GURU NANAK INSTITUTE OF TECHNOLOGY
An Autonomous Institute under MAKAUT
2022-2023
ADVANCED ENGINEERING MATHEMATICS
PGCSE102

TIME ALLOTTED: 3 HOURS **FULL MARKS:70**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable

GROUP – A
(Multiple Choice Type Questions)

Answer any ***ten*** from the following, choosing the correct alternative of each question: **10×1=10**

			Marks	CO No
1.	(i)	Order of convergence of Newton Raphson method is a) 1 b) 2 c) 0 d) 1.62	1	CO1
	(ii)	For a perfect matching the corresponding graph should be a) Tree b) Bipartite graph c) Complete Graph d) None of these	1	C02
	(iii)	Number of significant digits of 100200 is a) 2 b) 4 c) 3 d) 6	1	CO2
	(iv)	In a Poisson distribution if $2P(x=1)=P(x=2)$, then the variance is a) 1 b) -1 c) 4 d) 2	1	CO4
	(v)	If α is an eigen vector of the matrix A corresponding to the eigen value k if a) $A\alpha = k^2\alpha$ b) $A\alpha = \alpha$ c) $A\alpha - k\alpha = 0$ d) None of these	1	CO2
	(vi)	A system of linear equations be solved by a) LU- method b) SOR method c) Gauss Seidel Method d) All of these	1	CO1

(vii)	(M/M/1 : ∞ / FIFO) queue model with arrival and service rate λ and μ ($\lambda < \mu$) respectively, then the 'Average length of the non-empty queue' [E(m/m>0)] is given by a) $\lambda/(\mu-\lambda)$ b) $\mu/(\mu-\lambda)$ c) $1/(\mu-\lambda)$ d) $1/(\lambda-\mu)$	1	CO2
(viii)	Chromatic number of K_7 is a) 7 b) 6 c) 5 d) None of these	1	CO2
(ix)	Choose the correct statement a) every trail is a path b) every walk is a trail c) path is a open walk d) every vertex cannot appear twice in a walk	1	CO2
(x)	The function $f(z)=\frac{e^{z^2}}{z^4}$ has a) an essential singularity at $z=0$. b) b) a pole of order 4 at $z=0$. c) a simple pole at $z=0$. d) no singularity at $z=0$.	1	CO1
(xi)	Let A be an orthogonal matrix , then a) a) det A = 0 b) det A = ± 1 c) det A = 2 d) none of these	1	C01
(xii)	The residue of $\frac{z^2}{z^2+3^2}$ at $z = 3i$ is a) $\frac{-3i}{2}$ b) $\frac{3i}{2}$ c) 3 d) none of these	1	CO2

GROUP – B

(Short Answer Type Questions)

Answer any ***three***from the following:**3×5=15**

		Marks	CO No.
2.	Use Newton Raphson method to compute $\sqrt[5]{20}$, correct to 3 decimal places.	5	CO3

3		The matrix A is invertible if and only if every eigen value is nonzero.	5	CO2
4.		The marks obtained by 1000 students to a semester examination are found be approximately normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be between 60 and 75 both inclusive. [Given that area under normal curve between z=0 and z=1 is 0.3413 and between z=0 and z=2 is 0.4772]	5	CO3
5		Established the Chapman-Kolmogorov equation $p_{jk}^{(m+n)} = \sum_r p_{rk}^{(n)} p_{jr}^{(m)} = \sum_r p_{jr}^{(m)} p_{rk}^{(n)}$	5	CO2
6.		Use Stirling's formula to compute y(0.35) from the following table:	5	CO3

x	0.1	0.2	0.3	0.4	0.5
y	2.9456	2.5292	2.9643	3.3899	3.8904

GROUP – C
(Long Answer Type Questions)
Answer any ***three***from the following:**3×15=45**

			Marks	CO No.
7.	(a)	The probability density of a continuous distribution is given by $f(x) = \frac{3}{4}x(2-x), 0 < x < 2$. Compute mean and variance	5	CO4
	(b)	Consider the Markov chain having transition probability matrix (t.p.m.) draw transition graph of Markov chain	5	CO3

$$
\begin{matrix} -1 \\ -2 \\ -3 \\ -4 \\ -5 \\ \overline{6} \end{matrix}
\begin{pmatrix}
\frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\
0 & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\
\frac{2}{5} & 0 & \frac{3}{5} & 0 & 0 & 0 \\
0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\
0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4}
\end{pmatrix}
$$

	(c)	Expand the function $f(z) = \frac{1}{(z-1)(z-2)}$ between the annular region $\lvert z \rvert = 1$ and $\lvert z \rvert = 2$.	5	CO3
8.	(a)	Find y(1.4) from the following table	8	CO3

x:	0.00	0.50	1.00	1.50	2.00	2.50	3.00
y:	0.104	0.195	0.344	0.439	0.479	0.595	0.699

By using Bessel's formula correct to three decimal places.

	(b)	Find the chromatic polynomial of K_5.	7	CO3

9. (a) For Steady-State (M/M/1) Model prove that — 8 — CO2

- $p_0 = (1 - \rho)$
- $p_n = p_0\rho^n = (1 - \rho)\rho^n$
- A self service store employs one cashier at its counter 8 customer arrive on an average every 5 minutes while the cashier can serve 10 customers in the same time. Assuming Poisson distribution for arrival and exponential distribution for service rate, determine:

i. Average number of customers in the system
ii. Average time a customer spends in the system

(b) Find chromatic polynomial of any tree having n vertices — 7 — CO3

10. (a) Consider the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 5 & 3 & a \\ 4 & -2 & -1 \end{bmatrix}$ for some variable 'a'. — 6 — CO2

Find all values of 'a' which will prove that A has eigenvalues 0, 3, and −3.

(b) If $\{\alpha, \beta, \gamma\}$ be a basis of a real vector space V. Prove that the set $\{\alpha+\beta+\gamma, \beta+\gamma, \gamma\}$ is a basis of V. — 4 — CO2

(c) Examine if the set of vectors {(2,1,1),(1,2,2),(1,1,1)} is basis in R^3 — 5 — CO3

11. Write Hall's marriage theorem. — 15 — CO3

Define SDR (system of distinct representatives) with example.
Consider the family of four finite sets $S = \{A_1, A_2, A_3, A_4\}$ where $A_1 = \{a, b, d, e\}, A_2 = \{b, c, d, e, f\}, A_3 = \{c, f\}, A_4 = \{b, e, f\}$ Show whether S satisfies the marriage condition. If yes, find two valid SDR of S.