GURU NANAK INSTITUTE OF TECHNOLOGY

An Autonomous Institute under MAKAUT

2020-2021

MATHEMATICS-III (Backlog)

M(CSE)301

TIME ALLOTTED: 3 HOURS **FULL MARKS: 70**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable

GROUP – A
(Multiple Choice Type Questions)

Answer any ***ten*** from the following, choosing the correct alternative of each question: **10×1=10**

			Marks	CO No
1	i	For a Poisson distribution $P(x)$, $P(1)=P(2)$, then $P(0)$ is a) 1/e b) $1/e^2$ c) $1/e^3$ d) e.	01	C04
	ii	$\neg(p\vee q)\vee(p\wedge\neg q)\equiv$ a) $\neg p$ b) p c) $\neg q$ d) none of these	01	CO2
	iii	Let X be a Poisson Random Variate and $E(X)=\lambda$. Then $E[(X+1)^2]$ will be a) λ b) $\lambda^2+2\lambda$ c) $\lambda^2+2\lambda+1$ d) $\lambda^2+3\lambda+1$	01	CO2
	iv	If R is a ring without zero divisors, then $x.y=0$ implies a) $x=0\ or\ y=0$ b) $x=0\ and\ y=0$ c) $x=0, y\neq 0$ d) $x\neq 0, y=0$	01	CO 2
	v	In the Dijkstra's algorithm for a digraph if the edge AB is directed from A to B only then we take weight on the edge BA a) 0 b) ∞ c) – weight on the directed edge AB d) none	01	CO 1
	vi	Which of the following sets is closed under multiplication? a) $\{1,-1,0,2\}$ b) $\{1,i\}$ c) $\{1,\omega,\omega^2\}$ d) $\{\omega,1\}$	01	CO 2
	vii	The generating function for the sequence $\left\{1,1,\frac{1}{2!},\frac{1}{3!},\frac{1}{4!}......\right\}$ is a) e^x b) e^{-x} c) $\log(1+x)$ d) $(1-x)^{-1}$	01	CO2
	viii	A group contains 12 elements. Then the possible number of elements in a subgroup is a) 3 b) 5 c) 7 d) 11	01	CO 1
	ix	The number of generators of an infinite cyclic group is a) 1 b) 2	01	CO 2

	c) infinite	d) none of these		
x	The order of dihedral group D_4 is a) 4 b) 6 c) 8 d) 64		1	CO 3
xi	The minimum number of connected component of a graph with 16 vertices and 10 edges is a) 4 b) 5 c) 8 d) 6		1	CO1
xii	To make a graph G (with e edges, n vertices) free from any circuit the minimum number of edges to be removed from G is a) e – n b) e – n + 1 c) n – 1 d) e – 1		1	CO2

GROUP – B*
(Short Answer Type Questions)

Answer any ***three*** from the following: **3×5=15**

		Marks	CO No
2.	The probability density of a continuous distribution is given by $f(x) = \frac{3}{4}x(2-x), 0 < x < 2$. Compute mean and variance.	05	CO3
3	Use division algorithm to prove that the square of an odd integer is of the form $8k + 1$, where k is an integer.	05	CO4
4.	The minimum number of edges in a connected graph with n vertices is n-1.	05	CO4
5	Prove that the set of all even integers forms a commutative ring.	05	CO2
6	Show that $\{(p \wedge \sim q) \to r\} \to \{p \to (q \vee r)\}$ is a tautology	05	CO4

GROUP – C*
(Long Answer Type Questions)

Answer any ***three*** from the following: **3×15=45**

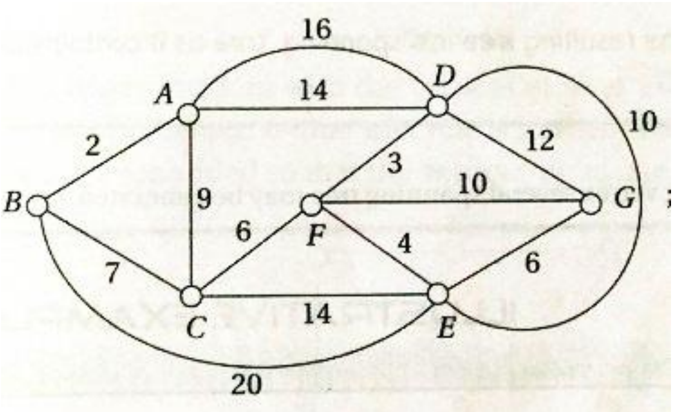
			Marks	CO No
7.	a.	A box contains 5 defective and 10 non defective lamps and 8 are drawn at random in succession without replacement. What is the probability that the 8th lamp is the 5th defective?	05	CO2
	b.	100 unbiased coins are tossed. Using normal approximation to binomial distribution calculate the probability to get (i) exactly 40 heads (ii) 55 heads or more. Given $\phi(2.1) = 0.9821,\ \phi(1.9) = 0.9713, \phi(0.9) = 0.8159$	05	CO3
	c.	Using Lagrange's theorem prove that every group of prime order is cyclic.	05	CO2
8.	a.	Prove that the order of each subgroup of finite group is a divisor of	05	CO1,2

the order of group

b. Prove that every cyclic group is commutative but the converse of above may not be true — 05 — CO2,3

c. Let G be a group. If $a, b \in G$ such that $a^4 = e$ the identity element of G and $ab = ba^2$ prove that $a = e$ — 05 — CO2,3

9. a. Find the shortest path and the length of the shortest path from the vertex B to G of the graph: — 08 — CO5



b. Solve the recurrence relation by using generating function $a_n - 5a_{n-1} + 6a_{n-2} = 2^n + n, \; n \geq 2$ with the boundary conditions $a_0 = 1, \; a_1 = 1$. — 07 — CO4

10. a. If in a ring R with unity, $(xy)^2 = x^2y^2$ for all $x, y \in R$ then show that R is commutative. — 05 — CO3

b. For any prime p, the ring Z_p of all integer module p is a field. Is it true? Justify your answer. — 05 — CO2,3

c. Show that all roots of the equation $x^4 = 1$ forms a commutative group under the operation usual multiplication. — 05 — CO2,3

11. a. Find the Principal Disjunctive Normal Forms(PDNF) and Principal Conjunctive Normal Forms(PCNF) of the statement — 05 — CO2,4

$$\sim ((p \wedge \sim q) \vee (p \wedge r)) \vee \sim p$$

b. The number of pendent vertices in a binary tree is (n+1)/2 where n is the number of vertices. — 05 — CO2,3

c. Find the remainder when the sum $1! + 2! + 3! + \ldots\ldots + 100!$ is divided by 5. — 05 — CO4