

**GURU NANAK INSTITUTE OF TECHNOLOGY**

An Autonomous Institute under MAKAUT

**2020-2021****MATHEMATICS-III (BACKLOG)****M(IT)301****TIME ALLOTTED: 3 HOURS****FULL MARKS: 70***The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable***GROUP – A****(Multiple Choice Type Questions)**Answer any *ten* from the following, choosing the correct alternative of each question: **10×1=10**

	<b>Marks</b>	<b>CO No.</b>
1. i. For the random experiment of a die is thrown until 3 occurs, number of sample points in the sample space is, a) 6 b) 18 c) 216 d) none of these	1	CO1
ii. For symmetric Group $S_3$ and alternating group $A_3$ , the index number $[S_3: A_3] = ?$ a) 1 b) 2 c) 3 d) 4	1	CO2
iii. How many even three-digit numbers can be formed from the digits 3,2,5,6 and 9 if each digit can be formed only once? a) 24 b) 60 c) 30 d) none of these	1	CO1
iv. Identity permutation $i$ is one of the following: a) even permutation b) odd permutation c) cyclic permutation d) transposition	1	CO3
v. Find the value of $c$ so that the function $f(x) = c(x+1)$ , $x = 0, 1, 2, 3$ is a probability mass function of a discrete random variable. a) 5 b) 1/5 c) 1/10 d) 10	1	CO2

- |      |  |   |     |
|------|--|---|-----|
| vi.  | The maximum degree of any vertex in a simple graph with n vertices is  |   |     |
|      | a) n-1   | 1 | CO3 |
|      | b) n+1   |   |     |
|      | c) 2n-1  |   |     |
|      | d) n   |   |     |
| vii. | For which distribution, $\sigma = \sqrt{\mu}$ ?  |   |     |
|      | a) Binomial Distribution   | 1 | CO1 |
|      | b) Poisson Distribution  |   |     |
|      | c) Uniform Distribution  |   |     |
|      | d) Normal Distribution   |   |     |
| viii | Circle has _____   |   |     |
|      | a) no vertices   | 1 | CO3 |
|      | b) only one vertex   |   |     |
|      | c) 8 vertices  |   |     |
|      | d) none of these   |   |     |
| ix.  | Which group is abelian but not cyclic?   |   |     |
|      | a) Symmetric group   | 1 | CO2 |
|      | b) Klein's 4-group   |   |     |
|      | c) $(Z_6, +)$  |   |     |
|      | d) all of the above  |   |     |
| x.   | A minimal spanning tree of a graph G is  |   |     |
|      | a) a spanning sub graph  | 1 | CO3 |
|      | b) a tree  |   |     |
|      | c) minimum weights   |   |     |
|      | d) all of the above  |   |     |
| xi.  | Order of element $\bar{4}$ for the group $(Z_5, +)$ is,  |   |     |
|      | a) 5   | 1 | CO2 |
|      | b) 2   |   |     |
|      | c) 4   |   |     |
|      | d) 3   |   |     |
| xii. | Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to |   |     |
|      | a) 3   | 1 | CO3 |
|      | b) 4   |   |     |
|      | c) 5   |   |     |
|      | d) 6   |   |     |

**GROUP – B\***

**(Short Answer Type Questions)**

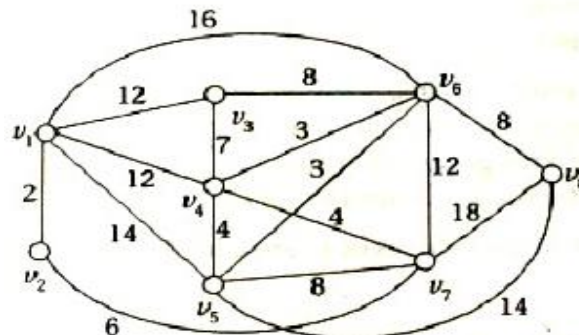
Answer any *three* from the following: **3×5=15**

- |    |   | <b>Marks</b> | <b>CO No.</b> |
|----|---|--------------|---------------|
| 2. | The probability density of a continuous distribution is given by $f(x) = \frac{3}{4}x(2-x), 0 < x < 2$ . Compute mean and variance.                                 | 05           | CO3           |
| 3. | Let $G = \{(a, b): a \neq 0, b \in R\}$ and * be a binary composition defined on G by $(a, b) * (c, d) = (ac, bc + d)$ . Show that $(G, *)$ is a non-abelian group. | 05           | CO2           |

- |    |   |    |     |
|----|---|----|-----|
| 4. | The minimum number of edges in a connected graph with $n$ vertices is $n-1$ .   | 05 | CO3 |
| 5. | In a normal distribution, 31% of the items are under 45 and 8% are above 64 . Find the mean and standard deviation<br>[Given $P(0 < Z < 1.405) = 0.42$ and $P(-0.496 < Z < 0) = 0.19$ ] | 05 | CO3 |
| 6. | A connected planar graph $G$ with $n$ vertices and $e$ number of edges determines $f = e - n + 2$ number of regions.  | 05 | CO4 |

**GROUP – C\***  
**(Long Answer Type Questions)**  
Answer any *three* from the following: **3×15=45**

- |    |   | <b>Marks</b> | <b>CO No.</b> |
|----|---|--------------|---------------|
| 7. | a. A box contains 5 defective and 10 non defective lamps and 8 are drawn at random in succession without replacement. What is the probability that the 8 <sup>th</sup> lamp is the 5 <sup>th</sup> defective?   | 05           | CO3           |
|    | b. 100 unbiased coins are tossed. Using normal approximation to binomial distribution calculate the probability to get<br>(i) exactly 40 heads<br>(ii) 55 heads or more.<br>Given $\phi(2.1) = 0.9821$ , $\phi(1.9) = 0.9713$ ,<br>$\phi(0.9) = 0.8159$   | 05           | CO3           |
|    | c. A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with average number of demand per day 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused. | 05           | CO2,<br>CO3   |
| 8. | a. Prove that the order of each subgroup of finite group is a divisor of the order of group   | 05           | CO1,<br>CO3   |
|    | b. Prove that every cyclic group is commutative but the converse of above may not be true   | 05           | CO1,<br>CO2   |
|    | c. Let $G$ be a group. If $a, b \in G$ such that $a^4 = e$ the identity element of $G$ and $ab = ba^2$ prove that $a = e$   | 05           | CO1,<br>CO3   |
| 9. | a. Find the shortest path and the length of the shortest path from the vertex $v_1$ to $v_8$ of the graph:  |              |               |



- |    |  |    |     |
|----|--|----|-----|
| b. | Prove that a tree with $n$ number of vertices has $n - 1$ number of edges. | 07 | CO2 |
|----|--|----|-----|

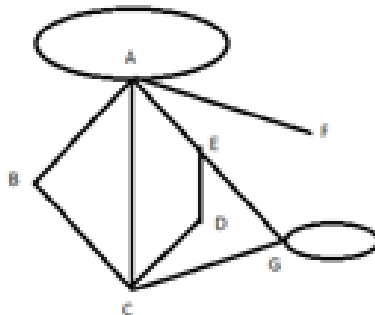
10. a. The weight of students in a college is normally distributed with  $m = 40kg$  and  $\sigma = 5kg$ . Find the percentage of the students that have weight
- (i) greater than  $40kg$
  - (ii) greater than  $50kg$  3+2+2=
  - (iii) between  $38kg$  and  $52kg$  07 CO3
- Given:  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^2 e^{-\frac{t^2}{2}} dt = 0.9772$ ,  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-\frac{t^2}{2}} dt = 0.6554$ ,  
 $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{2.4} e^{-\frac{t^2}{2}} dt = 0.9918$ ,
- b. Using Lagrange's theorem prove that every group of prime order is cyclic. 05 CO2, CO3
- c. G be a cyclic group of order 30 generated by a. Find the order of a cyclic subgroup generated by  $a^{18}$  03 CO3

11. a. Draw the graph of the following adjacency matrix.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

05 CO3

- b. If a simple regular graph G has n vertices and 24 edges, find all possible values of n. 05 CO3
- c. Find the dual of the following graph



05 CO2