5. Generally speaking, a tube will have a consistent OD and it's ID will change.
6. A tube is structural.

By having a consistent OD they can vary wall thickness, changing the ID, to increase
strength. Because they are consistent OD, they have predictable characteristics.

Pipe Fitting : Pipe fitting is the occupation of installing or repairing piping or tubing systems that convey liquid, gas, and occasionally solid materials. This work involves selecting and preparing pipe or tubing, joining it together by various means, and the location and repair of leaks.

1) For connecting different pipe sections :
a) Socket used for connecting two pipes

b) Union joint used assembling and dissembling of threaded or welded systems.

a) $90^{\circ}$ elbow:


$$
\mathrm{R}=1.5 \mathrm{X} \text { Dia. }
$$

b) $45^{\circ}$ elbow:

c) long radias elbow:

3) For changing the diameter of the pipe line :
a) Concentric:

b) Eccentric.
4) For branching of the pipeline
a) Tee
b) Cross
5) For termination of pipeline :
a) Plug fittings
b) Cap fittings
6) For connecting valves to the pipeline :
a) Short nipple
b) Reducing nipple
c) close nipple

## Flow Controlling Valves :

Valves regulate the flow of fluids. It isolates the piping or equipment for maintenance. The following valves are commonly used in chemical industries :
a) Gate Valve : It is used to minimize the pressure drop in open position and to shut the flow. This valve is generally used over the pressure ranging from 125 to 300 psi and temperature upto 230 ${ }^{\circ}$ for water, steam, oil etc.
b) Globe Valve : This is used for controlling the flow but pressure drop across the valve. The valves are installed with pressure side connected to the top of the desk. These valves are usually made up of elastic materials like gun metal, bronze or stainless steel.
c) Ball valve : The ball valve is used for complete shut off service.
d) Needle Valve:
e) Butterfly Valve :
f) Check Valve or non-return type valve

## Module III

## Lecture 17

Topics Covered: Introduction of Module III: Introduction to Heat Transfer: Conduction, Convection \& Radiation, Fourier's Law, Mean area of heat transfer, Thermal conductivity

## HEAT TRANSFER

We all know that heat can be transferred from source to receiver by three processes : 1) conduction, 2 ) convection, 3 ) radiation.

1) Conduction : The mechanism of conduction can be displayed easily by the transport of heat through solids. The basic law of conduction can be described as - Rate $=$ driving force/resistance. Here the driving force is the temperature drop through the solids.

## Fourier's Law :

Fourier's law describes that rate of heat flow through a uniform material is proportional to the area, the temperature drop and inversely proportional to the length of the path of flow. If a thin section of a wall dx, parallel to the area $A$, be taken at some point in the wall, with a temperature difference dT across such layer, then, Fourier's law may be represented by the equation : dQ $=-\mathrm{KA} \mathrm{dT} / \mathrm{dx}$ ....................(1)
[dQ = rate of heat transfer, $\mathrm{dT} / \mathrm{dx}=$ temperature gradient, $\mathrm{K}=$ proportionality constant which depends on the nature of the material and its temperature, designated as thermal conductivity, W/m.k].

Thermal conductivity of gases varies from 0.006 to $0.6 \mathrm{~W} / \mathrm{mk}$, while that of liquids vary from 0.09 to $0.7 \mathrm{~W} / \mathrm{mk}$. Materials with thermal conductivity $<0.2 \mathrm{~W} / \mathrm{mk}$ are usually used as insulators. The negative sign of thermal conductivity is due to the fact that with an increase in $x, T$ decreases.

Equation (1) can be integrated to get : $\mathrm{Q}=\mathrm{K} \mathrm{A}_{\mathrm{m}} \Delta \mathrm{T} / \mathrm{x}$
$\left[A_{m}=\right.$ mean area of heat transfer normal to the direction of heat flow $\left(m_{2}\right)$,
$\Delta \mathrm{T}=$ Temperature difference]
The quantity $\mathrm{KAm} / \mathrm{x}$ is characteristic of the solid and is called the thermal conductance. $\mathrm{x} / \mathrm{KA}_{\mathrm{m}}$ is called thermal resistance.

## Mean Area of Heat Transfer (Am)

Since the cross sectional area of heat transfer may vary along length of the conduction path, it is necessary to find the mean area for heat transfer rate calculation. The determination of $A_{m}$ depends on the shape of solid. Consider $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ as the limiting areas :

1) For solid of constant cross-section, e.g. a large flat plate and a cylinder wall where $A_{2} / A_{1}<2$ $\mathrm{A}_{\mathrm{m}}=\mathrm{A}_{1}+\mathrm{A}_{2} / 2$
2) For a solid, whose cross-sectional area of heat transfer is proportional to the radius. Example a long hollow cylinder. $\quad \mathrm{A}_{\mathrm{m}}=\mathrm{A}_{2}-\mathrm{A}_{1} / \ln \mathrm{A}_{2} / \mathrm{A}_{1}$
3) For a solid whose cross-sectional area of heat transfer is proportional to the square of the radius, example - a hollow sphere, $\quad \mathrm{A}_{\mathrm{m}}=\sqrt{ } \mathrm{A}_{1} \cdot \mathrm{~A}_{2}$

## Lecture 18

Topics Covered: Conduction through composite plane wall in series and parallel, Introduction of Convective heat transfer, Natural vs forced convection

## Conduction Through a Composite Plane Wall :

Many chemical engineering operations involve steady state heat flow through several layers of a plane wall. For example, a furnace wall may consist of several layers of materials for strength, insulation, outer appearance etc. The figure shows the temperature distribution for the steady state conduction of heat through a composite plane wall.
$\mathrm{Q}=\Delta \mathrm{T} / \mathrm{R}$, where, $\mathrm{R}=\mathrm{x} / \mathrm{KA}$
Therefore, $\mathrm{R}_{1}=\mathrm{x}_{1} / \mathrm{K}_{1} \mathrm{~A}_{1} ; \quad \mathrm{R}_{2}=\mathrm{x}_{2} / \mathrm{K}_{2} \mathrm{~A}_{2} ; \quad \mathrm{R}_{3}=\mathrm{x}_{3} / \mathrm{K}_{3} \mathrm{~A}_{3}$

So the total resistance $\Sigma \mathrm{R}=\mathrm{R} 1+\mathrm{R} 2+\mathrm{R} 3=\mathrm{x}_{1} / \mathrm{K}_{1} \mathrm{~A}_{1}+\mathrm{x}_{2} / \mathrm{K}_{2} \mathrm{~A}_{2}+\mathrm{x}_{3} / \mathrm{K}_{3} \mathrm{~A}_{3}$

Therefore, $\mathrm{Q}=\Delta \mathrm{T} / \Sigma \mathrm{R}=\Delta \mathrm{T} / \mathrm{x}_{1} / \mathrm{K}_{1} \mathrm{~A}_{1}+\mathrm{x}_{2} / \mathrm{K}_{2} \mathrm{~A}_{2}+\mathrm{x}_{3} / \mathrm{K}_{3} \mathrm{~A}_{3}$
For plane wall, $\mathrm{A}_{1}=\mathrm{A}_{2}=\mathrm{A}_{3}$

## Conduction Through Resistances in Parallel :

Let two plane solids be placed side by side in parallel as shown in following figure. The direction of heat flow is perpendicular to the plane of exposed surface of each solid. Total heat flow is the sum of the heat flow through solids A and B.

Therefore, $\mathrm{Q}_{\mathrm{t}}=\mathrm{q}_{\mathrm{a}}+\mathrm{q}_{\mathrm{b}}=\mathrm{K}_{\mathrm{A}} \mathrm{A}_{\mathrm{A}} / \mathrm{x}_{\mathrm{A}}\left(\mathrm{T}_{1}-\mathrm{T}_{2}\right)+\mathrm{K}_{\mathrm{B}} \mathrm{A}_{\mathrm{B}} / \mathrm{x}_{\mathrm{B}}\left(\mathrm{T}_{1}-\mathrm{T}_{2}\right)$ $\mathrm{Q}_{\mathrm{t}}=\left(\mathrm{T}_{1}-\mathrm{T}_{2}\right) / \mathrm{x}_{\mathrm{A}} / \mathrm{K}_{\mathrm{A}} \mathrm{A}_{\mathrm{A}}+\left(\mathrm{T}_{1}-\mathrm{T}_{2}\right) / \mathrm{x}_{\mathrm{B}} / \mathrm{K}_{\mathrm{B}} \mathrm{A}_{\mathrm{B}}=\left\{1 / \mathrm{R}_{\mathrm{A}}+1 / \mathrm{R}_{\mathrm{B}}\right\}\left(\mathrm{T}_{1}-\mathrm{T}_{2}\right)$

An insulated wall of brick (A) over where steel reinforcing members (B) are in parallel and penetrate the wall, is an example of resistances in parallel.

## CONVECTION

If a molecule is free to move and circulate after each collision, so that in its next collision it may exchange energy with a different molecule, the process of heat transfer is called convection. So convection is the motion of the hot body itself, carrying its heat with it. This may occur in case of both gas and liquid.

In the case of convection, heat is transferred from one place to another by the physical mixing of the hot and cold partitions of a fluid. When the ixing occurs due to the density difference alone, it is called natural convection. When it occurs due to the placement of a mechanically induced agitator within the fluid or by the introduction of a prime mover, such as pump, fan, blower etc. is called forced convection. The basic equation for the rate of heat transfer by convection under steady state condition is : $\mathrm{q}=\mathrm{hA}$. $\Delta \mathrm{T}[\mathrm{h}=$ heat transfer coefficient is a measure of the intensity of heat transfer between the surface of a body and its surroundings and its unit is $\mathrm{W} / \mathrm{m} 2-\mathrm{K}$. So it is the amount of heat transferred per unit time through unit area at a temperature difference of $1^{\circ}$ between the surfaces of the surroundings].

## Lecture 19

Topics Covered: Individual heat transfer coefficient and correlations between them, Heat transfer with change in phase, definition, classification \& derivation of equations

## Individual Heat Transfer Coefficient :

For convective heat transfer, both in laminar and turbulent flow, the equation generally used is : $\mathrm{q}=\mathrm{hA}\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\text {avg }}\right)\left[\mathrm{T}_{\mathrm{w}}=\right.$ temperature at the surface of the wall,, $\mathrm{T}_{\text {avg }}$ depend on the type of flow.]

Therefore, for fluid flowing inside a pipe, where heat transfer occurs from the heated wall of the pipe of the fluid : $q=h A\left(T_{w}-T_{i}\right)$, $\left[T_{i}=\right.$ average temperature of the fluid $]$ and for fluid flowing outside a heated pipe, $\mathrm{q}=\mathrm{hA}\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{0}\right)\left[\mathrm{T}_{0}=\right.$ temperature far from the surface $]$

For heat transfer in boiling, resistance is mainly offered by the film of vapour which forms immediately over the hot surface.

Therefore, resulting equation is : $\mathrm{q}=\mathrm{hA}\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\mathrm{l}}\right)[\mathrm{Tl}=$ standard liquid temperature on one side of the film, $\mathrm{Tw}=$ surface temperature on the other side, $\mathrm{h}=$ boiling coefficient.]. In condensation, the main resistance to heat transfer comes from the liquid film adjacent to the surface.

> Therefore, $\mathrm{q}=\mathrm{hA}\left(\mathrm{T}_{\mathrm{V}}-\mathrm{T}_{\mathrm{W}}\right) \quad\left[\mathrm{T}_{\mathrm{V}}=\right.$ standard vapour temperature, $\mathrm{h}=$ condensation coefficient]

## Correlation for Calculation of Heat Transfer Coefficient :

For heat transfer to a fluid in turbulent flow through a pipe, the heat transfer coefficient, h is a function of pipe of diameter D , thermal conductivity of the fluid K , velocity of the fluid v , density of the fluid $\rho$, heaty capacity of the fluid $\mathrm{C}_{\mathrm{P}}$ and viscosity of the fluid $\mu$. From dimensional analysis, we have :
$\mathrm{hD} / \mathrm{K}=\mathrm{f}\left(\mathrm{Dv} \rho / \mu, \mathrm{C}_{\mathrm{P}} \mu / \mathrm{K}\right)$
$\mathrm{Nu}=\mathrm{f}(\mathrm{Re}, \mathrm{Pr})$ $\qquad$

For heat transfer by natural convection, dimensional analysis gives :
$h D / K=f\left(D^{3} \rho^{2} g \beta \Delta T / \mu^{2}, C_{P} \mu / K\right)$ $\qquad$
$\mathrm{Nu}=\mathrm{f}(\mathrm{Gr}, \mathrm{Pr})[\mathrm{Gr}=$ Grasshof number, $\beta=$ coefficient of thermal expansion, $\Delta \mathrm{T}=$ temperature difference, $\mathrm{g}=$ acceleration due to gravity]

The Sieder-Tate equation for the calculation of the heat transfer coefficient for laminar flow of fluids inside horizontal tubes or pipes is : $\mathrm{Nu}=1.86\left[\right.$ Re.Pr.D/L] ${ }^{1 / 3} \cdot\left(\mu / \mu_{\mathrm{w}}\right)^{0.14}$

The equation is valid for $\mathrm{Re}>2100$ and (Re.Pr.D/L) > 100

All the fluid properties are evaluated at the arithmetic average bulk temperature of the fluid except $\mu_{\mathrm{w}}$, which is calculated at the average wall temperature.

The Dittus-Boelter equation for predicting the heat transfer coefficient for turbulent flow in the tubes or pipes is : $\mathrm{hD} / \mathrm{K}=0.023(\mathrm{Dv} \rho / \mu)^{0.8} \cdot\left(\mathrm{C}_{\mathrm{p}} \mu / \mathrm{K}\right)^{\mathrm{a}}$

The properties of the fluid are evaluated at the arithmetic mean bulk temperature of the fluid. When the fluid is heated, $\mathrm{a}=0.4$ and when the fluid is cooled, $\mathrm{a}=0.3$. For most gases $\operatorname{Pr}=1$, so the variation of a has no effect on it.

The Colburn equation for the evaluation of the heat transfer coefficient for the turbulent flow in tubes is: $\mathrm{h} / \mathrm{v} \mathrm{\rho C}_{\mathrm{p}}\left(\mathrm{C}_{\mathrm{p}} \mu / \mathrm{K}\right)^{2 / 3}=0.023(\mathrm{Dv} \rho / \mu)^{-0.2}$

Equation (8) can be rearranged : $\mathrm{hD} / \mathrm{K}=0.023(\mathrm{Dv} \mathrm{\rho} / \mu)^{0.8} \cdot\left(\mathrm{C}_{\mathrm{p}} \mu / \mathrm{K}\right)^{1 / 3}$
Cp is evaluated at the arithmetic mean bulk temperature and all other fluid properties are evaluated at the wall temperature.

The Sieder -Tate equation for turbulent flow in tubes takes into account the variation of the viscosity of the fluid near the wall with thermal gradients.

$$
\begin{equation*}
\mathrm{hD} / \mathrm{K}=0.023(\mathrm{Dv} \rho / \mu)^{0.8} \cdot\left(\mathrm{C}_{\mathrm{p}} \mu / \mathrm{K}\right)^{\mathrm{a}}\left(\mu / \mu_{\mathrm{w}}\right)^{0.14} . \tag{10}
\end{equation*}
$$

This equation is valid for $\operatorname{Re}>10000, \quad 0.7<\operatorname{Pr}<700$ and $\mathrm{L} / \mathrm{D}>60 \mu_{\mathrm{w}}$ is evaluated at the average wall temperature and all other fluid properties at arithmetic mean bulk temperature. When a fluid flows parallel to a flat plate and heat transfer occurs along the whole plate of length $L$, the average heat transfer coefficient is given by: $\mathrm{N}_{\mathrm{u}}=0.664 \mathrm{Re}_{\mathrm{L}}{ }^{0.5} \mathrm{Pr}^{1 / 3}$ (11). The equation holds for $\mathrm{Pr}>0.7$ and in laminar flow region, where $\operatorname{Re}_{\mathrm{L}}<3 \times 10^{5}$.

For $\mathrm{Pr}>0.7$ and in completely turbulent region where Reu $>3 \mathrm{X} 10^{5}$, the equation is: $\mathrm{Nu}=$ $0.0366 \operatorname{Re}_{\mathrm{L}}{ }^{0.8} \mathrm{Pr}^{1 / 3}$

Here the fluid properties are evaluated at the film temperature: $\mathrm{T}_{\mathrm{f}}=\left(\mathrm{T}_{\mathrm{w}}+\mathrm{T}_{\mathrm{b}}\right) / 2$, where $\mathrm{T}_{\mathrm{w}}=$ surface or wall temperature, $\mathrm{T}_{\mathrm{b}}=$ average bulk fluid temperature.

When a fluid flows past a single sphere, the heat transfer coefficient is given by :
$\mathrm{Nu}=2.0+0.6 \operatorname{Re}^{0.5} \mathrm{Pr}^{1 / 3}$ $\qquad$

The equation is valid for Re between 1 and 70000 and Pr between 0.6 and 400 .
For turbulent flow in pipes, the following simplified equation hold in SI units :
(a) For air at 1 atm . total pressure, $\mathrm{h}=3.5 \mathrm{v}^{0.8} / \mathrm{D}^{0.2}$
(b) For water in temperature range, $\mathrm{T}=4$ to $105^{\circ} \mathrm{C}, \mathrm{h}=1429\left(1+0.146 \mathrm{~T}^{\circ} \mathrm{C}\right) \mathrm{v}^{0.8} / \mathrm{D}^{0.2}$
(c) For organic liquids, $h=423 \mathrm{v}^{0.8} / \mathrm{D}^{0.2}$

In the case of heat transfer from a liquid in an agitated vessel to its jacketed walls: $\quad \mathrm{hDi}=$ $\mathrm{a}\left(\mathrm{Nd}^{2} \rho / \mu\right)^{\mathrm{b}}(\mathrm{Cp} \mu / \mathrm{k})\left(\mu / \mu_{\mathrm{w}}\right)^{\mathrm{m}} \ldots \ldots \ldots \ldots$. (14) Here, $\mathrm{h}=$ film coefficient for the inner wall, Di = internal diameter of the mixing vessel, $\mathrm{Nd}^{2} \rho / \mu=$ Reynold's number for mixing, $\mathrm{N}=$ speed, $\mathrm{d}=$ diameter of the agitator.

For heat transfer to helical coils in agitated vessels the equation for the calculation of the heat transfer coefficient are :

$$
\begin{equation*}
\mathrm{hDi} / \mathrm{K}=0.87\left(\mathrm{Nd}^{2} \rho / \mu\right)^{0.62}\left(\mathrm{C}_{\mathrm{p}} \mu / \mathrm{K}\right)^{1 / 3} \cdot\left(\mu / \mu_{\mathrm{w}}\right)^{1 / 4} \tag{16}
\end{equation*}
$$

When the flow of fluid is normal to a horizontal cylinder, the equation for the calculation of heat transfer coefficient for normal convection is :
$\mathrm{hD} / \mathrm{K}=0.53\left[\mathrm{D}^{3} \rho^{2} \mathrm{~g} \beta \Delta \mathrm{~T} / \mu^{2}\right]^{1 / 4} \cdot\left(\mathrm{C}_{\mathrm{p}} \mu / \mathrm{K}\right)^{1 / 4}$
The equation is valid for $\operatorname{Pr}>10^{4}, \mathrm{Gr}<10^{3}$
For vertical plates and cylinders, the natural convection coefficient can be predicted from the relations:
$\mathrm{Nu}=0.138 \mathrm{Gr}^{0.36}\left(\operatorname{Pr}^{0.175}-0.55\right)$
This equation is valid for $1<\operatorname{Pr}<40$ and $\mathrm{Gr}>10^{3}$ and :
$\mathrm{Nu}=0.683 \mathrm{Gr}^{0.25} \cdot \operatorname{Pr}^{0.25}[\operatorname{Pr} /(0.861+\operatorname{Pr})]^{0.25}$
This equation is valid for $1<\operatorname{Pr}<40$ and $\mathrm{Gr}<10^{3}$

## Lecture 20

Topics Covered: Mechanism of condensation heat transfer, Drop-wise \& film -type, Machanism of Boiling heat transfer, : Pool Boiling/Local Boiling, bulk boiling, Mechanism: Nucleate \& Film Boiling

## Heat Transfer with Change in Phase

Change in phase of a fluid occurs when a vapour is condensed or a liquid is vapourised. In many power or refrigeration cycles a vapour changes to a liquid or liquid changes into vapour depending on the particular part of the cycle under study. These changes are brought about by boiling or condensation. Usually high heat transfer rates are involved in boiling and condensation. Vapourisation of fluids such as gasoline, in the chemical industry, generation of steam I the tubes of a boiler and boiling of a refrigerant in the cooling coils of a refrigerator are examples of heat transfer processes involving change in phase.

## Mechanism of Condensation Heat Transfer

Two mechanisms are commonly observed during condensation of vapours in the chemical industry. They are drop-wise and film-type condensation.

Drop-wise Condensation : Let us consider a vertical flat plate exposed to a condensate vapour. If the temperature of the plate is below the saturation temperature of vapour, some of the vapour condenses and drops are formed on the plate if the condensing liquid does not wet the surface readily. The drops grow larger till their weight overcomes the surface tension forces holding them to the plate. Then the drops fall, clearing the plate for more condensation. This process, called drop-wise condensation, a large portion of the area of the plate is exposed directly to the vapour. The heat transfer coefficient is very high since most of the heat transfer takes place directly between the plate and the vapour. The heat transfer coefficient in drop-wise condensation is five to ten times larger than film type condensation and its value may be as high as $110,000 \mathrm{~W} / \mathrm{m}^{2}-\mathrm{K}$.

Drop-wise condensation normally occurs when impurities are present and the surface is contaminated.

## Film-type Condensation:

If the condensing liquid wets the surface readily, a film of the condensate is formed on the plate and flows due to action of gravity, the process is called film-type condensation. This film of liquid between the plate and the vapour offers the main resistance to heat transfer. For film-type condensation, the expression for the calculation of the average heat transfer coefficient is :
$\mathrm{hL} / \mathrm{K}=1.13\left[\rho_{1}\left(\rho_{1}-\rho_{v}\right) \mathrm{g} \cdot \lambda_{\mathrm{c}} \cdot \mathrm{K}^{3}{ }_{1} / \mu_{1} \mathrm{~K}_{1} \Delta \mathrm{~T}\right]$
$\rho_{1}=$ density of liquid $\left(\mathrm{kg} / \mathrm{m}^{3}\right) ; \rho_{\mathrm{v}}=$ density of vapour $\left(\mathrm{kg} / \mathrm{m}^{3}\right) ; \mathrm{g}=$ gravitational acceleration $\left(\mathrm{m} / \mathrm{s}^{2}\right) ; \lambda_{\mathrm{c}}=$ latent heat of condensation $(\mathrm{J} / \mathrm{kg})$ at $\mathrm{T}_{\text {sat }} ; \mathrm{L}=$ vertical height of the surface or tube $(\mathrm{m}) ; \mathrm{K}_{1}$ $=$ thermal conductivity of the liquid $(\mathrm{W} / \mathrm{m}-\mathrm{K}) ; \mu_{1}=$ viscosity of liquid in $\mathrm{P}_{\mathrm{a}}-\mathrm{S}: \Delta \mathrm{T}=\mathrm{T}_{\text {sat }}-\mathrm{T}_{\mathrm{w}}$

Here all the properties of the liquid except $\lambda \mathrm{c}$ are evaluated at the film temperature $\mathrm{T}_{\mathrm{f}}=\left(\mathrm{T}_{\text {sat }}+\right.$ $\left.T_{w}\right) / 2$. The equation is valid for $\mathrm{R}_{\mathrm{e}}<1800$. where Reynold's number is defined as :
$R_{e}=4 m / \pi D \mu 1$, for a vertical tube of diameter $D$ and
$\operatorname{Re}=4 \mathrm{~m} / \mathrm{W} \mu 1$, for a vertical plate of width W
For turbulent flow, $\mathrm{Re}>1800$, the equation (1) becomes: $\mathrm{hL} / \mathrm{K}=0.0077\left(\mathrm{~g} \rho_{1}{ }^{2} \mathrm{~L}^{3} / \mu_{1}{ }^{2}\right)^{1 / 3} .\left(\mathrm{R}_{\mathrm{e}}\right)^{0.4}$ .... (2)

For a vertical tier of N horizontal tubes of external diameterD placed one below the other, the equation for the average heat transfer coefficient is :
$h L / K=0.725\left[\rho_{1}\left(\rho_{1}-\rho_{v}\right) g . \lambda_{c} . D^{3} / N \mu_{1} K_{1} \Delta T\right]$
This equation is valid for laminar flow region.

## Mechanism of Boiling Heat Transfer :

If the temperature of a surface exposed to a liquid is higher than its saturation temperature, the liquid boils and the heat flux depends on the temperature difference between the surface and saturation temperature. When the heated surface is submerged below the free surface of a liquid, the process is called pool boiling.

If the temperature of the liquid is less than the saturation temperature, the process is called subcooled or local boiling.

When the temperature of the liquid is equal to the saturation temperature, the process is called saturated or bulk boiling.

Boiling usually occurs by two mechanisms :

1. Nucleate Boiling :- Here bubbles are formed by the expansion of entrapped gas or vapour in small cavities at the surface and the vapour is evolved as a multitude of bubbles. This type of mechanism is observed in the temperature range of 373 to 421.8 K . Bubbles ofvapour form at discreet sites on the heated surface and detouch almost immediately. Nucleate boiling is seen in kettle-type and naturalcirculation reboilers.
2. Film Boiling :- As the temperature of the surface rises bubbles form at more and more sites until they coalesce to form a continuous film of vapour covering the surface. The liquid is now separated from the surface by this film of vapour and the heat required to evaporate the liquid must pass through the film. This type of mechanism is called film boiling in which vaporisation occurs at the vapour-liquid interface.

The following simplified equations in SI units give the heat transfer coefficient for water boiling outside submerged surfaces at 1 atm. abs. pressure.

I the natural convection region, for a horizontal surface : $\mathrm{h}=1043(\Delta \mathrm{~T})^{1 / 3}$
For a vertical surface, $\mathrm{h}=537(\Delta \mathrm{~T})^{1 / 7} \ldots \ldots(\mathrm{~b})$, where $\Delta \mathrm{T}=\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\text {sat }}\right)$

Equation (a) is valid for $\mathrm{q} / \mathrm{A}<16$ and in equation (b) for $\mathrm{q} / \mathrm{A}<3$
In the forced convection region, for a horizontal surface : $h=5.56(\Delta T) 3$
Equation (c) holds for : $16<\mathrm{q} / \mathrm{A}<240$
For vertical surface, $h=7.95(\Delta \mathrm{~T})^{3}$
Equation (d) is valid for : $3<\mathrm{q} / \mathrm{A}<63$
For forced conversion boiling inside tubes, the equation for the calculation of heat transfer coefficient in SI unit is : $2.55(\Delta \mathrm{~T})^{3} . \mathrm{e}(\mathrm{p} / 1551) \quad$ [ p is in kpa] (e)

The Bromby equation for predicting the heat transfer coefficient for film boiling on horizontal tube is : $\mathrm{h}=0.62\left[\mathrm{~K}^{3}{ }_{\mathrm{v}} \rho_{\mathrm{v}}\left(\rho_{1}-\rho_{\mathrm{v}}\right) \mathrm{g}\left(\lambda_{\mathrm{v}}+0.4 \mathrm{C}_{\mathrm{pv}} \Delta \mathrm{T}\right) / \mathrm{D} \mu_{\mathrm{v}} \Delta \mathrm{T}\right]$ $\qquad$
$\mathrm{K}_{\mathrm{v}}=$ thermal conductivity of the vapour $(\mathrm{W} / \mathrm{m}-\mathrm{k}) ; \rho_{\mathrm{v}}=$ density of the vapour $\left(\mathrm{kg} / \mathrm{m}^{3}\right) ; \quad \rho_{\mathrm{l}}=$ density of the liquid $\left(\mathrm{kg} / \mathrm{m}^{3}\right) ; \quad \lambda_{\mathrm{v}}=$ latent heat of vaporisation $(\mathrm{J} / \mathrm{kg}) ; \quad \Delta \mathrm{T}=\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\text {sat }} ; \quad \mathrm{T}_{\text {sat }}=$ Temperature of saturated vapour in $\mathrm{K} ; \mathrm{D}=$ outer diameter of the tube $(\mathrm{m}) ; \mu_{\mathrm{v}}=$ viscosity of the vapour in $(\mathrm{Pa}-\mathrm{s}) ; \mathrm{g}=$ gravitational acceleration $\left(\mathrm{m} / \mathrm{s}^{2}\right)$.

Here all the physical properties of the vapour are evaluated at the film temperature, $\mathrm{T}_{\mathrm{f}}=\mathrm{T}_{\mathrm{w}}+$ $\mathrm{T}_{\text {sat }} / 2$ and $\lambda_{\mathrm{v}}$ at the saturation temperature.

## Lecture 21

Topics Covered: Introduction of Overall heat transfer coifficeint ( U), Derivation of equation of Uo \& Ui, Relation between overall and individual heat transfer coefficient, Calculation of LMTD

## Overall Heat Transfer Coefficient :

Heat normally flows from one fluid to another through a solid wall in a heat transfer equipment. The heat must flow through several resistances in series. There are, the hot fluid film resistance, solid wall resistance, cold fluid film resistance and dirt or fouling resistance which is the resistance offered by scale or dirt formed on the heat transfer surfaces. Thus for steady state heat flow conditions :

$$
\mathrm{q}=\mathrm{UA} \Delta \mathrm{~T}_{\mathrm{m}} \ldots \ldots \ldots \ldots . . \text { (1) }\left[\Delta \mathrm{T}_{\mathrm{m}}=\text { overall average temperature difference; } \mathrm{U}=\right.\text { overall }
$$ heat transfer coefficient]

The overall resistance to heat transfer, 1/UA is the sum of all the resistances to the flow of heat.

Let us consider a heat transfer problem involving conductive resistances in series. As shown in the following figure, heat flows from the inner surface of an insulated pipe to its outer surface.

The heat transfer coefficient for the inside surface is hi and that of outside surface is ho. Since the heat transfer coefficient is associated with an appropriately defined area, at steady state any one of the following equations may be used to calculate the rate of heat transfer through the pipe wall.

$$
\begin{align*}
& \mathrm{q}=\mathrm{h}_{\mathrm{i}} \mathrm{~A}_{\mathrm{i}}\left(\mathrm{~T}_{1}-\mathrm{T}_{2}\right) \ldots \ldots \ldots \ldots\left(\mathrm{K}_{\mathrm{m}} \cdot \mathrm{~A}_{\mathrm{m}}, \text { (mean) }\left(\mathrm{T}_{2}-\mathrm{T}_{3}\right) / \Delta \mathrm{r}_{\mathrm{m}}\right.  \tag{2}\\
& \mathrm{q}=\mathrm{K}_{1} \cdot \mathrm{~A}_{1}, \operatorname{mean}\left(\mathrm{~T}_{3}-\mathrm{T}_{4}\right) / \Delta \mathrm{r}_{1}  \tag{3}\\
& \mathrm{q}=\mathrm{h}_{0} \mathrm{~A}_{0}\left(\mathrm{~T}_{4}-\mathrm{T}_{5}\right) \ldots \ldots \ldots \ldots \ldots \tag{4}
\end{align*}
$$

$\qquad$
$\qquad$

Since it is difficult to measure the intermediate temperatures generally equation in terms of the overall temperature drop is used. Equation (2) to (5) may be written in terms of the appropriate temperature differences.

$$
\begin{align*}
& \mathrm{T}_{1}-\mathrm{T}_{2}=\mathrm{q} / \mathrm{h}_{\mathrm{i}} \mathrm{~A}_{\mathrm{i}} \ldots \ldots \ldots \ldots  \tag{6}\\
& \mathrm{~T}_{2}-\mathrm{T}_{3}=\mathrm{q} \Delta \mathrm{r}_{\mathrm{m}} / \mathrm{K}_{\mathrm{m}} \cdot \mathrm{~A}_{\mathrm{m} \text { (mean) }}  \tag{7}\\
& \mathrm{T}_{3}-\mathrm{T}_{4}=\mathrm{q} \Delta \mathrm{r}_{1} / \mathrm{K}_{1} \cdot \mathrm{~A}_{1 \text { (mean) }} \\
& \mathrm{T}_{4}-\mathrm{T}_{5}=\mathrm{q} / \mathrm{h}_{0} \mathrm{~A}_{0} \ldots \ldots \ldots \ldots \ldots \tag{9}
\end{align*}
$$

The overall temperature drop may be written as :
$\mathrm{T}_{1}-\mathrm{T}_{5}=\left(\mathrm{T}_{1}-\mathrm{T}_{2}\right)+\left(\mathrm{T}_{2}-\mathrm{T}_{3}\right)+\left(\mathrm{T}_{3}-\mathrm{T}_{4}\right)+\left(\mathrm{T}_{4}-\mathrm{T}_{5}\right)$

With the help of equations (6) to (10), we have :

$$
\begin{align*}
& \mathrm{q}=\mathrm{T}_{1}-\mathrm{T}_{5} / 1 / \mathrm{h}_{\mathrm{i}} \mathrm{~A}_{\mathrm{i}}+\Delta \mathrm{r}_{\mathrm{m}} / \mathrm{K}_{\mathrm{m}} \cdot \mathrm{~A}_{\mathrm{m} \text { (mean) }}+\Delta \mathrm{r}_{1} / \mathrm{K}_{1} \cdot \mathrm{~A}_{1 \text { (mean) }}+1 / \mathrm{h}_{0} \mathrm{~A}_{0} \\
& \mathrm{q}=(\Delta \mathrm{T}) \text { overall } / \Sigma \mathrm{R} \ldots \ldots \ldots \ldots(11) \tag{11}
\end{align*}
$$

The heat transfer coefficient may be expressed in terms of the inside area, Ai. Multiplying equation (11) by $\mathrm{Ai} / \mathrm{Ai}$ we get :

$$
\begin{equation*}
\mathrm{q}=\left[1 / 1 / \mathrm{h}_{\mathrm{i}}+\Delta \mathrm{r}_{\mathrm{m}} \mathrm{~A}_{\mathrm{i}} / \mathrm{K}_{\mathrm{m}} \cdot \mathrm{~A}_{\mathrm{m} \text { (mean) }}+\Delta \mathrm{r}_{1} \mathrm{~A}_{\mathrm{i}} / \mathrm{K}_{1} \cdot \mathrm{~A}_{1 \text { (mean) }}+\mathrm{A}_{\mathrm{i}} / \mathrm{h}_{0} \mathrm{~A}_{0}\right] \times(\Delta \mathrm{T})_{\text {overall }} \ldots .( \tag{12}
\end{equation*}
$$

The term within bracket in equation (12) is called $U_{i}=$ overall heat transfer coefficient based on inside area.

$$
\begin{equation*}
\mathrm{q}=\mathrm{U}_{\mathrm{i}} \mathrm{~A}_{\mathrm{i}}(\Delta \mathrm{~T})_{\text {overall }} \tag{13}
\end{equation*}
$$

Similarly the equation in terms of the overall heat transfer coefficient based on the outside area is $: q=U_{0} A_{0}(\Delta T)_{\text {overall }} \ldots \ldots \ldots \ldots$. (14) $\left[U_{0}=\right.$ overall heat transfer coefficient based on outside area.

So, $\quad \mathrm{q}=\left[1 / \mathrm{A}_{0} / \mathrm{h}_{\mathrm{i}} \mathrm{A}_{\mathrm{i}}+\mathrm{A}_{0} / \mathrm{h}_{\mathrm{f}} \mathrm{A}_{\mathrm{i}}+\Delta \mathrm{r}_{\mathrm{m}} \mathrm{A}_{0} / \mathrm{K}_{\mathrm{m}} \cdot \mathrm{A}_{\mathrm{m}(\text { mean })}+\Delta \mathrm{r}_{1} \mathrm{~A}_{0} / \mathrm{K}_{1} \cdot \mathrm{~A}_{1 \text { (mean) }}+1 / \mathrm{h}_{0}\right] \ldots \ldots$. (16)

In general, for heat transfer problems in which a number of conductive and convective resistances are involved, the resistance in the case of conduction may be written as : $\Delta \mathrm{x}_{1} \mathrm{~A}_{\mathrm{j}} / \mathrm{K}_{1} \mathrm{~A}_{\text {mean }}$ and for convection as : $A_{j} h_{l} A_{l}\left[\Delta x\right.$ is the appropriate thickness coordinate, $A_{j}=$ the area upon which $U_{j}$ is based; $\mathrm{A}_{\text {mean }}=$ appropriate mean area for conduction path.

Therefore for m conductive and n conductive resistances :

$$
\begin{equation*}
\mathrm{U}_{\mathrm{j}}=1 / \Sigma_{\mathrm{l}=1}^{\mathrm{m}}\left(\Delta \mathrm{x}_{1} \mathrm{~A}_{\mathrm{j}} / \mathrm{K}_{\mathrm{l}} \mathrm{~A}_{\text {mean }}\right)+\Sigma_{\mathrm{l}=1}^{\mathrm{n}}\left(\mathrm{~A}_{\mathrm{j}} / \mathrm{h}_{1} \mathrm{~A}_{\mathrm{l}}\right) \tag{17}
\end{equation*}
$$

## Logarithmic Mean Temperature Difference (LMTD) :

The temperature difference, $\Delta \mathrm{T}$, is not always constant but varies regularly during a period of time or regularly from point to point along a heat transfer surface. It is, therefore necessary to calculate the average temperature difference, $\Delta \mathrm{T}_{\text {avg }}$.

If we consider a parallel flow heat exchanger as shown in above figure. The heat transferred per unuit time from the hot fluid to the cold fluid through a surface element dA is :

$$
\begin{equation*}
\mathrm{dq}=\mathrm{U}\left(\mathrm{t}_{1}-\mathrm{t}_{2}\right) \mathrm{dA} \tag{1}
\end{equation*}
$$

During the process of heat transfer, let the fall in the temperature of the hot fluid 1 be $\mathrm{dt}_{1}$ and the corresponding rise of the cold fluid 2 be $\mathrm{dt}_{2}$. Then, from energy balance :

$$
\mathrm{dq}=-\mathrm{m}_{1} \mathrm{Cp}_{1} \mathrm{dt}_{1}=\mathrm{m}_{2} \mathrm{Cp}_{2} \mathrm{dt}_{2} \ldots \ldots \text {. (2) }\left[\mathrm{m}_{1}, \mathrm{~m}_{2} \text { and } \mathrm{Cp}_{1}, \mathrm{Cp}_{2}\right. \text { are the mass flow rates and }
$$ specific heats of the hot and cold fluids respectively]

$$
\mathrm{dt}_{1}=-\mathrm{dq} / \mathrm{m}_{1} \mathrm{Cp}_{1}, \quad \mathrm{dt} t_{2}=\mathrm{dq} / \mathrm{m}_{2} \mathrm{Cp}_{2}
$$

The change in temperature difference is, therefore, $\quad \mathrm{dt}_{1}-\mathrm{dt}_{2}=-\mathrm{dq}\left[1 / \mathrm{m}_{1} \mathrm{Cp}_{1}+1 / \mathrm{m}_{2} \mathrm{Cp}_{2}\right]$
$\mathrm{d}\left(\mathrm{t}_{1}-\mathrm{t}_{2}\right)=-\mathrm{Sdq}$
(3) [where, $\mathrm{S}=\left[1 / \mathrm{m}_{1} \mathrm{Cp}_{1}+1 / \mathrm{m}_{2} \mathrm{Cp}_{2}\right]$

Substituting the value of $d q$ from equation (1) in equation (3) : $d\left(t_{1}-t_{2}\right)=-S U\left(t_{1}-t_{2}\right) d A$
Or, $\mathrm{t}_{1}-\mathrm{t}_{2}=\Delta \mathrm{t}$
Therefore, $d(\Delta t) / \Delta t=$ SUdA
(4) $\left[\right.$ As $\left.\mathrm{dq}=\mathrm{U}\left(\mathrm{t}_{1}-\mathrm{t}_{2}\right) \mathrm{dA}\right]$

Or, $d(\Delta T)=-\operatorname{SU} \Delta T . d A=-\operatorname{Sdq}$
If $S$ and $U$ are constants, then integrating equation (4)
$\int \Delta \mathrm{t}_{2} \Delta \mathrm{t}_{1} \mathrm{~d}(\Delta \mathrm{t}) / \Delta \mathrm{t}=-\mathrm{SU} \int \mathrm{A}_{0} \mathrm{dA}$
$\ln \Delta \mathrm{t}_{2} / \Delta \mathrm{t}_{1}=\mathrm{e}-\mathrm{SUA}$
From equation (4) $d q=-d(\Delta t) / S$
Therefore, $\mathrm{q}=-1 / \mathrm{S}\left(\Delta \mathrm{t}_{2}-\Delta \mathrm{t}_{1}\right)$
Again, $q=U A \Delta t_{m}$
Therefore, $\Delta \mathrm{t}_{\mathrm{m}}=-1 / \operatorname{SUA}\left(\Delta \mathrm{t}_{2}-\Delta \mathrm{t}_{1}\right)=\Delta \mathrm{t}_{2}-\Delta \mathrm{t}_{1} / \ln \Delta \mathrm{t}_{2} / \Delta \mathrm{t}_{1}\left[\mathrm{As},-\mathrm{SUA}=\ln \Delta \mathrm{t}_{2} / \Delta \mathrm{t}_{1}\right]$
Therefore, Logarhythmic Mean Temperature Difference (LMTD) :-

$$
\Delta \mathrm{t}_{\mathrm{m}}=\Delta \mathrm{t}_{2}-\Delta \mathrm{t}_{1} / \ln \Delta \mathrm{t}_{2} / \Delta \mathrm{t}_{1}
$$

The ratio (r) of the arithmetic mean and logarithmic mean of two quantities $x$ and $y$ is given by : $\mathrm{r}=$ Arithmetic mean $(\mathrm{x}, \mathrm{y}) /$ Logarithmic mean $(\mathrm{x}, \mathrm{y})=(\mathrm{x}+\mathrm{y}) / 2 /(\mathrm{x}-\mathrm{y}) / \ln \mathrm{x} / \mathrm{y}$

$$
=(x / y+1) / 2(x / y-1) X \ln x / y .
$$

## Lecture 22

Topics Covered: Flow arrangement in heat exchangers, Variation of fluid temperature in heat exchangers



Heat exchange is an important unit operation that contributes to efficiency and safety of many processes. In this project you will evaluate performance of three different types of heat exchangers (tubular, plate, and shell \& tube). All these heat exchangers can be operated in both parallel- and counter-flow configurations. The heat exchange is performed between hot and cold water.
Degrees of Freedom and Performance Indicators
The following degrees of freedom can be varied in this experiment:

- Hot water feed temperature $\mathrm{T}_{\mathrm{h}, \mathrm{I}}$ and flow rate
-Cold water feed temperature $\mathrm{T}_{\mathrm{c}, \mathrm{I}}$ and flow rate
-Heat exchanger type (tubular, shell \& tube, or plate)
-Flow direction (parallel or counter-flow) You will investigate effects of these variable s on the following performance indicators:
- Return temperatures $\mathrm{T}_{\mathrm{h}, \mathrm{o}}$ and $\mathrm{T}_{\mathrm{c}, \mathrm{o}}$ of the hot and cold water
- Heat flow rate q
- Overall heat transfer coefficient U

Since all heat exchangers considered in this experiment have a single pass for both the hot and cold fluids, the discussion below is limited to single-pass heat exchangers. Qualitative dependence of the fluid temperature on position inside a single-pass heat exchanger is shown in Fig. 1-2. Logarithmic Mean Temperature Difference (LMTD) Method. The total heat transfer rate is

## Lecture 23

Topics Covered: Characteristics curves for co-current \& counter current flow, Heat transfer equipment: Double pipe \& shell \& tube heat Exchangers: Design and Principle

## Heat exchangers

A heat exchanger is any device that e_ects transfer of thermal energy between two _uids that are at different temperature. The two _uids do not come in direct contact but are separated by a solid surface or tube wall. Common heat exchangers include: Shell-and-tube (single pass or multi-pass) Flat-plate Finned tubes

Double pipe heat exchanger
The simplest form of an heat exchanger consists of two concentric cylindrical tubes, the double pipe heat exchanger: Heat transfer involves convection in each _uid and conduction through the wall separating the two fluids.


Direction of heat flow if fluild "a" is hotter than fluild "b"

Parallel or co-current flow
The fluids both flow in the same direction:


Counter-current flow
The fluids flow in opposite directions:


## Lecture 24

Topics Covered: Introduction of radiation heat transfer, Black body radiation, Problems of heat transfer

Thermal radiation is electromagnetic radiation generated by the thermal motion of charged particles in matter. All matter with a temperature greater than absolute zero emits thermal radiation. When the temperature of a body is greater than absolute zero, inter-atomic collisions cause the kinetic energy of the atoms or molecules to change. This results in charge-acceleration and/or dipole oscillation which produces electromagnetic radiation, and the wide spectrum of radiation reflects the wide spectrum of energies and accelerations that occur even at a single temperature.

Examples of thermal radiation include the visible light and infrared light emitted by an incandescent light bulb, the infrared radiation emitted by animals that is detectable with an infrared camera, and the cosmic microwave background radiation. Thermal radiation is different from thermal convection and thermal conduction-a person near a raging bonfire feels radiant heating from the fire, even if the surrounding air is very cold.

Sunlight is part of thermal radiation generated by the hot plasma of the Sun. The Earth also emits thermal radiation, but at a much lower intensity and different spectral distribution (infrared rather than visible) because it is cooler. The Earth's absorption of solar radiation, followed by its outgoing thermal radiation are the two most important processes that determine the temperature and climate of the Earth.

If a radiation-emitting object meets the physical characteristics of a black body in thermodynamic equilibrium, the radiation is called blackbody radiation. Planck's law describes the spectrum of blackbody radiation, which depends only on the object's temperature. Wien's displacement law determines the most likely frequency of the emitted radiation, and the Stefan-Boltzmann law gives the radiant intensity.

## The Stefan-Boltzmann law for blackbody

Josef Stefan based on experimental facts suggested that the total emissive power of a blackbody is proportional to the fourth power of the absolute temperature. Later, Ludwig Boltzmann derived the same using classical thermodynamics. Thus the eq. 7.12 is known as Stefan-Boltzmann law,

$$
E_{b}=\int_{0}^{\infty} E_{b \lambda}(\lambda, T) d \lambda
$$

$$
E_{b}=\sigma T^{4}
$$

where, $E_{b}$ is the emissive power of a blackbody, $T$ is absolute temperature, and $\sigma\left(=5.67 \times 10^{-8} \mathrm{~W} / \mathrm{m}^{2} / \mathrm{K}^{4}\right)$ is the Stefan-Boltzmann constant.

## Module IV

## Lecture 25

Topics Covered: Introduction of Module IV: Mechanical Operations/comminution, Size separation by settling: Free/hindered, Separation through filtration

## Mechanical Operations

## Size Reductions :

The term size reduction is applied to all ways in which particles of solids are cut or broken into smaller pieces. The most commonly used mechanisms for size reduction : (a) compression, (b) impact, (c) attrition or rubbing, (d) cutting.

The necessity of size reduction is - i) to increase the reactivity of the solis, ii) to permit separation of unwanted gradients, iii) to reduce the bulk of fibrous materials for easier handling, iv) to meet the stringent specifications regarding the size and shape of the particles of the commercial product, v) to achieve better mixing of the particles.

## Mechanism of Size Reduction :

For energy utilisation for size reduction only between 0.1 and $0.2 \%$ of the energy supplied to the machine appears as increased surface energy of the solids. The efficiency of the process depends on - i) the manner in which load is applied and its magnitude as well as ii) the nature of the force exerted (i.e., compression, impact etc.). If the applied force is insufficient for the elastic limit to be exceeded, the material is compressed and energy is stored in the particle. When the load is removed, the particles expands again to its original condition and energy appears as heat. When somewhat greater force is applied, the particles are first distorted and strained. As additional force is applied to the stressed particles, they are distorted beyond their ultimate strength and suddenly br5eak into fragments. As a result, new surfaces are formed. The extra energy required for the formation of new surface is supplied from the release of energy of stress, when particles break. The extra energy will appear as heat.

## Size Separation by Setting

1. Non-mechanical type - a) gravity settling tank, b) cone classifiers, c) liquid cyclone
2. Mechanical type - a) drug classifiers, b) rake classifiers, c) counter-current classifiers, d) hydroseparation, e) bowl classifiers
3. Magnetic separation
4. Electrostatic separation

## Lecture 26

Topics Covered: Rate equation, Incompressible \& compressible cakes, Relation between thickness of the cake \& volume of Filtrate

## Filtration :

In filtration, the flow of liquid is through a bed of granular material (cake). After some time the bed of cake will grow in thickness. Thus if filtration pressure is constant, the rate of flow will progressively diminish, whereas if the flow rate is constant, the pressure must be gradually increased. Since the particular farming, the cake are small and flow through the bed in slow, streamline conditions are almost invariably obtained and therefore the rate of filtration per unit area may be represented as: u $=1 / \mathrm{A} \cdot \mathrm{dv} / \mathrm{dt}=\mathrm{e} 3 / 5(1-\mathrm{e}) 2 \mathrm{~s} 2 .-\Delta \mathrm{p} / \mu \mathrm{l} \ldots \ldots \ldots \ldots \ldots$. (1) $[\mathrm{v}=$ the volume of filtrate which passed in time $t, A=$ cross-sectional area of filter cake, $u=$ superficial velocity of filtrate, $1=$ thickness of the cake, $\mathrm{S}=$ specific surface area of the particles, $\mathrm{e}=$ voidage, $\mu=$ viscosity of filtrate, $\Delta \mathrm{p}=$ applied pressure difference]

Some important reasons -
a) For any filtration pressure, the rate of flow is greatest at the beginning of the process, since the resistance is minimum.
b) High initial rates of filtration may cause the plugging of the pores of the filter cloth and cause a very high resistance to flow.
c) The orientation of the particles in the initial layers can appreciably influence the structure of the whole filter cake.

Filter cakes are of two different types - i) incompressible cakes, ii) compressible cakes.

For incompressible cakes, the resistance to flow for given volume of cake is not appreciably affected either by pressure difference across the cake or by the rate of deposition of material.

For incompressible cakes, e in equation (1) can be taken as constant and quantity e3/[5(1e)2s2] is then a property of the particles forming the cake and should be constant for a given material.

Therefore, $1 / \mathrm{A} . \mathrm{dv} / \mathrm{dt}=-\Delta \mathrm{p} / \mathrm{r} \mu \mathrm{l}$ (2) where, $\mathrm{r}=5(1-\mathrm{e}) 2 \mathrm{~s} 2 / \mathrm{e} 3$

## Relation Between Thickness of Cake and Volume of Filtrate :

In equation (2), the variables 1 and $v$ are connected and the relation between them can be obtained by making a material balance between the slurry and in the cake.

Mass of solids in the filter cake $=(1-\mathrm{e})$ A.1. $\rho_{\mathrm{s}}\left[\rho_{\mathrm{s}}=\right.$ density of solid $]$
Mass of the liquid retained in the filter cake $=$ e.A.l. $\rho[\rho=$ density of liquid]
If J is the mass fraction of solids in the original suspension, then -
$(1-\mathrm{e}) . \mathrm{A} .1 . \rho_{\mathrm{s}}=(\mathrm{V}+\mathrm{e} . \mathrm{A} .1) \cdot \rho \cdot \mathrm{J} /(1-\mathrm{J})$

Or, $(1-\mathrm{J})(1-\mathrm{e})$ A.l. $\rho_{\mathrm{s}}=$ V. ..J + e.A.1. $\rho . J ~$
Or, $(1-\mathrm{J})(1-\mathrm{e})$ A.l. $\rho_{\mathrm{s}}-$ e.A.l...J $=$ V.p.J
Or, e.A $\left[(1-\mathrm{J})(1-\mathrm{e}) . \rho_{\mathrm{s}}-\right.$ e.p.J] $=$ V.. J
Or, $1=$ V.p.J $/ \mathrm{A}\left[(1-\mathrm{J})(1-\mathrm{e}) \rho_{\mathrm{s}}-\right.$ e. $\left.\rho . \mathrm{J}\right]$
Or, $V=$ A.l. $\left[(1-J)(1-e) \rho_{s}-\right.$ e.p.J] $/ \mathrm{J} . \rho$

If $v$ is the volume of the cake deposited by unit volume of filtrate, then -
$\mathrm{v}=1 . \mathrm{A} / \mathrm{V}$ or, $1=\mathrm{vV} / \mathrm{A}$

Substituting the value of 1 in equation (5, we -
$\mathrm{V}=\mathrm{J} . \rho /(1-\mathrm{J})(1-\mathrm{e}) \rho_{\mathrm{s}}-$ e. $\rho . \mathrm{J}$
Substituting I in equation (2) - $1 / \mathrm{A} . \mathrm{dv} / \mathrm{dt}=(-\Delta \mathrm{p}) / \mathrm{r} \cdot \mu \mathrm{A} \cdot \mathrm{A} / \mathrm{VV}$
Or, dv/dt $=A^{2}(-\Delta p) / r . \mu . v . V$
Equation (8) shows the basic relation between $(-\Delta p), V$ and $t$.

Two important operation will now be considered : 1) filtration at constant rate and 2) filtration at constant pressure.

## Lecture 27

Topics Covered: Constant rate \& constant Pressure filtration, Mechanical Operations: Size reduction methods: definition

## Constant Rate Filtration

In this method a constant rate is maintained by starting at low pressure and continuously increasing the pressure to overcome the increasing resistance of the cake until maximum pressure is reached at the end of the run.

For filtration at constant rate : $\mathrm{dv} / \mathrm{dt}=\mathrm{V} / \mathrm{t}=$ constant

So, from equation (8), $\quad \mathrm{V} / \mathrm{t}=\mathrm{A} 2(-\Delta \mathrm{p}) / \mathrm{r} . \mu . \mathrm{v} . \mathrm{V}$

Or, $\quad \mathrm{t} / \mathrm{V}=\mathrm{r} . \mu . \mathrm{v} . \mathrm{V} / \mathrm{A} 2(-\Delta \mathrm{p})$
Here, $(-\Delta \mathrm{p})$ is directly proportional to V

## Constant Pressure Filtration

In this method, full pressure is applied at the start of filtration and this pressure is maintainred constant throughout the run.

Disadvantages : i) If the initial pressure is high, the first particle caught will be compacted into tight mass, that fill the pressure and results a low rate of filtration.
ii) If the sludge is non-homogenous, i.e., if it contains both crystalline and colloidal particles, then high initial pressure tends to force the colloidal portion of the sludge into the interstices between granular portions and greatly decrease the rate.

On the other hand, if the initial pressure is low, the initial layer of precipitate will be more open so that rates of filtration will be higher and this layer will not be crowded into the filter cloth and can be easily removed from the cloth.

For filtration at constant pressure, equation (8) becomes -

$$
\int_{\mathrm{vdv}}=\mathrm{A}^{2}(-\Delta \mathrm{p}) / \mathrm{r} . \mu . \mathrm{v} \cdot \int \mathrm{dt}
$$

$\mathrm{V}^{2} / 2=\mathrm{A}^{2}(-\Delta \mathrm{p}) . \mathrm{t} / \mathrm{r} \cdot \mu \cdot \mathrm{v}$
Or, $\quad V^{2} / t=2 A^{2}(-\Delta p) / r . \mu . v$ $\qquad$
Or, $\quad t / V^{2}=$ r. $\mu . v / 2 A^{2}(-\Delta p)$
Or, $\quad \mathrm{t} / \mathrm{V}=\mathrm{r} . \mu . \mathrm{v} . \mathrm{V} / 2 \mathrm{~A}^{2}(-\Delta \mathrm{p})$
So, for constant pressure, there is a linear relation between $V^{2}$ and $t$ or between $t / v$ and $V$.

## Lecture 28

Topics Covered: Classification \& mechanism, Crushing Efficiency: definition, working formula

## Crushing Efficiency

The ratio of the surface energy created by crushing to the energy absorbed by solid is called the crushing efficiency $\left(\mathrm{n}_{\mathrm{c}}\right)$.

So, $n_{c}=e_{s}\left(A w_{b}-A w_{a}\right)$

Or, $w_{n}=e_{s}\left(A w_{b}-A w_{a}\right) / n_{c} \ldots \ldots \ldots \ldots$ (1) $\left[e_{s}=\right.$ surface energy per unit area, $A w_{a}=$ area per unit mass of feed, $\mathrm{Aw}_{\mathrm{b}}=$ area per unit mass of product, $\mathrm{w}_{\mathrm{n}}=$ energy absorbed y unit mass of material]

The crushing efficiency is therefore low and cits value ranges from $0.06-1 \%$. The total energy absorbed by the solid (wn) < that feed to machine, because a part of total energy input (w) is used to overcome friction in bearings and other moving parts.

So, mechanical efficiency $\left(\mathrm{n}_{\mathrm{m}}\right)=\mathrm{w}_{\mathrm{n}} / \mathrm{w}$
Or, $w=w_{n} / n_{m}$
Or, $w=e_{s}\left(A w_{b}-A w_{a}\right) / n_{c} \cdot n_{m}$
If ' $m$ ' is the feed rate, the power required by the machine is -
$\mathrm{P}=\mathrm{w} \cdot \mathrm{m}=\mathrm{m}_{\mathrm{e}} \mathrm{e}_{\mathrm{s}}\left(\mathrm{Aw}_{\mathrm{b}}-\mathrm{Aw}_{\mathrm{a}}\right) / \mathrm{n}_{\mathrm{c}} \cdot \mathrm{n}_{\mathrm{m}}$
But, $\quad A w_{b}=6 / \Phi_{b} \cdot D_{\text {sb }} . \rho_{P}$ and $A w_{a}=6 / \Phi_{a} \cdot D s_{a} \cdot \rho_{p}\left[D_{s b}\right.$ and $D_{s a}=$ volume surface mean diameter at product and feed respectively, $\rho_{\mathrm{P}}=$ density of particle, $\Phi_{\mathrm{a}}, \Phi_{\mathrm{b}}=$ sphericity of feed and product respectively]

$$
\text { So, } P=6 \mathrm{me}_{s} / n_{\mathrm{c}} \cdot \mathrm{n}_{\mathrm{m}} \cdot \rho_{\mathrm{P}}\left[1 / \Phi_{\mathrm{b}} \cdot D_{\mathrm{sb}}-1 / \Phi_{\mathrm{a}} \cdot \mathrm{D}_{\mathrm{sa}}\right]
$$

Sphericity $\Phi=$ surface area of the sphere/actual surface are of the particle
For sphere $\mathrm{S}_{\mathrm{P}} / \mathrm{V}_{\mathrm{P}}=1 \pi \mathrm{D}_{\mathrm{p}}{ }^{2} / \Phi .1 / 6 \pi \cdot \mathrm{D}_{\mathrm{P}}{ }^{3}=6 / \Phi . \mathrm{D}_{\mathrm{P}}$

So, $\quad \mathrm{S}_{\mathrm{P}} / \mathrm{V}_{\mathrm{P}}=A \mathrm{w}_{\mathrm{b}} / \mathrm{V}_{\mathrm{P}} / \mathrm{m}_{\mathrm{P}}=6 / \Phi_{\mathrm{b}} \cdot \mathrm{D}_{\mathrm{sb}}$

Therefore, $A w_{b}=6 / \Phi_{b} \cdot D_{s b} \cdot \rho_{P}$

## Lecture 29

Topics Covered: Energy requirement for size reduction: Different laws, Rittinger's law, Kick's Law, Bond's Law, Calculation of work index

## Energy for Size Reduction

It is impossible to estimate accurately the amount of energy required in order to effect a size reduction of a given material, but a number of empirical laws have been put forward :

1. Rittinger's Law : "Work required for crushing is proportional to the new surface created". This law is equivalent to the statement that the crushing efficiency is constant and for a given machine and feed material is independent of the sizes of the feed and product.

If $\Phi_{\mathrm{a}}=\Phi_{\mathrm{b}}=1$ and mechanical efficiency is constant, the Rittinger's law can be written as -
$\mathrm{P} / \mathrm{m}=\mathrm{K}_{\mathrm{r}}\left[1 / \mathrm{D}_{\mathrm{sb}}-1 / \mathrm{D}_{\mathrm{sa}}\right]$

Or, $P / m=K_{r} f_{c}\left[1 / D_{s b}-1 / D_{s a}\right] \quad\left\{K_{r}=6 e_{s} / n_{c} n_{m} \rho_{\mathrm{P}}=\right.$ constant, $\mathrm{f}_{\mathrm{c}}=$ crushing strength of material\}

This law is applicable where, i) new surface is created and ii) holds for fine grinding where increase in surface area per unit mass is large.
2. Kick's Law : This law is based on the stress analysis of plastic deformation within the elastic limit, which states that the work required for crushing a given mass of material is constant for the same reduction ratio.

Reduction ratio for crusher is often expressed as the ratio of feed opening to the discharge opening or ratio of initial particle size to final particle size.

This leads to the relation -

$$
\mathrm{P} / \mathrm{m}=\mathrm{K}_{\mathrm{K}} \cdot \ln \cdot \mathrm{D}_{\mathrm{sa}} / \mathrm{D}_{\mathrm{sb}}
$$

Or, $P / m=K_{K} \cdot f_{c} \cdot \ln D_{s a} / D_{s b}\left[K_{K}=6 e_{s} / n_{c} n_{m} \rho_{P}=\right.$ constant, $f_{c}=$ crushing strength of material $]$
This law holds good for i) coarse crushing, ii) amount of surface produced is less.
So, a generalised relation in differential equation gives -
$\mathrm{d}(\mathrm{P} / \mathrm{m})=-\mathrm{KdD} / \mathrm{D}_{\mathrm{s}}{ }^{\mathrm{n}}$
Solution of equation (1) for $\mathrm{n}=1$ and $\mathrm{n}=2$ gives Kick's law and Ritinger's law respectively.

So, for $\mathrm{n}=1$
$\mathrm{d}(\mathrm{P} / \mathrm{m})=-\mathrm{KdD} / \mathrm{D}_{\mathrm{s}}{ }^{\mathrm{n}}$
$\int \mathrm{D}(\mathrm{P} / \mathrm{m})=-\mathrm{K} \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{dD}_{\mathrm{s}} / \mathrm{D}_{\mathrm{s}}$

This is Kick's law.

For, $\mathrm{n}=2$
$\int \mathrm{d}(\mathrm{P} / \mathrm{m})=-\mathrm{K} \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{dD}_{\mathrm{s}} / \mathrm{D}_{\mathrm{s}}{ }^{2}$
$\left.(\mathrm{P} / \mathrm{m})=\mathrm{K} \int 1 / \mathrm{D}_{\mathrm{sb}}-1 / \mathrm{D}_{\mathrm{sa}}\right]$
This is Rittinger's law.
3. Bond's Law : The law is more realistic for estimating the power required for crushing and grinding. It is defined as the work required to form particles of size $D_{P}$ from very large size of feed is proportional to the square root of surface-to-volume ratio of the product.

As because, $\mathrm{S}_{\mathrm{P}} / \mathrm{V}_{\mathrm{P}}=6 / \Phi_{\mathrm{s}} \mathrm{D}_{\mathrm{P}}$
Therefore, $\mathrm{P} / \mathrm{m}=\mathrm{K}_{\mathrm{b}} / / \mathrm{D}_{\mathrm{P}}$, where $\mathrm{K}_{\mathrm{b}}=6 \mathrm{e}_{\mathrm{s}} / \mathrm{n}_{\mathrm{c}} \mathrm{n}_{\mathrm{m}} \rho_{\mathrm{P}}=$ constant

Kb depends on type of machine and the material to be crushed.

The generalised differential equation leads to Bond's law for $\mathrm{n}=1.5$ is -
$\mathrm{d}(\mathrm{P} / \mathrm{m})=-\mathrm{KdD} / \mathrm{D}_{\mathrm{s}}{ }^{\mathrm{n}}$
$\int \mathrm{d}(\mathrm{P} / \mathrm{m})=-\mathrm{K} \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{dD}_{\mathrm{s}} / \mathrm{D}_{\mathrm{s}}{ }^{1.5}$
$\mathrm{a}=\mathrm{o}$, since feed is of infinite size.

So, $\mathrm{P} / \mathrm{m}=-\mathrm{K}\left[1 /(-0.5) \mathrm{D}_{\mathrm{s} 0.5}\right] \mathrm{b}_{0}$
Or, $\mathrm{P} / \mathrm{m}=\mathrm{K} / 0.5\left[1 / \sqrt{ } \mathrm{D}_{\mathrm{sb}}\right]$
Or, $\mathrm{P} / \mathrm{m}=\mathrm{K}_{\mathrm{b}} \cdot 1 / \sqrt{ } \mathrm{D}_{\mathrm{sb}}$
$\mathrm{P} / \mathrm{m}=\mathrm{K}_{\mathrm{b}} .1 / \sqrt{ } \mathrm{D}_{\mathrm{P}} ; \quad \mathrm{K}_{\mathrm{b}}=\mathrm{K} / 0.5$
The work index $\mathrm{W}_{\mathrm{i}}$ is defined as the gross energy requirement in kw-hours per ton of feed needed to reduce a very large feed to such a size that $80 \%$ of the product passes through $100 \mu \mathrm{~m}$ screen. So, $\mathrm{W}_{\mathrm{i}}$ $=\mathrm{KWH} / \mathrm{T}$

According to the Bond's law $\mathrm{P} / \mathrm{m}=\mathrm{K}_{\mathrm{b}} / \sqrt{ } \mathrm{D}_{\mathrm{P}}$ [Here $\mathrm{D}_{\mathrm{P}}$ is in $m m$ ]
So, $K_{b}=\sqrt{ } D_{\mathrm{P}} \times \mathrm{p} / \mathrm{m}$
Or, $\quad K_{b}=\sqrt{ } D_{P} \times K W / T /$ hour $=\sqrt{ } D_{P} \times K W H / T=\sqrt{ } D_{P} \times W_{i}$
But from the definition of $W_{i}$, product must pass through $100 \mu \mathrm{~m}$ screen. $\mathrm{D}_{\mathrm{P}}$ is taken in mm .
So, $\mathrm{K}_{\mathrm{b}}=\sqrt{ } 100 \times 10-3 \times \mathrm{W}_{\mathrm{i}}=0.3162 \mathrm{~W}_{\mathrm{i}}$
If $80 \%$ of the feed passes through a mesh of $D_{\mathrm{Pa}} \mathrm{mm}$ and $80 \%$ product passes through screen of $\mathrm{D}_{\mathrm{Pb}} \mathrm{mm}$, then, Bond's law becomes $-\mathrm{P} / \mathrm{m}=0.3162 \mathrm{~W}_{\mathrm{i}}\left[1 / \sqrt{ } \mathrm{D}_{\mathrm{Pb}}-1 / \sqrt{ } \mathrm{D}_{\mathrm{Pa}}\right.$

## Lecture 30

Topics Covered: Discussion on different size reduction equipment: Classification, Jaw crusher, Selection of crushing rolls, Angle of nip

## Size Reduction Equipment

Size reduction equipments are divided into crusher, grinder, ultrafine grinder, and cutteretc.

1. Coarse Crusher : A] Jaw crusher - i) Blake, ii) Dodge, ii)Overhead eccentric

B] Gyratory crusher i) Primary, ii)Secondary, iii) Cone
2. Intermediate and Fine Crusher : A] Heavy duty impact mills - i)hammer mill, ii) stamp battery, iii)squirrel cage impactors.

B \} Roll Crusher i)Smooth roll, ii)Toothed roll.

C]Tumbling mills i) Ball mill, ii)Tube mill, iii) Rod mill, iv)Compartment mill
3. Ultra-fine Grinders
i) Hammer mills with internal classification, ii)Fluid energy mill
4. Colloid mill
5. Cutting machine -i )knife cutters, ii) Slitters, iii) Dicers
$\mathrm{A}_{1}=$ centre of one roll, $\mathrm{A}_{2}=$ Centre of other roll, $\mathrm{B}=$ spherical particle caught between rolls, $r=$ force acting on the particles, $\alpha=$ angle of force $r$ with $A_{1}$ and $A_{2}$

Let t be the force which tends to pull the particle between the rolls. t depends on r and coefficient of friction between the material and roll surface.

As because OC is perpendicular to the direction of force $\mathrm{r}, \mathrm{LCOD}=\alpha$

So, $t=\mu . r \quad[\mu=$ coefficient of friction $]$
Again, $\mathrm{m}=\mathrm{r} \cdot \sin \alpha$ and $\mathrm{e}=\mathrm{t} \cdot \cos \alpha=\mu \cdot \mathrm{r} \cdot \cos \alpha$
Force e tends to draw the particle between the rolls, while force $m$ tends to eject the particle between the rolls and crush. So, e $>\mathrm{m}$.
$\mu$. R. $\cos \alpha>$ r $\cdot \sin \alpha$
Or, $\mu=\tan \alpha$

The angle OEF, called as angle of nip

So, angle OEF $=2 \alpha$
If, $\mathrm{R}=$ radius of feed particle, $\mathrm{r}=$ radius of the roll, $\mathrm{d}=$ radius of the largest possible particle in the product.

Then in $\triangle \mathrm{ABC}$, angle $\mathrm{CAB}=\alpha, \mathrm{AB}=\mathrm{r}+\mathrm{d}, \mathrm{AC}=\mathrm{r}+\mathrm{R}$
$\cos \alpha=\mathrm{AB} / \mathrm{AC}=\mathrm{r}+\mathrm{d} / \mathrm{r}+\mathrm{R}$

As for average condition $\alpha=16^{\circ}$
$\operatorname{Cos} \alpha=0.961$,

$$
\text { So, } 0.961=\mathrm{r}+\mathrm{d} / \mathrm{r}+\mathrm{R}
$$

## Lecture 31

Topics Covered: Gyratory crusher, Roll crusher, Fine crusher, Ultra fine grinders, Cutting machines, Introduction on Ball Mill, Speed of Rotation

Gyratory crusher


## Roll crusher



## Ball Mill

## Speed of Rotation

At low speeds, the balls simply roll over one one another and little crushing is obtained. At slightly higher speeds, the balls are lifted to more height. The minimum speed at which the balls are carried around in this manner is called the critical speed of the mill and under this conditions, there will be no resultant force acting on the ball, i.e., the centrifugal force will be exactly equal to the weight of the ball.

So, $\mathrm{RW}_{\mathrm{e}}{ }^{2}=\mathrm{g}$
Or, $\mathrm{W}_{\mathrm{c}}=\sqrt{ } / \mathrm{R} \quad\left[\mathrm{W}_{\mathrm{c}}=\right.$ angular velocity of rotating mill $=2 \pi \mathrm{nc}, \mathrm{R}=$ radius of the mill less the radius of particle.

## Lecture 32

Topics Covered: Problems on Jaw crusher \& Ball Mill, Separation of Solids by Screening: Material Balance, Screen Effectiveness. Discussion of Equipment

Mechanical screening, often just called screening, is the practice of taking granulated ore material and separating it into multiple grades by particle size.

This practice occurs in a variety of industries such as mining and mineral processing, agriculture, pharmaceutical, food, plastics, and recycling.

## Process



Model of Screening Process

A screening machine consist of a drive that induces vibration, a screen media that causes particle separation, and a deck which holds the screen media and the drive and is the mode of transport for the vibration.

There are physical factors that makes screening practical. For example, vibration, g force, bed density, and material shape all facilitate the rate or cut. Electrostatic forces can also hinder screening efficiency in way of water attraction causing sticking or plugging, or very dry material generate a charge that causes it to attract to the screen itself.

As with any industrial process there is a group of terms that identify and define what screening is. Terms like blinding, contamination, frequency, amplitude, and others describe the basic characteristics of screening, and those characteristics in turn shape the overall method of dry or wet screening.

In addition, the way a deck is vibrated differentiates screens. Different types of motion have their advantages and disadvantages. In addition media types also have their different properties that lead to advantages and disadvantages.

Finally, there are issues and problems associated with screening. Screen tearing, contamination, blinding, and dampening all affect screening efficiency.

## Physical principles

- Vibration - either sinusoidal vibration or gyratory vibration.
- Sinusoidal Vibration occurs at an angled plane relative to the horizontal. The vibration is in a wave pattern determined by frequency and amplitude.
- Gyratory Vibration occurs at near level plane at low angles in a reciprocating side to side motion.
- Gravity - This physical interaction is after material is thrown from the screen causing it to fall to a lower level. Gravity also pulls the particles through the screen media.
- Density - The density of the material relates to material stratification.
- Electrostatic Force - This force applies to screening when particles are extremely dry or is wet.


## Types of mechanical screening

There are a number of types of mechanical screening equipment that cause segregation. These types are based on the motion of the machine through its motor drive.

- Circle-throw vibrating equipment - This type of equipment has an eccentric shaft that causes the frame of the shaker to lurch at a given angle. This lurching action literally throws the material forward and up. As the machine returns to its base state the material falls by gravity to physically
lower level. This type of screening is used also in mining operations for large material with sizes that range from six inches to +20 mesh.
- High frequency vibrating equipment - This type of equipment drives the screen cloth only. Unlike above the frame of the equipment is fixed and only the screen vibrates. However, this equipment is similar to the above such that it still throws material off of it and allows the particles to cascade down the screen cloth. These screens are for sizes smaller than $1 / 8$ of an inch to +150 mesh.
- Gyratory equipment - This type of equipment differs from the above two such that the machine gyrates in a circular motion at a near level plane at low angles. The drive is an eccentric gear box or eccentric weights.
- Chemical Tumbler Screener
- Trommel screens - Does not require vibrations, instead, material is fed into a horizontal rotating drum with screen panels around the diameter of the drum.


## MODEL QUESTIONS

## 1. Multiple Choice Question (MCQ)

(i) When warm and cold liquids are mixed, the heat transfer is mainly by
(a) conduction (b) convection (c) radiation (d) both (a) and (b)
(ii) The value of Stefan- Boltzman constant in SI unit is
(a) $5.6697 \times 10^{-8} \mathrm{~W} /\left(\mathrm{m}^{2}{ }^{\circ} \mathrm{K}^{4}\right)$
(b) $0.1714 \times 10^{-8} \mathrm{~W} /\left(\mathrm{m}^{2}{ }^{\circ} \mathrm{K}^{4}\right)$
(c) $5.6697 \times 10^{-8} \mathrm{Kcal} /\left(\mathrm{m}^{2}{ }^{\circ} \mathrm{K}^{4}\right)$
(d) $0.1714 \times 10^{-8} \mathrm{Kcal} /\left(\mathrm{m}^{2}{ }^{\mathrm{o}} \mathrm{K}^{4}\right)$
(iii) Which of the following is a fine Crusher?
(a) Black Jaw Crusher (b) Gyratory Crusher (c) Toothhed Roll Crusher (d) Dodge Jaw Crusher.
(iv) Which of the following is directly concerned with heat transfer?
(a) Strouhal Number (b) Sherwood Number (c) Euler Number (d) Grashoff Number.
(v) Maximum rate of heat transfer is achieved by
(a) co-current flow (b) counter current flow (c) turbulent flow (d) laminar flow.
(vi) In counter flow compared to parallel flow
(a) LMTD is greater (b) Less surface is required for a given heat transfer rate (c) Both (a) and (b) (d) More surface is required for a given heat transfer rate
(vii) The heat transfer co-efficient in film type condensation is
(a) greater than that for drop wise condensation (b) less than that for drop wise condensation (c) same as that for drop wise condensation (d) half of that for drop wise condensation (vii) The ratio of kinematic viscosity to thermal diffusivity is called the
(a) Peclet number
(b) Prandtl number
(c) Stanton number
(d) Nusselt number
(ix) During a refrigeration cycle, heat is rejected by the refrigerant in a (a) evaporator, (b) condenser, (c) expansion valve, (d) compressor.
(x) The thickness of thermal and hydrodynamic boundary layer is equal if Prandtl number is (a) greater than one (b) less than one (c) equal to one (d) equal to Nusselt number.
(xi) Thermal diffusivity is a (a) function of temperature (b) all of these (c) physical property of a substance (d) dimensionless parameter.
(xii) A perfect black body is one which (a) is black in colour (b) absorbs heat radiations of all wave lengths falling on it (c) reflects all the heat radiations (d) transmits the heat radiations
(xiii) The unit of overall heat transfer coefficient is (a) $\mathrm{kcal} / \mathrm{m}^{2}$ (b) kcal/hr. ${ }^{0} \mathrm{C}$, (c) $\mathrm{kcal} / \mathrm{m}^{2} \mathrm{hr}{ }^{0} \mathrm{C}$, (d) kcal/m $\mathrm{hr}^{0} \mathrm{C}$.
(xiv) The process of increasing the rate of sedimentation in a suspension by adding some chemical is (a) Filtration, (b) crystallisation (c)loading (d) condensation.
(xv) Butter is separated from curd by the process of (a) filtration (b) heating (c) churning (d) sieving
(xvi) If in a process the total number of variables are $n$ and number of repeating variables are $j$ then the number of dimensionless groups (i) according to Buckingham pi method is
(a) $n+j$ (b) $n-j(c) n x j(d) n / j$
(xvii) Which one is dimensionless?
(a) $d / \rho$ (b) $d v / \mu$ (c) $d v \rho / \mu$ (d) All of these.
(xviii) Which one is the unit of viscosity?
(a) $\mathrm{kg} / \mathrm{m}-\mathrm{sec}$ (b) $\mathrm{m} / \mathrm{kg}-\mathrm{sec}$ (c) $\mathrm{kg} / \mathrm{m}-\sec ^{2}$ (d) none of these.
(xix) Kinematic viscosity is defined as
(a) $D / \rho$ (b) $\mu / \rho D$ (c) $\mu / \rho$ (d) $\rho / \mu$
( xx ) If the value of $\mathrm{N}_{\mathrm{Re}}$ is 3200 then the fluid flow is
(a) laminar (b) turbulent (c) transition (d) none of these.
(xxi) Hagen- Poiseuille's equation is applicable for friction losses through circular tubes in
(a) laminar flow (b) turbulent flow (c) steady flow (d) none of these.
(xxii) Fanning equation is applicable for
(a) laminar (b) turbulent (c) transition (d) none of these type of flow
(xxiii) The friction factor $f$ is a function of
(a) Reynold's number (b) relative roughness (c) both (a) and (b) (d) none of these
(xxiv) The kinematic viscosity of water having viscosity 0.8 cP is
(a) 8 stokes (b) 0.8 stokes (c) 0.08 stokes (d) 0.008 stokes
(xxv) The linear velocity of water flowing through a pipe of $0.1 \mathrm{~cm}^{2}$ area at a flow rate of $0.01 \mathrm{~L} / \mathrm{min}$ is
(a) $1 \mathrm{~cm} / \mathrm{min}$ (b) $10 \mathrm{~cm} / \mathrm{min}$ (c) $100 \mathrm{~cm} / \mathrm{min}$ (d) $1000 \mathrm{~cm} / \mathrm{min}$
(xxvi) The Fanning's friction factor is a
(a) dimensionless quantity (b) related to the pressure drop due to friction (c) both (a) and (b) are correct
(d) neither (a) nor (b) are correct.
(xxvii) Average fluid velocity is measured by
(a) venturimeter (b) pitot tube (c) both (a) and (b) (d) neither (a) nor (b).
(xxviii) The relation between Poise and Stoke is
(a) Stoke $=$ Poise $/ \operatorname{Density~(g/cc)~(b)~Stoke~}=$ Poisex Density (g/cc) (c) Stoke $=$ Density (g/cc) / Poise (d) Stoke $=$ Density $(\mathrm{g} / \mathrm{cc}) \times$ poise
(xxix) The local velocity of fluid is measured by
(a) orificemeter (b) venturimeter (c) pitot tube (d) all of these.
(xxx) Cavitation occurs in a centrifugal pump when
(a) the suction pressure < vapour pressure of the liquid at that temperature
(b) the suction pressure > vapour pressure of the liquid at that temperature
(c) the suction pressure $=$ vapour pressure
(d) the suction pressure $=$ developed head

## Short Question (SQ)

2. A thick-walled cylinder tubing of hard rubber having an inside radius of 20 mm is being used as temporary cooling coil in a bath. Ice water is flowing rapidly through inside tube and the inside wall temperature is 274.9 K . The outside surface temperature is 297.1 K . A total of 14.65 W must be removed from the bath by cooling coil. How many $m$ of tubing are required? 5
3. Explain the mechanism of boiling heat transfer.
4. What do you understand by mean area of heat transfer for conduction with different cross sectional areas?
5. Explain the variation of fluid temperature in heat exchangers.
6. A cold storage room is constructed of an inner layer of 12.7 mm of pine, a middle layer of 101.6 mm of cork board and an outer layer of 76.2 mm of concrete. The wall surface temperature is 255.4 K inside the cold room and 297.1 K at the outside surface of the concrete. Thermal conductivities of pine, cork board \& concrete are $0.151,0.0433,0.762 \mathrm{~W} / \mathrm{mK}$. Calculate the heat loss in W for $1 \mathrm{~m}^{2}$. 5
7. Hot water flowing through a tube with a diameter of 16 mm . and a length of 2 m . transfers heat through the wall of the tube to the surrounding medium. The rate of flow of water through the tube is $0.01 \mathrm{Kg} / \mathrm{s}$, the water inlet temperature is $80^{\circ} \mathrm{C}$ \& outlet temperature is $36^{\circ} \mathrm{C} \&$ the mean temperature of the wall of the tube is $24^{\circ} \mathrm{C}$. given, $\mathrm{C}_{\mathrm{p}}=4.178 \mathrm{KJ} / \mathrm{Kg} \mathrm{K}$ for water.

Calculate the heat transfer coefficient based on
(i) the arithmetic mean difference
(ii) the logarithmic mean difference between the temperature of the water \& the wall of the tube. 5
9. Define the non Newtonian fluids with Power law. The space between horizontal and parallel plates 1.25 cm apart is filled with an oil of viscosity 14 poise. Calculate the shear stress in the oil, if the upper plate is moving with a velocity of $25 \mathrm{~cm} / \mathrm{sec}$.
10. Define Laminar and Turbulent flow. An oil having kinematic viscosity 21.4 stokes is flowing through a pipe of 300 mm diameter. Determine the type of flow if the discharge through the pipe is $1.5 \mathrm{~L} / \mathrm{sec}$. $2+3=5$
11. What is critical velocity? Calculate the critical velocity of water flowing through 25 mm id pipe. Take viscosity of water $=0.008$ poise. What are the limitations of Bernoulli's equation? $1+2+2=5$
12. Water flows through an orifice of diameter 25 mm installed in a 75 mm diameter pipe at a rate of 500 $\mathrm{cc} / \mathrm{sec}$. What will be the difference in level of carbon tetrachloride manometer. Take density of carbon $\begin{array}{lllllll}\text { tetrachloride } & 1.6 & \mathrm{~g} / \mathrm{cc} & \text { and } & \text { coefficient } & \text { of } & \text { orificemeter }\end{array}$ 5
13. Prove that $\mathrm{f}=16 / \mathrm{N}_{\mathrm{Re}}$ for laminar flow. Find the type of flow of an oil of specific gravity $0.9 \&$ dynamic viscosity 20 poise flowing through a pipe of diameter 20 cm \& giving a discharge of $10 \mathrm{lit} / \mathrm{sec}$. $2+3$
14. How loss of pressure in pipe fittings are calculated? In the flow system there are 3 gate valves, each equivalent to 10 pipe diameter and fittings equivalent to 100 pipe diameter. What will be the total equivalent length of piping system if the pipe diameter is 40 mm and pipeline is 200 m long. $2+3=5$
15. Water is flowing at velocity $2.5 \mathrm{~m} / \mathrm{sec}$ through 20 mm id pipe. Find out friction factor if viscosity of water is 0.008 poise. What are Thixotropic and Rheopectic fluids?

$$
3+2=5
$$

## Long Answer Type Question (LQ)

16. (a) Name the different heat exchangers used in food industries.
(b) A shell and tube heat exchanger is using tubes of 114 mm outside diameter. If the clearance between the tubes is 6 mm , calculate the tube pitch.
(c) A process is under consideration in which large meat sausages are to be processed in an autoclave. The sausage may be taken as thermally equivalent to a cylinder of 30 cm long and 10 cm in diameter. If the sausages are initially at a temperature of $21^{\circ} \mathrm{C}$ and the temperature in the autoclave at $116^{\circ} \mathrm{C}$, estimate the temperature of the sausage at its centre 2 hr after it has been placed in the autoclave. Assume that the thermal conductivity of the sausage is $0.48 \mathrm{~J} / \mathrm{m}-\mathrm{s}^{-}{ }^{0} \mathrm{C}$, that its specific gravity is 1.07 and its specific heat is $3350 \mathrm{k} / \mathrm{kg}-{ }^{0} \mathrm{C}$. The surface heat transfer coefficient in the autoclave to the surface of the sauge is $1200 \mathrm{~J} / \mathrm{m}^{2}-\mathrm{s}-{ }^{0} \mathrm{C}$. \{Given $\left(\mathrm{T}-\mathrm{T}_{\mathrm{o}}\right) /\left(\mathrm{T}_{\mathrm{i}}-\mathrm{T}_{\mathrm{o}}\right)=0.175$ for cylinder and $\left(\mathrm{T}-\mathrm{T}_{\mathrm{o}}\right) /\left(\mathrm{T}_{\mathrm{i}}-\mathrm{T}_{\mathrm{o}}\right)=0.98$ for slab\} $3+2+10$
17. Discuss the mechanism of condensation heat transfer. Determine the heat transfer coefficient for water flowing in a tube of 16 mm . diameter at a velocity of $3 \mathrm{~m} / \mathrm{s}$. The temperature of the tube is $24^{\circ} \mathrm{C}$, the water enters at $80^{\circ} \mathrm{C}$ \& leaves at $36^{\circ} \mathrm{C}$ using the Dittus - Boelter equation.
where $\mathrm{a}=0.3$ and the properties of water at the arithmetic mean bulk temperature are $\mathrm{P}=984.1$ $\mathrm{kg} / \mathrm{m}^{3}, \mathrm{C}_{\mathrm{p}}=4178 \mathrm{~J} / \mathrm{Kg} \mathrm{K}, \mu=485 \times 10^{-6} \mathrm{P}_{\mathrm{a}}-\mathrm{S}, \mathrm{K}=0.657 \mathrm{~W} / \mathrm{m}-\mathrm{k} \quad 5+10$
18. Mention the name of different comminuting equipments. What do you understand by crushing efficiency?

A certain crusher accepts a feed of rock having volume- surface mean diameter of 0.75 inches \& discharges a product of diameter of 0.20 inches. The power required to crush $15 \mathrm{~T} / \mathrm{hr}$ in $12 \mathrm{~h} . \mathrm{p}$. What
should be the power consumption if the capacity is reduced to $10 \mathrm{~T} / \mathrm{hr}$ \& volume surface mean diameter to 0.15 inches by using Rittinger's law. $3+4+8$
19. Explain the terms Constant rate and constant pressure filtration. A plate and frame filter press, filtering a slurry and gave a total of 8 m 3 of filtrate in 1800 sec and $11 \mathrm{~m}^{3}$ in 3600 sec when filtration was stopped. Estimate the washing time in sec if $3 \mathrm{~m}^{3}$ of wash water was used. The resistance of the cloth can be neglected and constant pressure is used throughout. 8+7
20. (a) State and explain bond's law.
(b) Derive the expression for angle of nip of crushing rolls.
(c) Calculate the power required in hp to crush $150 \times 10^{3} \mathrm{~kg}$ of feed, if $80 \%$ of it passes through 2.5 inchs screen and $80 \%$ of the product passes through $1 / 8$ inch screen. (given bond's constant=4.784). 5+5+5
21. Derive Hagen-Poiseuille Equation of Friction losses in laminar flow through a circular tube. Derive the expression of Fanning friction factor for laminar flow. A pipeline 600 m long and of 15 cm diameter is discharging an oil with velocity of $50 \mathrm{~cm} / \mathrm{sec}$. If the kinematic viscosity of oil is $19 \mathrm{~cm}^{2} / \mathrm{sec}$, find the loss of head due to friction. $7+3+5=15$
22. Derive Kozeny-Carman equation. A water softner consists of a vertical tube of 50 mm diameter and packed to a height of 0.5 m with ion exchange resin particles. The particles may be considered spherical with a diameter of 1.25 mm . Water flows over the bed because of gravity as well as pressure difference at a rate of $300 \mathrm{ml} / \mathrm{sec}$. The bed has a porosity of 0.30 . Calculate the pressure gradient. $9+6=15$
23. Orifice meter and venturimeter are called variable head meters but rotameter is known as variable area meter- Explain.

Water at $20^{\circ} \mathrm{C}$ is being pumped from a tank to an elevated tank at the rate of $5 \mathrm{lit} / \mathrm{sec}$. Pump has an efficiency of $65 \%$. All of the piping is of uniform diameter of 0.1023 m and total length of the piping system is 170 m . If there are two elbows in the whole piping system, calculate the power needed for the pump. Given: Density of water is $998.2 \mathrm{~kg} / \mathrm{m} 3$, Viscosity 1.005 cP . Fanning friction factor 0.0051 and equivalent resistance due to elbow is 15 d where $\mathrm{d}=$ diameter of pipe. $5+10$
24. The frictional pressure drop $\Delta \mathrm{p}$ for the flow of a fluid through a long, straight and round pipe depends upon the length 1 , diameter d , average height of the wall roughness e of the pipe, average fluid velocity v , density $\rho$, viscosity $\mu$ of the fluid. Use the Buckingham method to make a dimensional analysis of the system.

How can you calculate the Reynold's number of a system when a fluid is flowing through the annulus between two concentric circular pipe of diameter D1 and D2 while D1>D2? 10+5
25. Prove that frictional pressure drop in a circular tube arranged in the form of coil is greater than the straight pipe of equal length.

Derive the expression of volumetric flow rate $(\mathrm{Q})$ of any fluid passing through Rotameter.

A rotameter with a stainless steel float has a maximum capacity of $1.2 \mathrm{~L} / \mathrm{sec}$ of water at $28^{\circ} \mathrm{C}$. What will be the maximum capacity for kerosene in $\mathrm{L} / \mathrm{sec}$ for the same rotameter and the same float? Specific gravity of stainless steel and kerosene are 7.92 and 0.82 respectively. $3+7+5=15$

