GURU NANAK INSTITUTE OF TECHNOLOGY

An Autonomous Institute under MAKAUT

2020-2021

MATHEMATICS-III (BACKLOG)

M301

TIME ALLOTTED: 3 HOURS **FULL MARKS: 70**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable

GROUP – A

(Multiple Choice Type Questions)

Answer any ***ten*** from the following, choosing the correct alternative of each question: **10×1=10**

		Question	Marks	CO No
1.	i)	A function is said to be even if (a) $f(-x) = f(x)$ (b) $f(-x) = -f(x)$ (c) $f(x+a) = f(x)$ (d) $f(x-a) = f(x)$	1	CO1
	ii)	The period of the function $f(x) = sinx$is (a) 2π (b) $\frac{\pi}{2}$ (c) 3π (d) π	1	CO.1
	iii)	Find a_0 of the function $f(x) = x, -\pi \le x \le \pi$ in Fourier series expansion. (a) 0 (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$	1	CO2
	iv)	The poles for the function $\frac{z}{(z-2)(z-3)}$ are a) -2,-3 b) 2,3 c) 2,-3 d) -2,3	1	CO1
	v)	Find the real part of $\bar{z} where z = x - iy$ (a) x (b) $-x$ (c) y (d) $-y$	1	CO1
	vi)	The value of the integral $\int_C \frac{e^z}{z+1}$ where $C: \lvert Z-1 \rvert = 4$ is a) 0 b) $2\pi i$ c) $2\pi i\, e^{-1}$ d) $2\pi i\, e^{-2}$	1	CO1

vii)	Which one is Legendre differential equation (a) $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$ (b) $(1+x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$ (c) $(1-x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+n(n+1)y=0$ (d) $(1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+n(n+1)y=0$	1	CO1
viii)	Which one is the one dimensional heat equation? (a) $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ (c) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ (d) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0$	1	CO1
ix)	Consider the differential equation $(x-2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\frac{1}{x}y=0$. Then $x=0$ is (a) an ordinary point (b) a singular point but not a regular singular point (c) a regular singular point (d) none of these	1	CO2
x)	$J_{\frac{1}{2}}(x)$ is given by (a) $\sqrt{\frac{2\pi}{x}}\sin x$ (b) $\sqrt{\frac{2\pi}{x}}\cos x$ (c) $\sqrt{\frac{2}{\pi x}}\sin x$ (d) $\sqrt{\frac{2}{\pi x}}\cos x$	1	CO2
xi)	The expectation of the following distribution is: (a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) 1 (d) $\frac{3}{2}$	1	CO2
xii)	For the ordinary differential equation $P_0(x)\ddot{y}+P_1(x)\dot{y}+P_2(x)y=0$, the point $x=a$ is said to be ordinary point if (a) $P_0(a)=0$ (b) $P_0(a)\neq 0$ (c) $P_1(a)=0$ (d) $P_1(a)\neq 0$	1	CO1

Table for xi):

x_i	0	1	2	3
f_i	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

GROUP – B*
(Short Answer Type Questions)

Answer any ***three*** from the following: **3×5=15**

		Marks	CO No
2.	If the wages of 10000 workers in a factory follows normal distribution with mean 70 and standard deviation 5, find the expected number of workers whose weekly wages are between Rs 66 and Rs. 72.	5	CO3
3.	Find the value of 'a' for which $\int_C \left(\frac{z+1}{z^2-3z+2} + \frac{a}{z-1} \right) dz = 0$ where C is the circle in the complex plane that is oriented in the counter clockwise direction.	5	CO3
4.	Prove that $xJ'_n(x) = -nJ_n(x) + xJ_{n-1}(x)$	5	CO4
5.	Find Fourier sine transform of $\frac{e^{-ax}}{x}$	5	CO4
6.	Find the bilinear transformation which maps $z = 1, 0, -1$ onto the points $w = i, 0, -i$.	5	CO3

GROUP – C*
(Long Answer Type Questions)

Answer any ***three*** from the following: **3×15=45**

			Marks	CO No
7.	a)	Find the Fourier series expansion of the periodic function of period 2π, $f(x) = x^2, -\pi \le x \le \pi$. Hence deduce $\frac{1}{1^2} - \frac{1}{1^2} + \frac{1}{1^2} - \frac{1}{1^2} + \cdots\cdots\cdots += \frac{\pi^2}{12}$	5+3=8	CO4
	b)	Find the Fourier Transform of the function $f(x) = 1, \ \|x\| \le a$ $= 0, \|x\| > a$ Hence evaluate $\int_{-\infty}^{\infty} \frac{sinsacossx}{s} ds$	4+3=7	CO5
8.	a)	Prove that $\int_{-1}^{1} P_0(x) dx = 1.$	04	CO4
	b)	Show that $\frac{d}{dx}\{x^{-p} J_p(x)\} = -x^p J_{p+1}(x)$, where $J_p(x)$ is the Bessel's function of degree p	07	CO2
	c)	Find the mean and variance of a Binomial distribution X with parameter n and p	04	CO4

9. a)	Define Milne Thomson method. Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is a harmonic function and determine the corresponding analytic function $u + iv$.	06	CO3
b)	Expand the function $f(z) = \frac{1}{(z-1)(z+3)}$ in the region $1 < \|z\| < 3$.	03	CO2
c)	State Cauchy's Residue Theorem. Using it find the value of the integral $\int_c \frac{e^z dz}{z-2}$, where c is $\|z-2\| = 4$	06	CO3
10. a)	Solve the one dimensional Heat Equation by method of separation of variable $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $u(0,t) = 0, u(L,t) = 0$, $u(x,0) = f(x)$.	09	CO5
b)	A variable X has the following density function $f(x) = \frac{x}{2}, \quad 0 \le x \le 1$; $= \frac{1}{2}, \quad 1 < x \le 2$. Find the mean and variance.	06	CO4
11. a)	Using Cauchy's residue theorem, prove that $\int_0^{2\pi} \frac{d\theta}{2+cos\theta} = \frac{2\pi}{\sqrt{3}}$	07	CO2
b)	Find the power series solution of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2+2)y = 0$ about $x = 0$.	08	CO5