GURU NANAK INSTITUTE OF TECHNOLOGY

An Autonomous Institute under MAKAUT

YEAR: 2020-2021

Mathematics III

M(EE) 301

TIME ALLOTTED: 3 HOURS **FULL MARKS: 70**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable

GROUP – A
(Multiple Choice Type Questions)

1. Answer any ***ten*** from the following, choosing the correct alternative of each question: **10×1=10**

			Marks	CO No.
1.	i	The variance of a random variable X is i) $E(X)-E(X^2)$ ii) $(E(X))^2-E(X^2)$ iii) $E(X^2)-E(X)$ iv) $E(X^2)-(E(X))^2$	1	CO2
	ii	The arithmetic mean of a set of 10 numbers is 20. If each number is first multiplied by 2 and then increased by 5, then what is the mean of new numbers? i) 20 ii) 25 iii) 40 iv) 45	1	CO2
	iii	The relation between mean, median and mode is i) Mode=3 Median – 2 Mean ii) Mode=2 Median – 3 Mean iii) Median=2Mode– 2 Mean iv) None of the above	1	CO2
	iv	The partial differential equation $5\frac{\partial^2 z}{\partial x^2}+6\frac{\partial^2 z}{\partial y^2}=xy$ is classified as i) Elliptic ii) Parabolic iii) Hyperbolic iv) None of the above	1	CO 3
	v	The following partial differential equation: $\frac{\partial^2 u}{\partial t^2}+\frac{\partial^4 u}{\partial x^4}-6\frac{\partial^2 u}{\partial x^2}=0$ *is* i) Linear; 4^{th} order ii) Nonlinear; 4^{th} order iii) Linear; 2^{nd} order iv) Nonlinear; 2^{nd} order	1	CO 3
	vi	A population has normal distribution with parameter m=5 and $\sigma = 0.1$. Then $\bar{X}$, *the sample mean with sample size* 25 is a normal variable with mean and s.d	1	CO 2

i) 5, 0.02
ii) 5, 0.1
iii) 5, 0.2
iv) None of these

vii The percentage error in approximated 4/3 to 1.3333 is
i) 0.0025%
ii) 25%
iii) 0.000025%
iv) 0.25%
1 CO 1

viii The degree of precision of Simpson's one third rule is
i) *3*
ii) 2
iii) 1
iv) 5
1 CO 1

ix In Simpson's 1/3 rule of finding $\int_a^b f(x)\,dx$, $f(x)$ is approximated by
i) line segment
ii) parabola
iii) circular sector
iv) part of ellipse
1 CO 1

x Lagrange's interpolation formula is used for
i) equispaced arguments only
ii) unequispaced arguments only
iii) both equispaced and unequispaced argumentsd
iv) None of these
1 CO 1

xi From an unknown population of a sample size 5 is drawn whose variance is 13.2. Then unbiased estimate of the population mean is
i) 26
ii) 6
iii) 8
iv) 10
1 CO 2

xii Regula-Falsi method is
i) Conditionally convergent
ii) linearly convergent
iii) divergent
iv) none of these
1 CO 1

GROUP – B
(Short Answer Type Questions)

Answer any ***three*** from the following: **3×5=15**

Marks CO No.

2. From the following table, find by Lagrange's formula, the value of y when x=102.

x	93.0	96.2	100.0	104.2	108.7
y	11.38	12.80	14.70	17.07	19.91

5 CO2

			Marks	CO No.
3.	a.	Solve the differential equation by Euler method $\frac{dy}{dx}=(x-y), y(0)=1$ at x=0.2, taking h=0.1, correct upto three decimal places.	3	CO3
	b.	Prove that $E \equiv \mathrm{I}+\Delta$	2	CO3
4.		Solve the system of equations by Gauss Elimination method: $3x_1+9x_2-2x_3=11$ $4x_1+2x_2+13x_3=24$ $4x_1-2x_2+x_3=-8$	5	CO3
5.		A normal population has a mean 0.1 and S.D 2.1. Find the probability that the mean of a sample of size 900 will be negative. Given that P($\|Z\|$<1.43)=0.847.	5	CO5
6.		Obtain the series solution of the equation $(1+x^2)y''+xy'-y=0$	5	CO4

GROUP – C
(Long Answer Type Questions)

Answer any ***three*** from the following: **3×15=45**

			Marks	CO No.
7.	a.	For finding the square root of ' a '($a>0$), derive the Newton Raphson iteration formula $x_{n+1}=\frac{1}{2}(x_n+\frac{a}{x_n}), n=0,1,2,3,....$ Hence find $\sqrt{2}$, correct to five significant figures.	6	CO3
	b.	Find the 5^{th} root of 2 by Newton Raphson method, which is correct upto four decimal places	5	CO3
	c.	Compute one positive root of $x+\ln x-2=0,$ correct to four decimal places by method of Newton Raphson method.	4	CO3
8.	a.	Solve the system of equations by LU factorization method: $8x_1-3x_2+2x_3=20$ $4x_1+11x_2-x_3=33$ $6x_1+3x_2+12x_3=36$ Correct to three significant figures.	8	CO3
	b.	Solve the system of equations by Gauss Seidel method: $10x-5y-2z=3$ $4x-10y+3z=-3$ $x+6y+10z=-3$ Correct to four significant figures.	7	CO3
9.	a.	Show that $\frac{d}{dx}\{x^{-p}J_p(x)\}=-x^pJ_{p+1}(x)$, where $J_p(x)$ is the Bessel's function of degree p.	6	CO4
	b.	A population consists of the three numbers 1,3,4. Consider all possible samples of size two with replacement. Find the mean of the sampling distribution of the sample variance and verify the result $E(S^2)=\frac{n-1}{n}\sigma^2$. Find the standard deviation of the sampling	4	CO5

distribution of variances.

c. The height of male students in a large university is normally distributed with s.d =2.1 inches. The average height of a randomly chosen sample of 100 male students is found to be 68.6 inches. Find the 98% confidence interval for the mean height of the male students of the university[given that P(Z>2.327)=0.01,where Z is a standard normal variate] — 5 — CO5

10. a. A particle moves in a path such that its velocity is proportional to the square of its displacement measured from a fixed point on the path. Describe the motion mathematically and find the distance of the particle at the time 1.02 unit from the fixed point by Euler's method, where the particle started its journey from the distance 2 unit from the fixed point. Given that the proportionality constant is 2. — 5 — CO2

b. Solve by Euler's modified method the following differential equation for x=0.02 taking step length h=0.01,

$$\frac{dy}{dx} = x^2 + y,\ \text{y=1 when x=0.}$$

— 6 — CO2

c. Compute $y(0.4)$ by Runge Kutta method of fourth order for the differential equation $\frac{dy}{dx} = xy, y(0) = 1$, taking h=0.2. — 4 — CO2

11. a. A particle travels in a path at a maximum velocity 20Km/h satisfying the points (2,4), (3, 7), (5,13), (6,28), (7,39), (9, 55). It started its journey from (2, 4) at a velocity 15 Km/h. What will be its height at a distance 4 unit from origin? — 5 — CO3

b. Find the power series solution of the differential equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 + 2\right)y = 0 \text{ about } x = 0.$$

— 5 — CO4

c. Solve the one dimensional Heat Equation by method of separation of variable

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$
$$u(0,t) = 0, u(L,t) = 0$$
$$u(x,0) = f(x).$$

— 5 — CO4