Online Course Ware

Stream: ECE

Paper Name: EM WAVE PROPAGATION & ANTENNA

Paper Code: EC 601 Contacts: 3L Credits: 3

Semester: 6^{th} Total Contact: 33

Prepared by: Ms.Antara Ghosal

Module - 1
Maxwell equation, Boundary between media interface, Helmholtz's equation, Plane Wave in lossy dielectric, loss-less dielectric, good conductor, free-space; Poynting theorem, power flow, Poynting Vector, Skin Depth, Surface Resistance.

Maxwell equations

The general form of time- varying Maxwell equations, can be written in differential, form as

$$\nabla \times \bar{\mathcal{E}} = \frac{-\partial \bar{\mathcal{B}}}{\partial t} - \bar{\mathcal{M}},$$

$$\nabla \times \bar{\mathcal{H}} = \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{J}},$$

$$\nabla \cdot \bar{\mathcal{D}} = \rho,$$

$$\nabla \cdot \bar{\mathcal{B}} = 0.$$

These four equations form the basis of all electromagnetic theory. They are partial differential equations and relate the electric and magnetic fields to each other and their sources, charge and current density.

The auxiliary equations are:

$\mathbf{D} = \varepsilon\, \mathbf{E}$

$\mathbf{B} = \mu \mathbf{H}$ $\mathbf{J}$

$= \sigma \mathbf{E}$

$\mathbf{D} = {}_0\mathbf{E} + \mathbf{P}$

The boundary conditions on Electric field and Magnetic field at interfaces are given as:

For tangential components and normal components:

$E_{t1} = E_{t2}$

$H_{t1} = H_{t2}$

$D_{N1} - D_{N2} = \rho_S$ B_{N1}
$= B_{N2}$

It is often desirable to idealize a physical problem by assuming a perfect conductor for which σ is infinite but J is finite. From Ohm's law, then, in a perfect conductor, E = 0 and it follows from the point form of Faraday's law that for time-varying fields. H = 0

Wave Propagation in Free Space

When considering electromagnetic waves in free space, we note that the medium is source-less (ρ = J = 0). Under these conditions, Maxwell's equations may be written in terms of E and H only as

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

Manipulation of the above equations lead us to the Helmholtz's equation:

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 H_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2}$$

In phasor form these equations can be simplified as

$$\nabla^2 \bar{E} + \omega^2 \mu \epsilon \bar{E} = 0$$

$$\nabla^2 \bar{H} + \omega^2 \mu \epsilon \bar{H} = 0$$

Wave propagation in lossy dielectric

If the medium is conductive, with a conductivity σ, Maxwell's curl equations can be written as

$$\nabla \times \bar{E} = -j\omega\mu\bar{H},$$
$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} + \sigma\bar{E}.$$

The resulting wave equation for $\bar{E}$ then becomes

$$\nabla^2\bar{E} + \omega^2\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)\bar{E} = 0.$$

A complex propagation constant for the medium as

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$$

Where α is the attenuation constant and β is the phase constant.

Assuming an electric field with only an x component and uniform in x and y, the wave equation of reduces to

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0.$$

Which has solutions

$$E_x(z) = E^+e^{-\gamma z} + E^-e^{\gamma z}$$

The positive traveling wave then has a propagation factor of the form

$$e^{-\gamma z} = e^{-\alpha z}e^{-j\beta z}$$

which has solutions

$$e^{-\alpha z}\cos(\omega t - \beta z)$$

This represents a wave traveling in the +z direction with a phase velocity $v_p = \omega/\beta$, a wavelength $\lambda = 2\pi/\beta$, and an exponential damping factor.

If the loss is removed, $\sigma = 0$, and we have $\gamma = jk$ and $\alpha = 0, \beta = k$.

Loss can also be treated through the use of a complex permittivity. With $\sigma = 0$ but $\varepsilon = \varepsilon' - j\,\varepsilon''$ complex, we have that

$$\gamma = j\omega\sqrt{\mu\epsilon} = jk = j\omega\sqrt{\mu\epsilon'(1 - j\tan\delta)},$$

Where $\tan\delta = \epsilon''/\epsilon'$ is the loss tangent of the material.

The associated magnetic field can be calculated as

$$H_y = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} = \frac{-j\gamma}{\omega\mu}(E^+e^{-\gamma z} - E^-e^{\gamma z})$$

The intrinsic impedance of the conducting medium is now complex,

$$\eta = \frac{j\omega\mu}{\gamma},$$

Plane Waves in a Good Conductor

A good conductor is a special case of the preceding analysis, where the conductive current is much greater than the displacement current, which means that σ >> ω ε. In terms of a complex ε, rather than conductivity, this condition is equivalent to ε''>> ε'.

The propagation constant

$$\gamma = \alpha + j\beta \simeq j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}.$$

The skin depth, or characteristic depth of penetration, is defined as

$$\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}.$$

Thus the amplitude of the fields in the conductor will decay by an amount 1/e, or 36.8%, after traveling a distance of one skin depth.

Poynting's theorem and Poynting Vector

$$-\oint_{\text{area}} (\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S} = \int_{\text{vol}} \mathbf{J}\cdot\mathbf{E}\,dv + \frac{d}{dt}\int_{\text{vol}} \frac{1}{2}\mathbf{D}\cdot\mathbf{E}\,dv + \frac{d}{dt}\int_{\text{vol}} \frac{1}{2}\mathbf{B}\cdot\mathbf{H}\,dv$$

Poynting's theorem relates total electromagnetic power flowing to power dissipation. On the righthand side, the firstintegral is the total (but instantaneous) ohmic power dissipated within the volume. The second integral is the total energy stored in the electric field, and the third integral is the stored energy in the magnetic field. Since time derivatives are taken of the second and third integrals, those results give the time rates of increase of energy stored within the volume, or the instantaneous power going to increase the stored energy. The sum of the expressions on the right must therefore be the total power flowing into this volume, and so the total power flowing out of the volume is

$$\oint_{area} (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} \quad \text{W}$$

where the integral is over the closed surface surrounding the volume. The cross product E × H is known as the Poynting vector, S, which is interpreted as an instantaneous power density, measured in watts per square meter (W/m^2). The direction of the vector S indicates the direction of the instantaneous power flow at a point.

Module - 2
Concept of lumped parameters, Transmission line equation & their solution, Propagation constant, characteristic Impedance, wavelength, velocity of propagation for distortion less line and loss-less line; Reflection and Transmission coefficients, Standing Wave, VSWR, Input Impedance; Smith Chart; Some impedance techniques- Quarter wave matching, Single stub matching; Reflection in miss-matched load; T-line in time domain, Lattice diagram calculation, Pulse propagation on T-line.

Concept of lumped parameters

When the impedance of a transmission line or device at the operating frequency may be considered as equivalent to devices concentrated at one point and the parameters of the system including the line or device is not substantially independent of the load devices connected thereto, the transmission line or device may be said to have lumped parameters. Lumped impedances are also used to include devices such as capacitors, inductors, and resistors which have their impedance concentrated at the terminals thereof.

What is the difference between Lumped and Distributed systems?

The elements building a lumped system are thought of being **concentrated at singular points** in space. The classical example is an electrical circuit with passive elements like resistor, inductance and capacitor. The physical quantities current and voltage are functions of **time (only)**. E. g. the current at a capacitor with capacity C is given by $i(t)=Cdv(t)/dt$

Where C is a constant (and so are R and L). This leads to ordinary differential equations.

In contrast, the elements in distributed systems are thought of being **distributed in space**, so that physical quantities **depend on both time and space**. The classical example is the electrical line where inductance, capacity and resistance are not constant but functions of length x. This leads to partial derivatives of $i(t,x)$ and $v(t,x)$ in t and x.

It is important to realize that the terms *lumped* or *distributed* are not properties of the system itself. These properties are related to the size of the system *compared to* the wavelength of the voltages and currents passing through it. So a resistor is or isn't a lumped element (even though it is usually meant to be one), depending on the frequency of the applied signals.

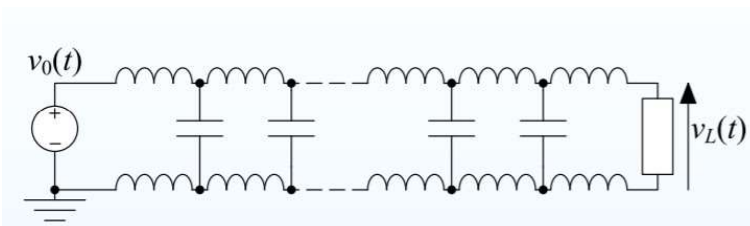
An element can be considered as *lumped* if its size is much smaller than the wavelength of the applied voltages and currents. In this case wave propagation effects may be neglected and a lumped element can be treated as a black box which is completely described by the behaviour at its terminals.

Transmission line equation & their solution

Previously assume that any change in $v_0(t)$ appears instantly at $v_L(t)$.

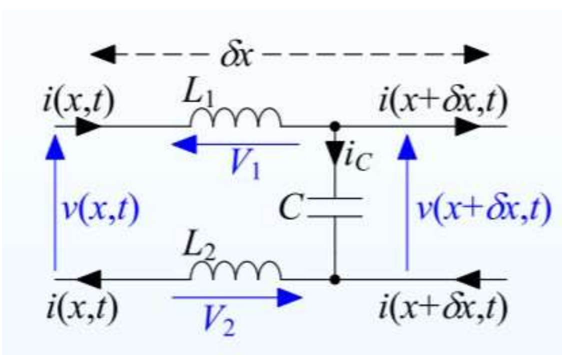
This is not true. If fact signals travel at around half the speed of light (c = 30 cm/ns).

Reason: all wires have capacitance to ground and to neighbouring conductors and also selfinductance. It takes time to change the current through an inductor or voltage across a capacitor. A transmission line is a wire with a uniform geometry along its length: the capacitance and inductance of any segment is proportional to its length. We represent as a large number of small inductors and capacitors spaced along the line. The signal speed along a transmission line is predictable.



Transmission Line Equations

A short section of line δx long: v (x, t) and i (x, t) depend on both position and time.



Small δx ⇒ ignore 2nd order derivatives:

$\partial v(x,t)/\partial t = \partial v(x+\delta x,t)/\partial t = \partial v/\partial t$.

Basic Equations

KVL: $v(x, t) = V_2 + v(x + \delta x, t) + V_1$

KCL: $i(x, t) = i_C + i(x + \delta x, t)$

Capacitor equation: $C\,\partial v/\partial t = i_C = i(x, t) - i(x + \delta x, t) = -(\partial i/\partial x)\,\delta x$

Inductor equation (L_1 and L_2 have the same current): $(L1 + L2) \partial i/ \partial t = V_1 + V_2 = v (x, t) - v (x + \delta x, t) = -(\partial v /\partial x) \delta x$

Transmission Line Equations

$C_0 \, \partial v/\partial t = - \partial i /\partial x$

$L_0 \, \partial i /\partial t = -\partial v/ \partial x$

where $C_0 = C/ \delta x$ is the capacitance per unit length (Farads/m) and $L_0 =(L_1+ L_2)/ \delta x$ is the total inductance per unit length (Henries/m).

Solution to Transmission Line Equations

Transmission Line Equations: $C_0 \, \partial v/\partial t = - \partial i /\partial x$; $L_0 \, \partial i /\partial t = -\partial v/ \partial x$

General solution: $v (t, x) = f (t - x /u) + g (t + x/ u)$

$$i (t, x) = (f(t - x/ u) - g(t + x /u)) /Z_0$$

where $u = \sqrt{(1/L_0C_0)}$ and $Z_0 = \sqrt{(L_0/C_0)}$. u is the propagation velocity and Z_0 is the characteristic impedance. f() and g() can be any differentiable functions.

Verify by substitution:

$$- \partial i/\partial x = - ((- f' (t - x/ u) - g' (t + x /u))/ Z_0 \times (1/u) = C_0(f' (t - x /u) + g' (t + x /u)) = C_0\partial v/\partial t$$

Propagation, attenuation and phase constants

The propagation constant is an important parameter associated with transmission lines. It is a complex number denoted by Greek lower case letter γ (gamma), and is used to describe the behavior of an electromagnetic wave along a transmission line.

Propagation, attenuation and phase constants

The propagation constant is separated into two components that have very different effects on signals:

$\gamma=\alpha+j\beta$ α= attenuation constant β= phase constant

The real part of the propagation constant is the **attenuation constant** and is denoted by Greek lowercase letter α (alpha). It causes a signal amplitude to decrease along a transmission line. The natural units of the attenuation constant are Nepers/meter, but we often convert to dB/meter in microwave engineering. To get loss in dB/length, multiply Nepers/length by 8.686. Note that attenuation constant is always a positive number.

The **phase constant** is denoted by Greek lowercase letter β (beta) adds the imaginary component to the propagation constant. It determines the sinusoidal amplitude/phase of the signal along a transmission line, at a constant time. The phase constant's "natural" units are radians/meter, but we often convert to degrees/meter. A transmission line of length "l" will have an electrical phase of βl, in radians or degrees. To convert from radians to degrees, multiply by $180/\pi$.
The two parts of the propagation constant have radically different effects on a wave. The amplitude of a wave (frozen in time) goes as cosine(βl). In a lossless transmission line, the wave would propagate

as a perfect sine wave. In real life there is some loss to the transmission line, and that is where the attenuation constant comes in. The amplitude of the signal decays as Exp(-αl). The composite behaviour of the propagation constant is observed when you multiply the effects of α and β.

Phase constant

Recall that there are 2π radians in a wavelength, therefore the relationship between phase constant and wavelength is simply:
$ß=2\pi/\lambda$(radian/length)

Distortion less line and Loss-less line

A transmission line is said to be distortionless when attenuation constant 'α' is frequency independent and the phase shift constant 'β' is linearly dependent on the frequency.

Condition for line to be distortionless

$R/L=G/C$

(a) **Propagation constant**

$\gamma = (R+j\omega L)(G+j\omega C)$

Or $\gamma = RG(1+j\omega L/R)(1+j\omega C/G)$

If $R/L=G/C$

Put value of R/L in equation of γ

Thus $\gamma = RG(1+j\omega C/G)(1+j\omega C/G)$

Or $\gamma = RG(1+j\omega C/G)$

Also $\gamma = \alpha+j\beta$

Comparing Real and Imaginary parts, we get

$\alpha=RG$

and $j\beta= RGj\omega C/G$

$=j\omega RC/G$

Thus $\beta=\omega LC$

The above results show that α is frequency independent and β is frequency dependent

(b) **Characteristic impedance**

$Z_0= \sqrt{(R+j\omega L)/(G+j\omega C)}$

$Z_0 =\sqrt{R/G}=\sqrt{L/C}$

(c) **Phase velocity:-**

$V_p=\omega/\beta$

Substituting value of β in above expression,we get

$V_p=\omega/\omega\sqrt{LC}$

Thus $v_p=1/\sqrt{LC}$

LOSSLESS TRANSMISSION LINE AND ITS CONDITION

CHARACTERISTIC IMPEDANCE ,Z_0

DEFINITION I:- Z_0 is defined as the ratio of the square root of series impedance per unit length ,Z to the square root of shut admittance per unit length,Ẏ

$Z_0=Z/Y =\sqrt{((R+j\omega L)/(G+j\omega c))}$

DEFINITION 2:- The characteristic impedance , Z_0 of a line is defined as the ratio of the forward voltage wave ,$V_f$to the forward current wave ,$I_f$at any point on the line.

$Z_0=V_f/I_f$

DEFINITION 3:- z_0 IS defined as the minus of the ratio of the reflected voltage wave . V_r to the reflected current wave , I_r at any point on the line,

$Z_0= -V_r/I_r$

Characteristic impedance ,Z_0 is also called **Surge imperdance.**

LOSSLESS TRANSMISSION LINES

A transmission line is said to be lossless if the conductors of line are perfect that is cnductivity $\sigma_c=\infty$ and the dielectric medium between the lines is lossless that is conductivity $\sigma_d=0$

Condition for a line to be lossless

R=0=G

For loss less line,

(a) **Attenuation Constant** **$\alpha=0$**

(b) **Propagation constant**

$\gamma = \alpha + j\beta = j\beta$ ($\alpha=0$)

Also as $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

As $R=0, G=0$

Thus propagation constant $\gamma = j\omega LC$

(c) **Phase shift constant**

By comparing imaginary parts of γ, we get

Phase shift constant $\beta=\omega LC$

(d) **Characteristic impedance,**

$Z_0=\sqrt{(R+j\omega L)/(G+j\omega C)}$

As $R=0=G$

$Z_0=\sqrt{L/C}$

Thus Z_0 is **pure reactance (**that is in the form of inductance or capacitance).

(e) **Phase velocity or the velocity of propagation in lossless line,**

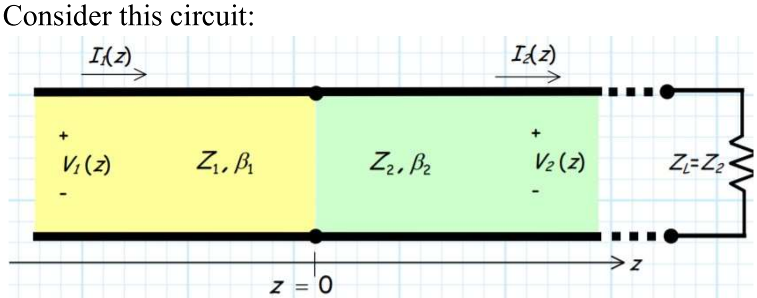
$V_p=\omega/\beta$

By putting value of β,we get

Thus $v_p=\omega/\omega \sqrt{LC}$ Or

$v_p=1/\sqrt{LC}$

Consider this circuit:



I.E., a transmission line with characteristic impedance Z_1 transitions to a different transmission line at location $z = 0$. This second transmission line has different characteristic impedance Z_2 ($Z_1 \neq Z_2$). This second line is terminated with a load $Z_L = Z_2$ (i.e., the second line is matched). $z < 0$ We know that the voltage along the first transmission line is:

$$V_2(z) = V_{02}^{+} e^{-j\beta_2 z} + V_{02}^{-} e^{+j\beta_2 z} \qquad [\text{for } z > 0]$$

while the current along that same line is described as:

$$I_1(z) = \frac{V_{01}^{+}}{Z_1} e^{-j\beta_1 z} - \frac{V_{01}^{-}}{Z_1} e^{+j\beta_1 z} \qquad [\text{for } z < 0]$$

$z > 0$ We likewise know that the voltage along the second transmission line is:

$$V_2(z) = V_{02}^{+} e^{-j\beta_2 z} + V_{02}^{-} e^{+j\beta_2 z} \qquad [\text{for } z > 0]$$

while the current along that same line is described as:

$$I_2(z) = \frac{V_{02}^{+}}{Z_2} e^{-j\beta_2 z} - \frac{V_{02}^{-}}{Z_2} e^{+j\beta_2 z} \qquad [\text{for } z > 0]$$

Moreover, since the second line is terminated in a matched load, we know that the reflected wave from this load must be zero:

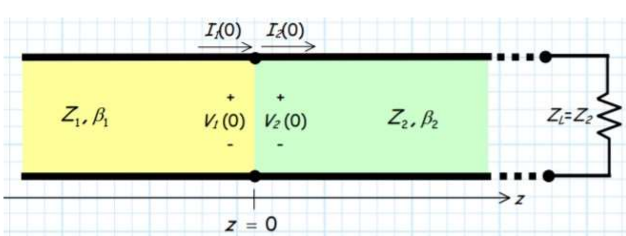
$$V_2^{-}(z) = V_{02}^{-} e^{-j\beta_2 z} = 0$$

The voltage and current along the second transmission line is thus simply:

$$V_2(z) = V_2^{+}(z) = V_{02}^{+} e^{-j\beta_2 z} \qquad [\text{for } z > 0]$$

$$I_2(z) = I_2^{+}(z) = \frac{V_{02}^{+}}{Z_2} e^{-j\beta_2 z} \qquad [\text{for } z > 0]$$

$z=0$ At the location where these two transmission lines meet, the current and voltage expressions each must satisfy some specific boundary conditions:



The first boundary condition comes from KVL, and states that:

$$V_1(z=0) = V_2(z=0)$$
$$V_{01}^{+}\, e^{-j\beta_1(0)} + V_{01}^{-}\, e^{+j\beta_1(0)} = V_{02}^{+}\, e^{-j\beta_2(0)}$$
$$V_{01}^{+} + V_{01}^{-} = V_{02}^{+}$$

We can therefore define a transmission coefficient, which relates v_{02}^{+} to V_{01}:

$$T_0 \doteq \frac{V_{02}^{+}}{V_{01}^{+}} = \frac{2Z_2}{Z_1 + Z_2}$$

We can therefore define a reflection coefficient, which relates V_{01}^{-} to V_{01}^{+}

$$\Gamma_0 \doteq \frac{V_{01}^{-}}{V_{01}^{+}} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

Standing wave in transmission line

Whenever there is a mismatch of impedance between transmission line and load, reflections will occur. If the incident signal is a continuous AC waveform, these reflections will mix with more of the oncoming incident waveform to produce stationary waveforms called *standing waves*

VSWR

VSWR and System Efficiency

In an ideal system 100% of the energy is transmitted from the power stages to the load. This requires an exact match between the source impedance, i.e., the characteristic impedance of the transmission line and all its connectors, and the load's impedance. The signal's AC voltage will be the same from end to end since it passes through without interference.

In real systems, however, mismatched impedances cause some of the power to be reflected back toward the source (like an echo). Reflections cause constructive and destructive interference, leading to peaks and valleys in the voltage at various times and distances along the line. VSWR measures these voltage variances. It is the ratio of the highest voltage anywhere along the transmission line to the lowest voltage.

Since the voltage does not vary in an ideal system, its VSWR is 1.0 or, as commonly expressed as a ratio of 1:1. When reflections occur, the voltages vary and VSWR is higher, for example 1.2, or 1.2:1.

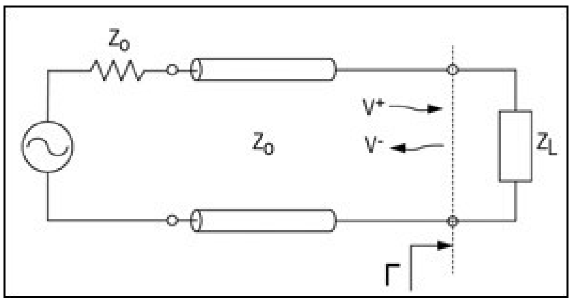
Reflected Energy

When a transmitted wave hits a boundary such as the one between the lossless transmission line and load (**Figure 1**), some energy will be transmitted to the load and some will be reflected. The reflection coefficient relates the incoming and reflected waves as:

$\Gamma = V^-/V^+$ (Eq. 1)

Where V^- is the reflected wave and V^+ is the incoming wave. VSWR is related to the magnitude of the voltage reflection coefficient (Γ) by:

$VSWR = (1 + |\Gamma|)/(1 - |\Gamma|)$ (Eq. 2)



Transmission line circuit illustrating the impedance mismatch boundary between the transmission line and the load. Reflections occur at the boundary designated by Γ. The incident wave is V^+ and the reflective wave is V^-.

VSWR can be measured directly with an SWR meter. An RF test instrument such as a vector network analyzer (VNA) can be used to measure the reflection coefficients of the input port (S_{11}) and the output port (S_{22}). S_{11} and S_{22} are equivalent to Γ at the input and output port, respectively. The VNAs with math modes can also directly calculate and display the resulting VSWR value.

The return loss at the input and output ports can be calculated from the reflection coefficient, S_{11} or S_{22}, as follows:

$RL_{IN} = 20\log10|S_{11}|$ dB (Eq. 3)
$RL_{OUT} = 20\log10|S_{22}|$ dB (Eq. 4)

The reflection coefficient is calculated from the characteristic impedance of the transmission line and the load impedance as follows:

$\Gamma = (Z_L - Z_O)/(Z_L + Z_O)$ (Eq. 5)

Where Z_L is the load impedance and Z_O is the characteristic impedance of the transmission line (Figure 1).

VSWR can also be expressed in terms of Z_L and Z_O. Substituting Equation 5 into Equation 2, we obtain:

$VSWR = [1 + |(Z_L - Z_O)/(Z_L + Z_O)|]/[1 - |(Z_L - Z_O)/(Z_L + Z_O)|] = (Z_L + Z_O + |Z_L - Z_O|)/(Z_L + Z_O - |Z_L - Z_O|)$

For $Z_L > Z_O$, $|Z_L - Z_O| = Z_L - Z_O$

Therefore:

$$VSWR = (Z_L + Z_O + Z_L - Z_O)/(Z_L + Z_O - Z_L + Z_O) = Z_L/Z_O. \qquad \text{(Eq. 6)}$$

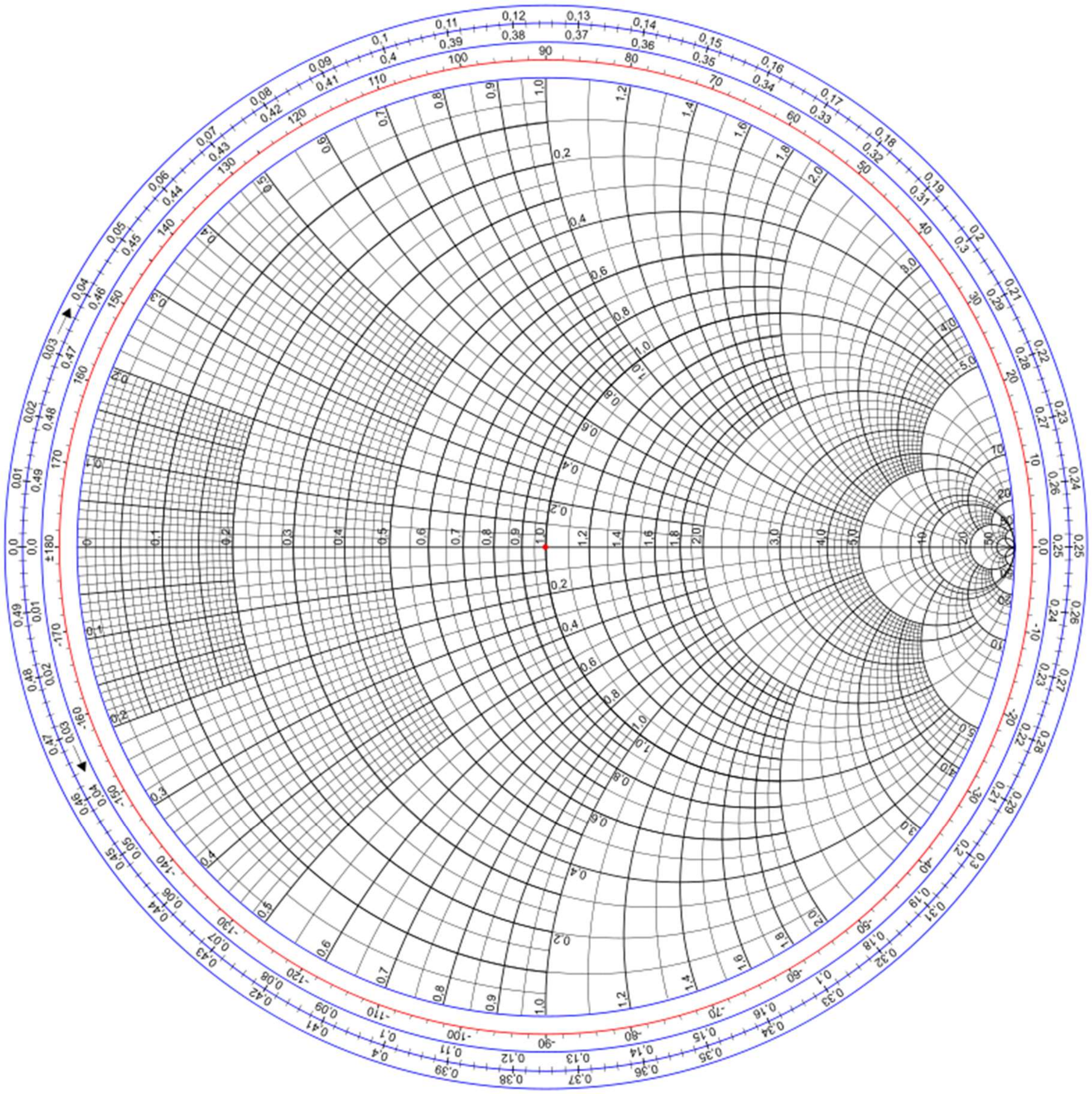
For $Z_L < Z_O$, $|Z_L - Z_O| = Z_O - Z_L$

Smith chart

The Smith chart, invented by Phillip H. Smith (1905–1987), is a graphical aid or nomogram designed for electrical and electronics engineers specializing in radio frequency (RF) engineering to assist in solving problems with transmission lines and matching circuits. The Smith chart can be used to simultaneously display multiple parameters including impedances, admittances, reflection coefficients, scattering parameters, noise figure circles, constant gain contours and regions for unconditional stability, including mechanical vibrations analysis.

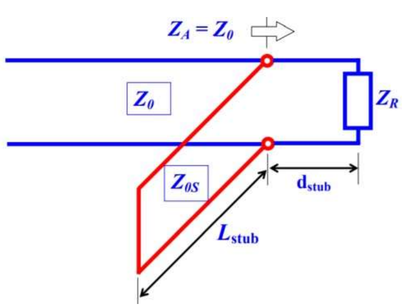
The Smith chart is most frequently used at or within the unity radius region. However, the remainder is still mathematically relevant, being used, for example, in oscillator design and stability analysis.

While the use of paper Smith charts for solving the complex mathematics involved in matching problems has been largely replaced by software based methods, the Smith chart display is still the preferred method of displaying how RF parameters behave at one or more frequencies, an alternative to using tabular information. Thus most RF circuit analysis software includes a Smith chart option for the display of results and all but the simplest impedance measuring instruments can display measured results on a Smith chart display.



Single stub impedance matching

Impedance matching can be achieved by inserting another transmission line (stub) as shown in the diagram below



<u>There are two design parameters for single stub matching</u>: ‰

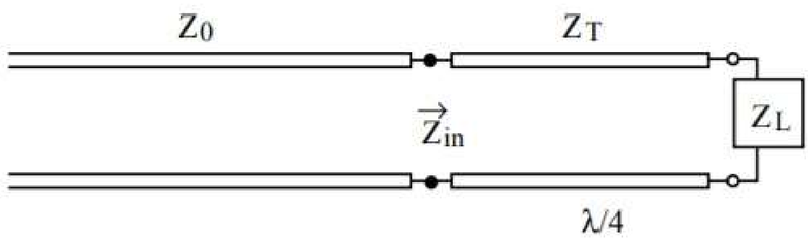
- The location of the stub with reference to the load dstub ‰

- The length of the stub line Lstub

Any load impedance can be matched to the line by using single stub technique. The drawback of this approach is that if the load is changed, the location of insertion may have to be moved. The transmission line realizing the stub is normally terminated by a short or by an open circuit. In many cases it is also convenient to select the same characteristic impedance used for the main line, although this is not necessary. The choice of open or shorted stub may depend in practice on a number of factors. A short circuited stub is less prone to leakage of electromagnetic radiation and is somewhat easier to realize. On the other hand, an open circuited stub may be more practical for certain types of transmission lines, for example microstrips where one would have to drill the insulating substrate to short circuit the two conductors of the line.

Quarter Wave Transforme

A quarter wave transformer, like low-frequency transformers, changes the impedance of the load to another value so that matching is possible.



A quarter-wave transformer uses a section of line of characterstic impedance Z_T of $\frac{\lambda}{4}$ long. To have a matching condition, we want $Z_{in} = Z_0$. From Equation (6.11) we have

$$Z_{in} = Z_T \frac{Z_L + jZ_T \tan\frac{\pi}{2}}{Z_T + jZ_L \tan\frac{\pi}{2}} = \frac{Z_T^2}{Z_L}, \quad (1)$$

since $\tan\beta l = \tan\frac{2\pi}{\lambda}\frac{\lambda}{4} = \tan\frac{\pi}{2} = \infty$. In order for $Z_{in} = Z_0$, we need that

$$Z_T^2 = Z_0 Z_L \Rightarrow Z_T = \sqrt{Z_0 Z_l}. \quad (2)$$

If Z_0 and Z_L are both real, then Z_T is real, and we can use a lossless line to perform the matching. If Z_L is complex, it can be made real by adding a section of line to it.

Module - 3 a) Antenna Characteristics: Radiation Pattern, Beam width, Radiation resistance, Directivity, Gain, Efficiency, Impedance, Polarization, Noise temperature; Friis transmission equation.

b) Radiation characteristics of Herzian dipole antenna; Duality principle.

c) Properties and Typical application:- Half-wave Dipole, Mono pole, Loop antenna, Parabolic & Corner Reflector antenna, Helical antenna, Pyramidal Horn antenna, Micro-Strip patch antenna, Array: Yagi-Uda, Log-Periodic.

A. **Antenna Introduction:** Radiation of EM waves

Electromagnetic (EM) radiation is a form of energy that is all around us and takes many forms, such as radio waves, microwaves, X-rays and gamma rays. Sunlight is also a form of EM energy, but visible light is only a small portion of the EM spectrum, which contains a broad range of electromagnetic wavelengths.

Electromagnetic theory

Electricity and magnetism were once thought to be separate forces. However, in 1873, Scottish physicist James Clerk Maxwell developed a unified theory of electromagnetism. The study of electromagnetism deals with how electrically charged particles interact with each other and with magnetic fields.

There are four main electromagnetic interactions:

- The force of attraction or repulsion between electric charges is inversely proportional to the square of the distance between them.
- Magnetic poles come in pairs that attract and repel each other, much as electric charges do.
- An electric current in a wire produces a magnetic field whose direction depends on the direction of the current. A moving electric field produces a magnetic field, and vice versa.
- Time varying current in a conductor is a source of Electromagnetic (EM) radiation.

Waves and fields

EM radiation is created when an atomic particle, such as an electron, is accelerated by an electric field, causing it to move. The movement produces oscillating electric and magnetic fields, which travel at right angles to each other in a bundle of light energy called a photon. Photons travel in harmonic waves at the fastest speed possible in the universe: 186,282 miles per second (299,792,458 meters per second) in a vacuum, also known as the speed of light. The waves have certain characteristics, given as frequency, wavelength or energy.

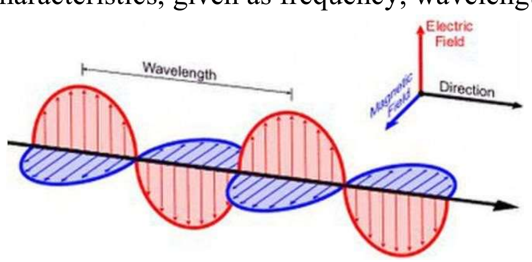


Figure 1-1 Electromagnetic (EM) radiation in free space

The EM spectrum

EM radiation spans an enormous range of wavelengths and frequencies. This range is known as the electromagnetic spectrum. The EM spectrum is generally divided into seven regions, in order of decreasing wavelength and increasing energy and frequency. The common designations are: radio waves, microwaves, infrared (IR), visible light, ultraviolet (UV), Xrays and gamma rays. Typically, lower-energy radiation, such as radio waves, is expressed as frequency; microwaves, infrared, visible and UV light are usually expressed as wavelength; and higher-energy radiation, such as X-rays and gamma rays, is expressed in terms of energy per photon.

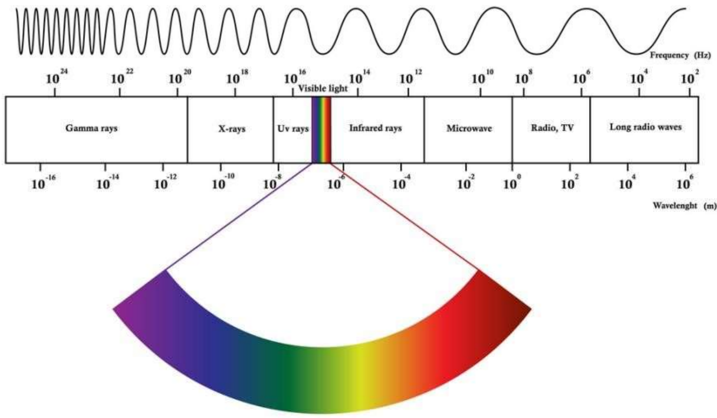


Figure 1-2 The EM spectrum

Introducing Antenna in communication system:

An antenna is a metallic structure that captures and/or transmits radio electromagnetic waves. Antennas come in all shapes and sizes from little ones that can be found on your roof to watch TV to really big ones that capture signals from satellites millions of miles away. Another definition of antenna- It is a transducer that converts radio frequency (RF) fields into alternating current or vice versa. There are both receiving and transmission antennas for sending or receiving radio transmissions. Antennas play an important role in the operation of all radio equipment. They are used in wireless local area networks, mobile telephony and satellite communication.

Theoretically, any structure can radiate EM waves, but not all structure can serve as efficient radiation mechanisms.

Application of antenna:
U.S. Navy's ELF system

- Operates at 76 Hz
- 80 miles of wire
- Penetrates to underwater subs
- One-way system

VHF and UHF Antennas
Wireless Communications

B. **Antenna Characteristics:** Radiation Pattern, Beam Width:

The angular dependence of the radiating and receiving properties of an antenna in the farfield is often referred to as the antenna radiation pattern. Thus, a pattern is a normalized plot of the directivity, gain, or effective aperture as a function of angle and is often given in dB scale. Typically, the radiated normalized radiated power density or radiated field is plotted in dB (for the infinitesimal or ideal dipole, the power density sin2θ is plotted in dB). A typical antenna pattern has a main lobe, sidelobes, minor lobes, a backlobe, and several nulls, as illustrated in Figure 1-3, in a φ = const. plane. The half-power or 3 dB beamwidth of the main lobe (or main beam) is indicated in the drawing. If the pattern of an antenna is given in a plane parallel to the **E** field vector, the corresponding pattern is referred to as an **E** plane pattern. Alternatively, if the pattern cut is in a plane parallel to the **H** field polarization, it is called an **H** plane pattern.

There are many types of antenna radiation patterns, but the most common are

- Isotropic (Ideal case)
- Omnidirectional (azimuthal-plane)
- Pencil beam
- Fan beam
- Shaped beam

Isotropic radiation is the radiation from a point source, radiating uniformly in all directions, with same intensity regardless of the direction of measurement in figure 1-4. The **omnidirectional** pattern is most popular in communication and broadcast applications in figure 1-4. The azimuthal pattern is circular, but the elevation pattern has some directivity to increase the gain in the horizontal direction. The term pencil beam is applied to a highly directive antenna pattern consisting of a major lobe contained within a cone of a small solid angle. Highly directive antenna patterns can be employed for point-to-point communication links and help reduce the required transmitter power. A fan beam is narrow in one direction and wide in the other. A fan beam is typically used in search or surveillance radars. Shaped beam patterns are adapted to the requirements of particular applications.

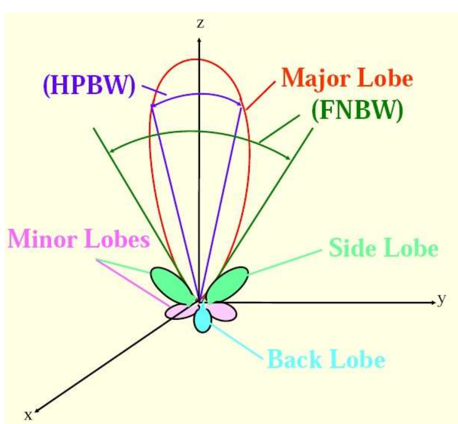


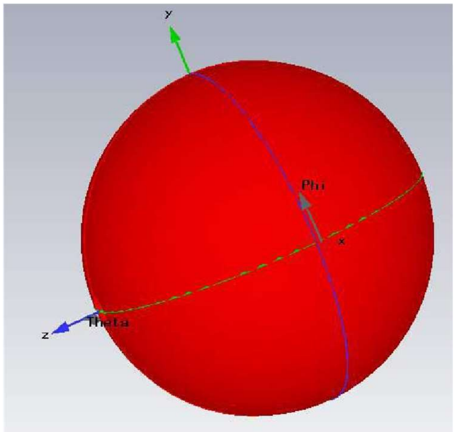
Figure 1-3 Antenna pattern in plane φ = const

Beamwidth between first nulls (FNBW) ~ 2.25 × HPBW (Half Power Beamwidth)

Side Lobe Level (SLL)< 20 dB for satellite and high power applications

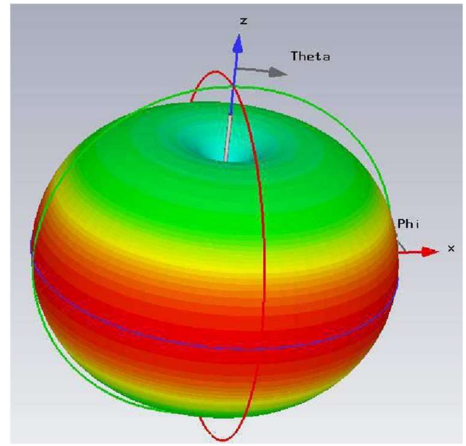
Front to Back Ratio (F/B) > 20 dB

Figure 1-4
Isotropic, Omnidirectional radiation,



Isotropic Radiation Pattern D = 1 = 0dB

Omni-Directional Radiation Pattern of λ/2 Dipole Antenna D = 1.64 = 2.1dB



Radiation Resistance

The power flowing through a circuit is P=V×I, where V is the voltage (defined as energy per unit charge) and I is the current (defined as charge flow per unit time), so P has dimensions of energy per unit time. The physicist George Simon Ohm observed that the current flowing through most materials is proportional to the applied voltage, so many (but not all) objects have a well-defined resistance defined by R=V/I (**Ohm's law**). For them, $P=V\times I=I^2R=V^2/R$. From Ohm's law for time-varying currents,

$$\langle P\rangle = \langle I^2\rangle R$$

If $I=I_0\cos(\omega t)$, then $\langle P\rangle = I_0^2 R/2$

The radiation resistance of an antenna is defined by

$$R_{rad} = 2\frac{\langle P\rangle}{I_0^2}$$

For our short dipole, the radiation resistance is

$$R_{rad} = \frac{2\pi^2}{3c}\left(\frac{l}{\lambda}\right)^2$$

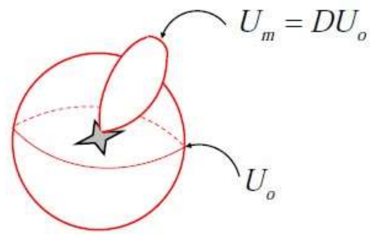
Where c is velocity of EM wave in free space, λ is wave length of operating frequency, l is length of antenna.

Directivity:

Directivity of an antenna is the ratio of radiation density in the direction of maximum radiation to the radiation density averaged over all the directions.

An antenna that radiates equally in all directions would have effectively zero directionality, and the directivity of this type of antenna would be 1 (or 0 dB).

$$D = \frac{maximum\ radiation\ intensity}{average\ radiation\ intensity} = \frac{U_{max}}{U_0}$$



$$D = \frac{U_{max}}{\frac{P_{rad}}{4\pi}} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi U_{max}}{U_{max}\Omega_A} = \frac{4\pi}{\Omega_A}$$

Where Ω_A is beam soild angale

$$\Omega_A = \frac{1}{F(\theta,\varphi)|_{max}} \int_0^{2\pi} \int_0^{\pi} F(\theta,\varphi) \sin\theta \, d\theta d\varphi$$

$F(\theta,\varphi) \cong \left[|E_\theta^0(\theta,\varphi)|^2 + |E_\varphi^0(\theta,\varphi)|^2 \right]$; $E_\theta^0(\theta,\varphi)$ and $E_\varphi^0(\theta,\varphi)$ are radiation field component of θ dircetion and φ direction repectively.

Directivity also calculated $D \cong 4\pi/(\theta_E\theta_H)$; where θ_E *and* θ_H are Half Power Beamwidth in radian at E-plane and H-Plane repectively

Example: For Infinitesimal Dipole

$$\theta_E = \frac{\pi}{2}, \theta_H = 2\pi \rightarrow D = 4\pi/(\theta_E\theta_H) = 4\pi/((\pi/2) \times 2\pi) \cong 1.3 \neq 1.5$$

Directivity is proportional to the Effective Aperture Area of Antenna $D = 4\pi A_{eff}/\lambda^2$ where A_{eff} is the effective area of antenna, λ is wave length of operating frequency.

Gain of Antenna:

Antenna is passive element, so gain of antenna does not mean that ratio between output power and input power. Definition of gain for antenna is that ratio between the powers produced by the antenna from a far-field source on the antenna's beam axis to the power produced by a hypothetical lossless isotropic antenna, which is equally sensitive to signals from all directions.

Antenna gain is more commonly quoted than directivity.

Gain(G) = Efficiency (η) × Directivity (D)

Value of the efficiency of antenna is always less than one due to ohmi loss of antenna and radiation loss of EM wave.

Efficiency:

The efficiency of an antenna is a ratio of the power delivered to the antenna relative to the power radiated from the antenna. A high efficiency antenna has most of the power present at the antenna's input radiated away. A low efficiency antenna has most of the power absorbed as losses within the antenna, or reflected away due to impedance mismatch.

The antenna efficiency (or radiation efficiency) can be written as the ratio of the radiated power to the input power of the antenna:

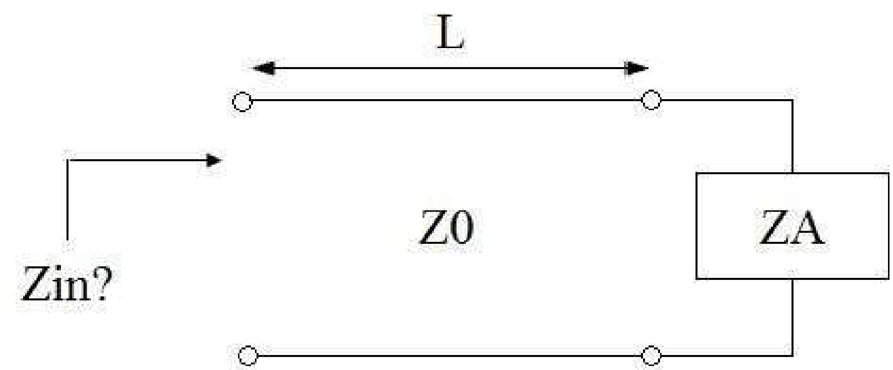
$$\eta = \frac{P_{raditated}}{P_{input}}$$

Impedance:

Total impedance of antenna is $Z_A=R_A+jX_A$

R_A represents power loss from the antenna and X_A gives the power stored in the near field of the antenna.

$R_A = R_{rad} + R_L$ (R_{rad} is radiation resistance, R_L is ohmi loss of antenna)



VSWR:

We see that an antenna's impedance is important for minimizing impedance-mismatch loss. A poorly matched antenna will not radiate power. This can be somewhat alleviated via impedance matching, although this doesn't always work over a sufficient bandwidth.

Reflection coefficient $\Gamma = \frac{Z_A+Z_0}{Z_A-Z_0}$

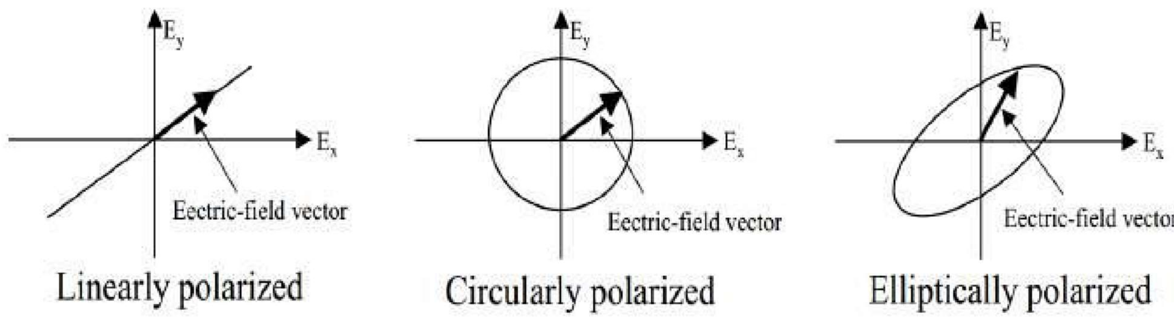
$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

A common measure of how well matched the antenna is to the transmission line or receiver is known as the Voltage Standing Wave Ratio (VSWR). VSWR is a real number that is always greater than or equal to 1. A VSWR of 1 indicates no mismatch loss (the antenna is perfectly matched to the transmission line). Higher values of VSWR indicate more mismatch loss. As an example of common VSWR values, a VSWR of 3.0 indicates about 75% of the power is delivered to the antenna (1.25 dB of mismatch loss); a VSWR of 7.0 indicates 44% of the power is delivered to the antenna (3.6 dB of mismatch loss). A VSWR of 6 or more is pretty high and will generally need to be improved.

Polarization:

Orientation of radiated electric field vector in the main beam of the antenna.

Any electromagnetic wave can be decomposed into two orthogonal polarized components. For example, the transverse electric field can be resolved into horizontal and vertical components, or horizontal and vertical linear polarizations. If the horizontal and vertical electric fields are equal in amplitude and 90^o out of phase, the radiation is circularly polarized. Any radio wave can also be decomposed into left- and right-circular polarizations.



EM field vary with time as $E = \widehat{a_\theta} E_\theta \csc \omega t + \widehat{a_\varphi} E_\varphi \csc(\omega t + \alpha)$

If case1: $\alpha = 0$ or π wave linear polarized.
If case2: $\alpha = \pm\pi/2$ $E_\theta = E_\varphi$ and wave circularly polarized.
If case3: $\alpha = \pm\pi/2$ $E_\theta \neq E_\varphi$ and wave elliptically polarized.

If the wave is essentially random (noise generated by blackbody radiation for example), the two orthogonal components will vary rapidly in intensity but have equal powers when averaged over long times. Such radiation is called unpolarized. Blackbody radiation is unpolarized. Most radio astronomical sources are unpolarized or nearly so.
Any antenna with a single output collects only one of the two polarizations from an electromagnetic wave. For example, a linear dipole antenna collects radiation only from the linear polarization whose electric field is parallel to the antenna wires. Electric fields perpendicular to the dipole antenna do not produce currents in the antenna, so the linear dipole is completely insensitive to the linear polarization perpendicular to its wires. A pair of crossed dipoles is needed to collect power from both orthogonal polarizations simultaneously.

Noise Temperature of Antenna:

A convenient practical unit for the power output per unit frequency from a receiving antenna is the antenna temperature TA. Antenna temperature has nothing to do with the physical temperature of the antenna as measured by a thermometer; it is only the temperature of a matched resistor whose thermally generated power per unit frequency equals that produced by the antenna. It is widely used because:
1 K of antenna temperature is a conveniently small power. TA=1 K corresponds to $P_v = kT_A = 1.38\times10^{-23}$ J $K^{-1}\times1$ K $= 1.38\times10^{-23}$ W Hz^{-1}.
It can be calibrated by a direct comparison with hot and cold loads (another word for matched resistors) connected to the receiver input.

The units of receiver noise are also K, so comparing the signal in K with the receiver noise in K makes it easy to decide if a signal will be detectable.

$$T_A \equiv \frac{P_v}{k}$$

An unpolarized point source of flux density S increases the antenna temperature by

$$T_A = \frac{P_v}{k} = \frac{A_{eff} S}{2k}$$

Where A_{eff} is the effective collecting area.

Effective height or effective length:

In telecommunication, the effective height, or effective length, of an antenna is the height of the antenna's centre of radiation above the ground. It is defined as the ratio of the induced voltage to the incident field.

Receive Aperture:

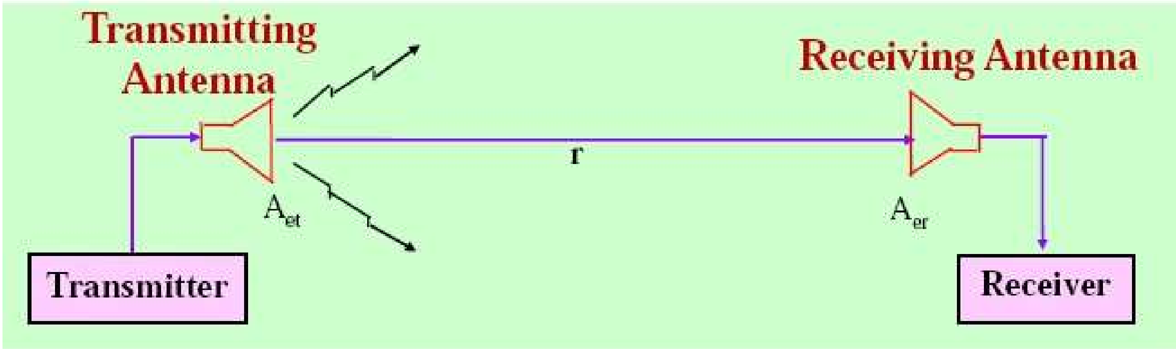
How can we characterize antennas used for receiving, as in radio astronomy, rather than for transmitting? The receiving counterpart of transmitting power gain is the effective area or effective collecting area of an antenna.
Imagine an ideal antenna that collects all of the radiation falling on it from a distant point source and converts it to electrical power—a "rain gauge" for collecting photons. The total spectral power that it collects will be the product of its geometric area A and the incident spectral power per unit area, or flux density S. By analogy, if any real antenna collects spectral power Pv, its effective area Aeff is defined by

$$A_{eff} = \frac{P_v}{S_{(matched)}}$$

Where S(matched) is the flux density in the "matched" polarization.

Friis Equation/ Link Budget:

Consider two antennae separated by a distance r. The transmitting antenna transmits a total power Pt. ($A_{eff} \sim A_e$)



The time-average power density at the receiving antenna is Pavg

$$P_{avg} = \frac{P_t}{4\pi r^2} G_{Dt}$$

Where G_{Dt} is gain of transmitting antenna.

The power received to the load is Pr

$$P_r = P_{avg} \times A_{er} = P_{avg} \times \left(\frac{\lambda^2 G_{Dr}}{4\pi}\right) = \left(\frac{P_t}{4\pi r^2} G_{Dt}\right) \times \left(\frac{\lambda^2 G_{Dr}}{4\pi}\right) = \frac{\lambda^2}{(4\pi r)^2} G_{Dt} G_{Dr} P_t$$

A_{er} is effective area of receiving antenna, G_{Dr} is gain of receiving antenna.

$$\therefore \frac{P_r}{P_t} = \frac{\lambda^2}{(4\pi r)^2} G_{Dt} G_{Dr}$$

This above equetion is called **Friis Equation**. Using this equetion we are define Link Budget as-

What should be the gain of antennas?

What should be the transmitted power?

What should be the sensitivity of receiving antenna?

Radiation Hazards:

Microwave Heating Principle

Microwave radiation causes vibration in the water molecules, which leads to friction and heating. The radiation effects are classified as:

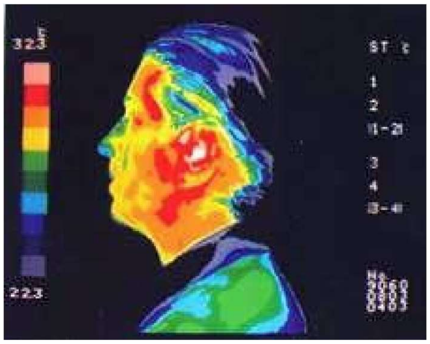
- Non-thermal
- Thermal

Current exposure safety standards are mainly based on the thermal effects, which are inadequate.

Non-thermal effects are several times more harmful than thermal effects.

Cell Phone - Ear Warming?

Have you ever noticed warm sensation in ear after using mobile phone for a long time?



Temperature of ear lobes increases by 10oC when cell phone is used for approx. 20 minutes.

Warm sensation -> pain -> irreve	**All these effects lead to Ear Tumor**

SAR and Cell Phone Use Time Limit

A cell phone transmits 1 to 2 watts of power for calling time.

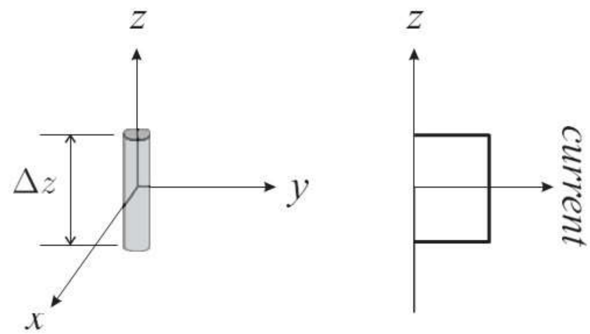
Cell Phone Use Time Limit: 6 minute/day usage

SAR (Specific absorption rate) - Rate at which radiation is absorbed by human body, measured in watts per kg (W/kg).

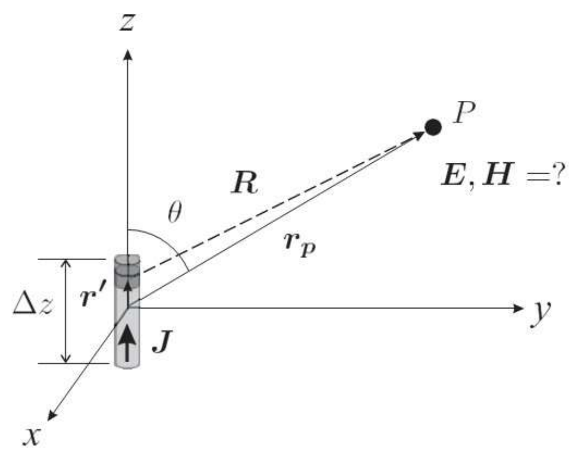
In USA, max. SAR limit for cell phones is 1.6W/Kg which is for 6 minutes. It has a safety margin of 3 to 4, so a person should not use cell phone for more than 18 to 24 minutes per day.
This information is not given to people in India.

Ideal (Hertzian) Dipole

A very simple radiating element we can study is the ideal dipole, also known as Hertzian dipole and infinitesimal dipole. It is very short (length λ), and as such has current uniformly distributed along its length.



Although it is difficult to implement in practice (having a current distribution that is difficult to realize since it is discontinuous), it is highly useful for helping analyze larger wire antennas which can be subdivided into short sections having uniform current (i.e., ideal dipoles). Then, much in the same way as we derived vector potential for a continuous current distribution, we can use superposition to find the fields of a long wire antenna.
Let's orient the ideal dipole along the z-axis and denote the current flowing through the dipole as I. The current has an associated surface current density J.



In this illustration, R is the distance from the current element to the field point P , and r is the distance from the origin to P .
First, we need to derive the vector potential of the line source. It is a continuous current distribution over its length Δl = Δz. Since we only have a z-component of current, A will only have a z-component as well.

Recall

$$\boldsymbol{A} = \int_V \mu \boldsymbol{J} \frac{e^{-jkR}}{4\pi R} dv' = \iiint \mu \boldsymbol{J} \frac{e^{-jkR}}{4\pi R} dx' dy' dz'$$

in Cartesian coordinates. Here,

$$\boldsymbol{J}(\boldsymbol{r}') = \begin{cases} I_0 \delta(x')\delta(y')\hat{\boldsymbol{z}} & \Delta z/2 < z' < \Delta z/2 \\ 0 & \text{elsewhere} \end{cases}$$

since the dipole is infinitely thin. Therefore,

$$\begin{aligned} \boldsymbol{A} &= \hat{\boldsymbol{z}}\mu I_0 \int_{-\infty}^{\infty} \delta(x')dx' \int_{-\infty}^{\infty} \delta(y')dy' \int_{\Delta z/2}^{\Delta z/2} \frac{e^{-jkR}}{4\pi R} dz' \\ &= \hat{\boldsymbol{z}}\mu I_0 \int_{\Delta z/2}^{\Delta z/2} \frac{e^{-jkR}}{4\pi R} dz'. \end{aligned}$$

Evaluating the integral, we first notice that since Δz is small, R does not change significantly as we move along the length of the dipole, (i.e. $r \approx R$). So we can effectively say that R is not a function of z', making the integral simple to evaluate:

$$\boldsymbol{A} = \hat{\boldsymbol{z}}\mu I_0 \frac{e^{-jkr}}{4\pi r} \int_{\Delta z/2}^{\Delta z/2} dz' = \frac{\mu I_0 e^{-jkr}}{4\pi r} \Delta z\, \hat{\boldsymbol{z}}.$$

Now we can find the radiated magnetic field of the dipole:

$$\boldsymbol{H} = \frac{1}{\mu} \boldsymbol{\nabla} \times \boldsymbol{A} = \frac{1}{\mu} \boldsymbol{\nabla} \times A_z\, \hat{\boldsymbol{z}}.$$

Since we know the analysis of point sources revealed spherical wave solutions, it is best to evaluate this curl in spherical coordinates. So first we need to convert A to spherical coordinates:

$$\begin{aligned} A_r &= \boldsymbol{A} \cdot \hat{\boldsymbol{r}} = A_z\, \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{r}} = A_z \cos\theta && (7) \\ A_\theta &= \boldsymbol{A} \cdot \hat{\boldsymbol{\theta}} = A_z\, \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{\theta}} = -A_z \sin\theta && (8) \\ A_\phi &= \boldsymbol{A} \cdot \hat{\boldsymbol{\phi}} = A_z\, \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{\phi}} = 0. && (9) \end{aligned}$$

Reminder: curl in spherical coordinates is

$$\nabla \times \boldsymbol{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{\boldsymbol{r}} & r\,\hat{\boldsymbol{\theta}} & r\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \tag{10}$$

$$\nabla \times \boldsymbol{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{\boldsymbol{r}} & r\,\hat{\boldsymbol{\theta}} & r\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \frac{\mu I_0 e^{-jkr}}{4\pi r}\Delta z\cos\theta & -\frac{\mu I_0 e^{-jkr}}{4\pi}\Delta z \sin\theta & 0 \end{vmatrix} \tag{11}$$

$$\begin{aligned} &= \frac{1}{r^2 \sin\theta}\Big\{\hat{\boldsymbol{r}}\left[\frac{\partial}{\partial\theta}(0) + \frac{\partial}{\partial\phi}\frac{\mu I_0 e^{-jkr}}{4\pi}\Delta z \sin\theta\right] - \\ &\qquad r\,\hat{\boldsymbol{\theta}}\left[\frac{\partial}{\partial r}(0) - \frac{\partial}{\partial\phi}\frac{\mu I_0 e^{-jkr}}{4\pi r}\Delta z\cos\theta\right] + \\ &\qquad r\sin\theta\,\hat{\boldsymbol{\phi}}\left[-\frac{\partial}{\partial r}\frac{\mu I_0 e^{-jkr}}{4\pi}\Delta z\sin\theta - \frac{\partial}{\partial\theta}\frac{\mu I_0 e^{-jkr}}{4\pi r}\Delta z\cos\theta\right]\Big\} \end{aligned} \tag{12}$$

$$= \frac{1}{r^2\sin\theta}\cdot r\sin\theta\,\hat{\boldsymbol{\phi}}\left[\frac{jk\mu I_0 e^{-jkr}}{4\pi}\Delta z\sin\theta + \frac{\mu I_0 e^{-jkr}}{4\pi r}\Delta z\sin\theta\right] \tag{13}$$

$$= \frac{\mu I_0 \Delta z e^{-jkr}}{4\pi}\sin\theta\left(\frac{jk}{r} + \frac{1}{r^2}\right)\hat{\boldsymbol{\phi}}. \tag{14}$$

Now,

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A} = \frac{I_0\Delta z}{4\pi}\left(\frac{jk}{r} + \frac{1}{r^2}\right)e^{-jkr}\sin\theta\,\hat{\boldsymbol{\phi}} \tag{15}$$

$$= \frac{I_0\Delta z}{4\pi}jk\left(1 + \frac{1}{jkr}\right)\frac{e^{-jkr}}{r}\sin\theta\,\hat{\boldsymbol{\phi}}. \tag{16}$$

Next, we find the electric field from Maxwell's curl equation:

$$\boldsymbol{E} = \frac{1}{j\omega\varepsilon}\nabla\times\boldsymbol{H} \tag{17}$$

$$\nabla\times\boldsymbol{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{\boldsymbol{r}} & r\,\hat{\boldsymbol{\theta}} & r\sin\theta\,\hat{\boldsymbol{\phi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ H_r = 0 & H_\theta = 0 & r\sin\theta\frac{I_0\Delta z}{4\pi}\left(\frac{jk}{r} + \frac{1}{r^2}\right)e^{-jkr}\sin\theta \end{vmatrix}. \tag{18}$$

Evaluating the curl in the same manner as for the magnetic field case, we arrive at the final solution for E

$$\boldsymbol{E} = \frac{I_0\Delta z}{2\pi}\eta\left(\frac{1}{r} - \frac{j}{kr^2}\right)\frac{e^{-jkr}}{r}\cos\theta\,\hat{\boldsymbol{r}} + \frac{I_0\Delta z j\omega\mu}{4\pi}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]\frac{e^{-jkr}}{r}\sin\theta\,\hat{\boldsymbol{\theta}}. \tag{19}$$

Now, let's interpret the meaning of all these fields. The first situation we wish to consider is the so-called far field of the antenna, which is analytically defined as when r is large (r » λ)1. Then, all the terms with r in the denominator tend to zero, and we are left with

$$\boldsymbol{E}_{\text{ff}} = \frac{I_0 \Delta z j\omega\mu}{4\pi}\frac{e^{-jkr}}{r}\sin\theta\,\hat{\boldsymbol{\theta}} \quad (20)$$

$$\boldsymbol{H}_{\text{ff}} = \frac{I_0 \Delta z}{4\pi} jk \frac{e^{-jkr}}{r}\sin\theta\,\hat{\boldsymbol{\phi}}. \quad (21)$$

The ratio of E_θ / H_ϕ is

$$\frac{E_\theta}{H_\phi} = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \quad (22)$$

which is also what we found for a plane wave. We shall see that this is a property of radiated fields.

What is the power radiated by the antenna? First we compute the Poynting vector of the far fields components,

$$\boldsymbol{P} = \frac{1}{2}\boldsymbol{E} \times \boldsymbol{H}^* = \frac{1}{2} E_\theta H_\phi^* \hat{\boldsymbol{r}}, \quad (23)$$

since E and H are orthogonal ($\hat{\theta} \times \hat{\varphi} = \hat{r}$). Then,

$$P_r = \frac{1}{2}\frac{I_0 \Delta z j\omega\mu}{4\pi}\frac{e^{-jkr}}{r}\sin\theta \cdot \frac{I_0 \Delta z}{4\pi}(-jk)\frac{e^{jkr}}{r}\sin\theta \quad (24)$$

$$\boldsymbol{P} = \frac{I_0^2 \Delta z^2 \omega\mu k}{2(4\pi r)^2}\sin^2\theta\,\hat{\boldsymbol{r}}. \quad (25)$$

An important observation is that P rolls off as $1/r^2$, indicating that a square-law in power density with distance (i.e. double the distances gives quadruple the loss [-6 dB]). Now we surround the dipole with an imaginary sphere of radius r and compute the power by taking the surface integral of the (radiated) power density:

$$W_{rad} = \int_S \boldsymbol{P} \cdot d\boldsymbol{s}' = \int_0^\pi \int_0^{2\pi} \boldsymbol{S} \cdot r^2 \sin\theta\,\hat{\boldsymbol{r}}\, d\phi\, d\theta \quad (26)$$

$$= 2\pi \int_0^\pi \left(\frac{I_0 \Delta z}{4\pi}\right)^2 \frac{\omega\mu k}{2}\sin^3\theta\, d\theta \quad (27)$$

$$= \frac{(I_0 \Delta z)^2}{12\pi}\omega\mu k, \quad (28)$$

Where

$\int_0^\pi \sin^3\theta\, d\theta = 4/3.$

Since P is real, it is dissipated or radiated power (versus stored [imaginary] power).

Let's focus on the structure of the electric field expression in the far field for a moment, since the magnetic field is readily computed knowing the intrinsic impedance of the medium. We observe that the electric field can be expressed as follows:

$$\boldsymbol{E} = \underbrace{\frac{I_0 \Delta z}{4\pi} j\omega\mu}_{\text{strength factor}} \cdot \underbrace{\frac{e^{-jkr}}{r}}_{\text{distance factor}} \cdot \underbrace{\sin\theta}_{\text{shape/element factor}} \cdot \hat{\boldsymbol{\theta}}. \tag{29}$$

The expression can be separated into the product of three components:

- Strength factor – determined solely by material parameters, magnitude of excitation current, and dipole length
- Distance factor – purely the amplitude decay and phase shift incurred with distance
- Shape factor – determined the radiation pattern of the antenna, or the part that is a function of θ, φ

At this point it is worth comparing the far field electric and magnetic fields to the vector potential in (5). Notice that in the far field, $E_\theta = -j\omega A_\theta$. The dipole only radiates a θ polarized E-field, but it can be shown that if it radiated in the φ-polarization as well, in the far field, $E_\varphi = -j\omega A_\varphi$.

Also, there is no radial component of E in the far-field, nor is there a radial component in the vector potential. Hence, just for far-field electric and magnetic fields, we can say:

$$\boldsymbol{E}_{\text{ff}} \approx -j\omega\boldsymbol{A} \tag{30}$$

$$\boldsymbol{H}_{\text{ff}} \approx \frac{\hat{\boldsymbol{r}}}{\eta} \times \boldsymbol{E}_{\text{ff}} = -j\frac{\omega}{\eta}\hat{\boldsymbol{r}} \times \boldsymbol{A}. \tag{31}$$

These equations form a fast an easy way to determine the far-field radiated electric field, without going through two curl operations as we had to do before.

We have considered the far field quantities to this point. What about the other fields? Since they are not in the far field, they are in the so-called near field of the antenna, or where r $\ll$λ. Examining the expressions for E and H, under this condition the 1/rn terms dominate and we have:

$$\boldsymbol{H}_{\text{nf}} = \frac{I_0 \Delta z e^{-jkr}}{4\pi jkr^2} jk \sin\theta\,\hat{\boldsymbol{\phi}} = \frac{I_0 \Delta z e^{-jkr}}{4\pi r^2}\sin\theta\,\hat{\boldsymbol{\phi}} \tag{32}$$

$$\begin{aligned}\boldsymbol{E}_{\text{nf}} &= \frac{I_0\Delta z}{4\pi} j\omega\mu\left[\frac{1}{jkr} - \frac{1}{(kr)^2}\right]\frac{e^{-jkr}}{r}\sin\theta\,\hat{\boldsymbol{\theta}} + \\ &\quad \frac{I_0\Delta z}{2\pi}\eta\left[\frac{1}{r} - j\frac{1}{kr^2}\right]\frac{e^{-jkr}}{r}\cos\theta\,\hat{\boldsymbol{r}}.\end{aligned} \tag{33}$$

In the expression for $\boldsymbol{E}_{\text{nf}}$, the $1/r^3$ terms dominate for small r, so

$$\boldsymbol{E}_{\text{nf}} = \frac{-I_0\Delta z}{4\pi} j\omega\mu\frac{e^{-jkr}}{k^2 r^3}\sin\theta\,\hat{\boldsymbol{\theta}} - j\frac{I_0\Delta z}{2\pi}\eta\frac{e^{-jkr}}{kr^3}\cos\theta\,\hat{\boldsymbol{r}}. \tag{34}$$

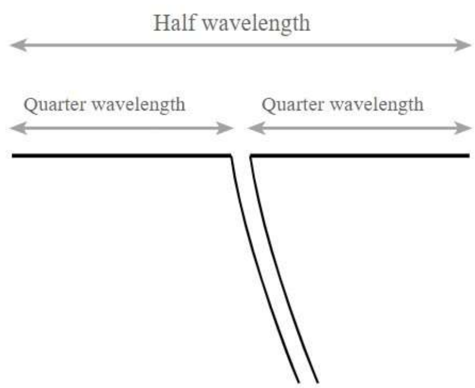
Since $\omega\mu/k = \eta$,

$$\boldsymbol{E}_{\text{nf}} = \frac{-jI_0\Delta z}{4\pi k}\eta\frac{e^{-jkr}}{r^3}\sin\theta\,\hat{\boldsymbol{\theta}} - j\frac{I_0\Delta z}{2\pi}\eta\frac{e^{-jkr}}{kr^3}\cos\theta\,\hat{\boldsymbol{r}}. \tag{35}$$

Half wave dipole basics

The half wave dipole is formed from a conducting element which is wire or metal tube which is an electrical half wavelength long. The half wave dipole is normally fed in the middle where the impedance falls to its lowest. In this way, the antenna consists of the feeder connected to two quarter wavelength elements in line with each other.

It should be remembered that the length of the half wave dipole is an electrical half wavelength for the wave travelling in the antenna conductors. This is slightly shorter than the equivalent length of a wave travelling in free space as the antenna conductors affect the wavelength.

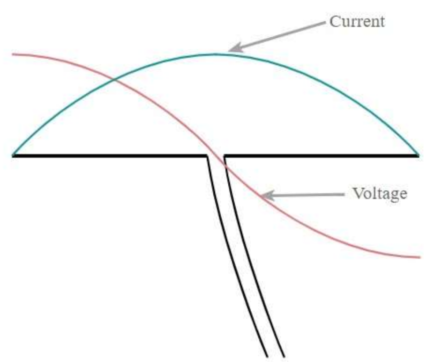


Basic half wave dipole antenna

The voltage and current levels vary along the length of the radiating section of the antenna. This occurs because standing waves are set up along the length of the radiating element. As the ends are open circuit current at these points is zero, but the voltage is at its maximum. As the point at which these quantities is measured moves away from the ends, it is found that they vary sinusoidally: the voltage falling, but the current rising. The current then reaches a maximum and the voltage a minimum at a length equal to an electrical quarter wavelength from the ends. As it is a half wave dipole, this point occurs in the centre.

Half wave dipole feed impedance

One of the major considerations with any antenna is the feed arrangements – how to transfer the power from the feeder / transmission line in to the antenna itself. Impedance matching, balanced or unbalanced and many other aspects need to be considered.
In many aspects the half wave dipole is very easy to feed. The feeder is normally connected to the centre point is where there is a current maximum and a voltage minimum. This results in the antenna presenting a low impedance to the feeder. This is much easier to feed because the high RF voltages associated with high impedance feed arrangements can present many problems for feeders and matching units.

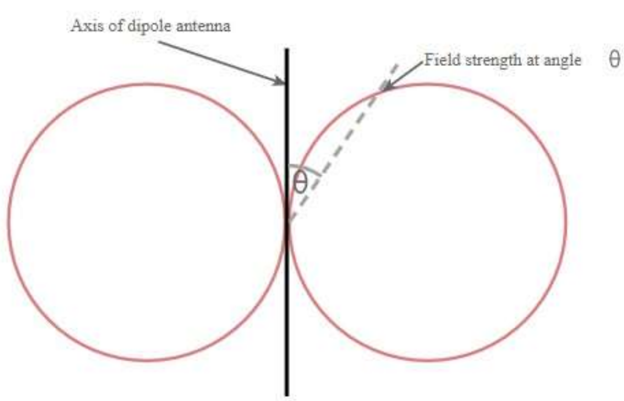


Current and voltage waveforms on a half wave dipole

Half wave dipole radiation pattern & directivity

It is possible to calculate the radiation pattern and hence determine the directivity. As might be expected the maximum half wave dipole directivity shows the maximum radiation at right angles to the main radiator.

At other angles, the angle θ in the half wave dipole formula above can be used to determine the field strength.



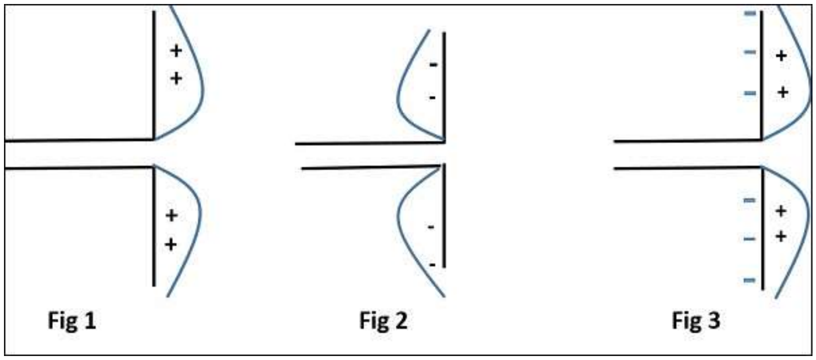
Half wave dipole polar diagram

It is also possible to view the radiation pattern in terms of the plane looking around the dipole antenna, i.e. in the plane cutting the dipole in its field of maximum radiation.

Construction & Working of Half-wave Dipole

It is a normal dipole antenna, where the frequency of its operation is half of its wavelength. Hence, it is called as half-wave dipole antenna.

The edge of the dipole has maximum voltage. This voltage is alternating (AC) in nature. At the positive peak of the voltage, the electrons tend to move in one direction and at the negative peak, the electrons move in the other direction. This can be explained by the figures given below.



The figures given above show the working of a half-wave dipole.
Fig 1 shows the dipole when the charges induced are in positive half cycle. Now the electrons tend to move towards the charge.
Fig 2 shows the dipole with negative charges induced. The electrons here tend to move away from the dipole.
Fig 3 shows the dipole with next positive half cycle. Hence, the electrons again move towards the charge.

Applications

The following are the applications of half-wave dipole antenna − •
Used in radio receivers.

- Used in television receivers.
- When employed with others, used for wide variety of applications.

Loop Antennas

A loop antenna is a type of a radio antenna, which consists of a loop (circular electrical conductor) with ends connected to the transmission line. There are different types of shapes. They are triangular, circular, elliptical, and square shape antennas.
Depends on loop's circumference the loop antenna is classified as two types electrically small and electrically large. The Schematic diagram of the small circular loop antenna with radius a in xyz plane is shown below.

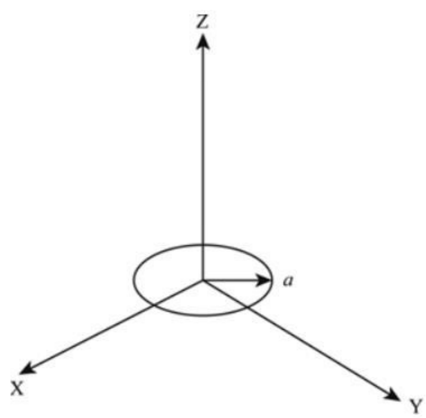


Figure 1

Large loop antennas

Large loop antennas are also called as resonant antennas. They have high radiation efficiency. These antennas have length nearly equal to the intended wavelength.

$$L=\lambda$$

Where,
L is the length of the antenna λ
is the wavelength

The main parameter of this antenna is its perimeter length, which is about a wavelength and should be an enclosed loop. It is not a good idea to meander the loop so as to reduce the size, as that increases capacitive effects and results in low efficiency.

Small loop antennas

Small loop antennas are also called as magnetic loop antennas. These are less resonant. These are mostly used as receivers.
These antennas are of the size of one-tenth of the wavelength.
$L=\lambda/10$

Where,
L is the length of the antenna λ
is the wavelength

The current flowing through the small circular is loop is constant and the value is given by,
$I = I_0$.
The electric field around the loop antenna is computed by following expression.

$$E_\phi = \frac{\eta k^2 a^2 I_0 e^{-jkr} \sin\theta}{4r}$$

The formula to convert electric field intensity to the magnetic field intensity is given as follows.

$$H_\phi = \frac{E_\phi}{\eta}$$

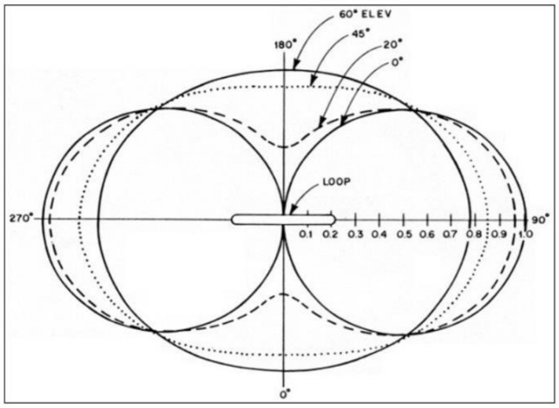
Rewrite the equation.

$$H_\phi = \frac{k^2 a^2 I_0 e^{-jkr} \sin\theta}{4r}$$

The following features are available in small loop antenna.
▪ Very High radiation resistance
▪ Very less radiation frequency due to high losses present in the antennal. ▪
Simple in construction and available in smaller size with less weight.

Radiation Pattern

The radiation pattern of these antennas will be same as that of short horizontal dipole antenna.



The radiation pattern for small, high-efficiency loop antennas is shown in the figure given above. The radiation patterns for different angles of looping are also illustrated clearly in the figure. The tangent line at 0° indicates vertical polarization, whereas the line with 90° indicates horizontal polarization.

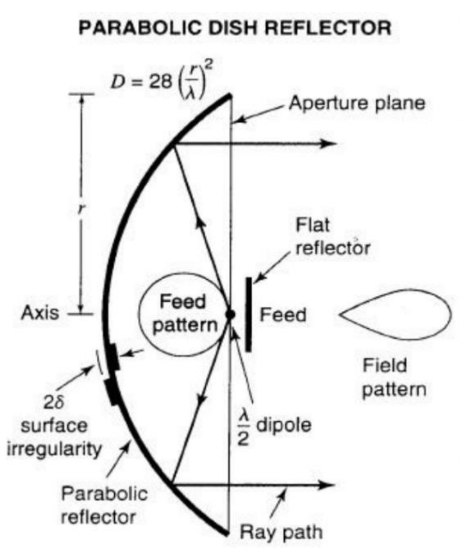
Applications

The following are the applications of Loop antenna −

- Used in RFID devices
- Used in MF, HF and Short wave receivers
- Used in Aircraft receivers for direction finding
- Used in UHF transmitters

Parabolic Reflectors

- A parabolic reflector operates much the same way a reflecting telescope does.
- Reflections of rays from the feed point all contribute in phase to a plane wave leaving the antenna along the antenna bore sight (axis)
- Typically used at UHF and higher frequencies



Applications

The following are the applications of Parabolic reflector antenna −

- The cassegrain feed parabolic reflector is mainly used in satellite communications.
- Also used in wireless telecommunication systems.

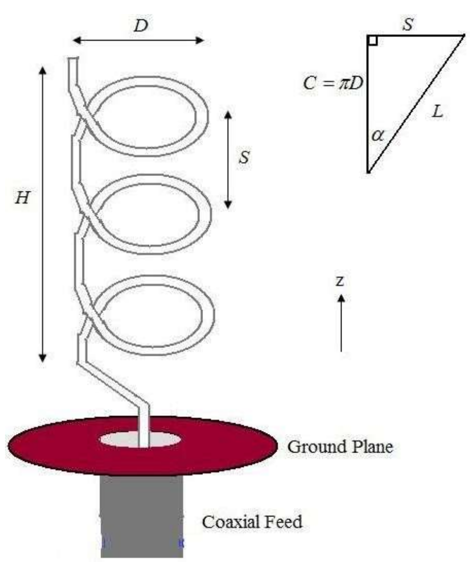
Helical antenna

Helical antenna or helix antenna is the antenna in which the conducting wire is wound in helical shape and connected to the ground plate with a feeder line. It is the simplest antenna, which provides circularly polarized waves. It is used in extra-terrestrial communications in which satellite relays etc., are involved.
Pitch angle is the angle between a line tangent to the helix wire and plane normal to the helix axis.

$$\alpha = \tan^{-1}(\frac{S}{\pi D})$$

where,

- D is the diameter of helix.
- S is the turn spacing (centre to centre).
- α is the pitch angle.



Applications

The following are the applications of Helical antenna −

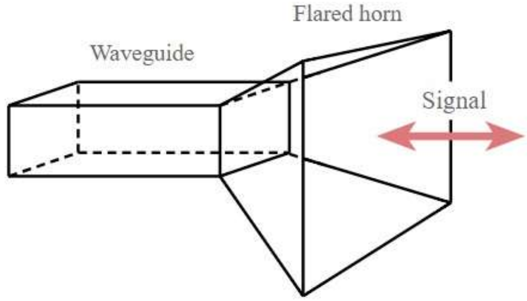
- A single helical antenna or its array is used to transmit and receive VHF signals

- Frequently used for satellite and space probe communications
- Used for telemetry links with ballastic missiles and satellites at Earth stations
- Used to establish communications between the moon and the Earth
- Applications in radio astronomy

Pyramidal Horn antenna

Basic horn antenna concept

The horn antenna may be considered as an RF transformer or impedance match between the waveguide feeder and free space which has an impedance of 377 ohms. By having a tapered or having a flared end to the waveguide the horn antenna is formed and this enables the impedance to be matched. Although the waveguide will radiate without a horn antenna, this provides a far more efficient match.



Microwave horn antenna

In addition to the improved match provided by the horn antenna, it also helps suppress signals travelling via unwanted modes in the waveguide from being radiated.
However the main advantage of the horn antenna is that it provides a significant level of directivity and gain. For greater levels of gain the horn antenna should have a large aperture. Also to achieve the maximum gain for a given aperture size, the taper should be long so that the phase of the wave-front is as nearly constant as possible across the aperture. However there comes a point where to provide even small increases in gain, the increase in length becomes too large to make it sensible. Thus gain levels are a balance between aperture size and length. However gain levels for a horn antenna may be up to 20 dB in some instances. When the horn needs to be used with coax, a small section of waveguide is required in which a waveguide to coax transition is located.

There are several horn configurations out of which, three configurations are most commonly used.

Sectoral horn

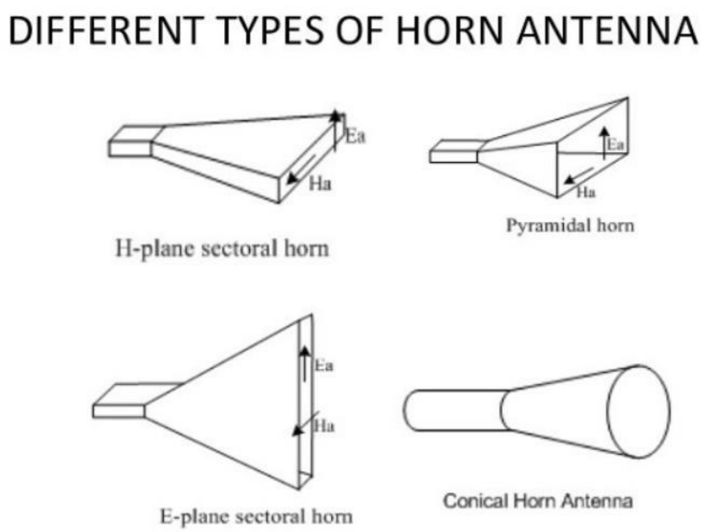
This type of horn antenna, flares out in only one direction. Flaring in the direction of Electric vector produces the sectorial E-plane horn. Similarly, flaring in the direction of Magnetic vector, produces the sectorial H-plane horn.

Pyramidal horn

This type of horn antenna has flaring on both sides. If flaring is done on both the E & H walls of a rectangular waveguide, then pyramidal horn antenna is produced. This antenna has the shape of a truncated pyramid.

Conical horn

When the walls of a circular wave guide are flared, it is known as a conical horn. This is a logical termination of a circular wave guide.



Applications

The following are the applications of Horn antenna − •
Used for astronomical studies

- Used in microwave applications

Yagi-Uda antenna

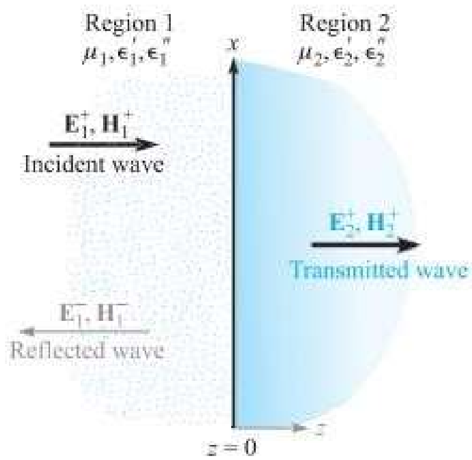
The Yagi-Uda antenna or Yagi Antenna is one of the most brilliant antenna designs. It is simple to construct and has a high gain, typically greater than 10 dB. The Yagi-Uda antennas typically operate in the HF to UHF bands (about 3 MHz to 3 GHz), although their bandwidth is typically small, on the order of a few percent of the center frequency. You are probably familiar with this antenna, as they sit on top of roofs everywhere. TV antennas are still a major application of the Yagi antenna.

- Driven element induces currents in parasitic elements
- When a parasitic element is slightly longer than $\lambda/2$, the element acts inductively and thus as a reflector -current phased to reinforce radiation in the maximum direction and cancel in the opposite direction .

- The director element is slightly shorter than λ/2, the element acts inductively and thus as a director -current phased to reinforce radiation in the maximum direction and cancel in the opposite direction • The elements are separated by ≈0.25

Module –4
Reflection of plane wave at Normal and Oblique incidence; Diffraction and Scattering Phenomena, multipath fading and its characteristics.

Reflection of plane wave at Normal and Oblique incidence



We define region 1 (ε_1, μ_1) as the half-space for which z < 0; region 2 (ε_2, μ_2) is the half-space for which z > 0. Initially we establish a wave in region 1, traveling in the +z direction, and linearly polarized along x.

$$\mathcal{E}_{x1}^+(z,t) = E_{x10}^+ e^{-\alpha_1 z}\cos(\omega t - \beta_1 z)$$

Associated magnetic field in the y direction

$$H_{ys1}^+(z) = \frac{1}{\eta_1} E_{x10}^+ e^{-jk_1 z}$$

This uniform plane wave in region l that is traveling toward the boundary surface at z = 0 is called the incident wave. Since the direction of propagation of the incident wave is perpendicular to the boundary plane, we describe it as normal incidence. Energy may be transmitted across the boundary surface at z = 0 into region 2 by providing a wave moving in the +z direction in that medium. The phasor electric and magnetic fields for this wave are

$$E_{xs2}^+(z) = E_{x20}^+ e^{-jk_2 z}$$

$$H_{ys2}^+(z) = \frac{1}{\eta_2} E_{x20}^+ e^{-jk_2 z}$$

This wave, which moves away from the boundary surface into region 2, is called the transmitted wave. Note the use of the different propagation constant k_2 and intrinsic impedance η_2. To satisfy the boundary conditions at z = 0 with these assumed fields. With E polarized along x, the field is tangent to the interface, and therefore the E fields in regions l and 2 must be equal at z = 0. Setting z = 0 and would require that $E_{x10}+ = E_{x20}+$. H, being y-directed, is also a tangential field, and must be continuous across the boundary (no current sheets are present in real media). When we let z = 0 we find that we must have $E_{x10}+/\eta_1 = E_{x20}+/\eta_2$. Since $E_{x10}+ = E_{x20}+$, then $\eta_1 = \eta_2$. But this is a very special condition that does not fit the facts in general, and we are therefore unable to satisfy the boundary conditions with only an incident and a transmitted wave.

We require a wave traveling away from the boundary in region 1, as shown in Figure; this is the reflected wave,

$$E_{xs1}^{-}(z) = E_{x10}^{-}\, e^{jk_1 z}$$

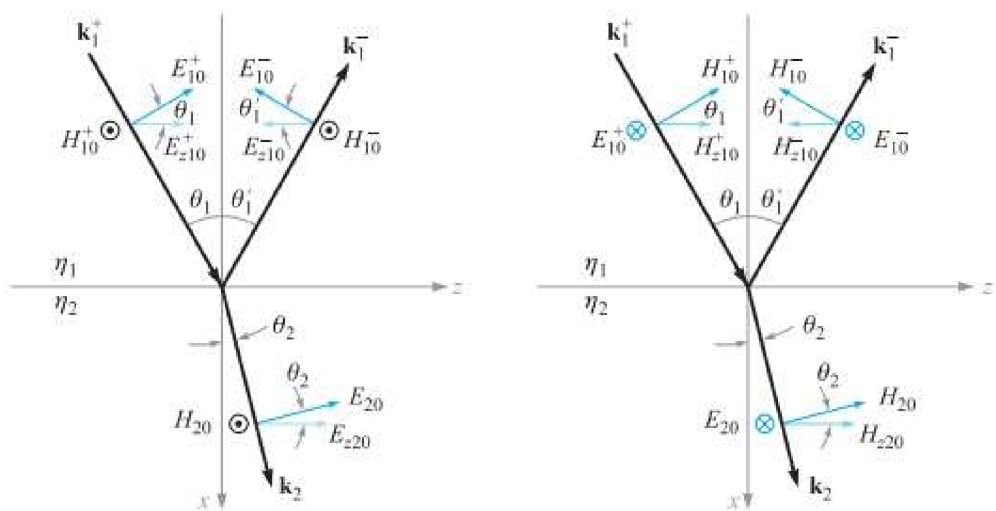
$$H_{xs1}^{-}(z) = -\frac{E_{x10}^{-}}{\eta_1}\, e^{jk_1 z}$$

where $E_{x10}-$ may be a complex quantity. Because this field is traveling in the –z direction.

The ratio of the amplitudes of the reflected and incident electric fields defines the reflection coefficient, designated by

$$\Gamma = \frac{E_{x10}^{-}}{E_{x10}^{+}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| e^{j\phi}$$

Plane Wave Reflection at Oblique Incidence Angle



The situation is illustrated in Figure in which the incident wave direction and position-dependent phase are characterized by wavevector **k**+ 1 . The angle of incidence is the angle between **k**+ 1 and a line that is normal to the surface (the x axis in this case). The incidence angle is shown as θ_1. The reflected wave, characterized by wavevector **k-** 1 , will propagate away from the interface at angle θ_1. Finally, the transmitted wave, characterized by **k**2, will propagate into the second region at angle θ_2 as shown. One would suspect (from previous experience) that the incident and reflected angles are equal (θ_1 = θ_1), which is correct. We need to show this, however, to be complete. The two media are lossless dielectrics, characterized by intrinsic impedances $\eta 1$ and $\eta 2$. We will assume, as before, that the materials are nonmagnetic, and thus have permeability $\mu 0$. Consequently, the materials are adequately described by specifying their dielectric constants, $r1$ and $r2$, or their refractive indices, $n1 = \sqrt{\varepsilon_{r1}}$ and $n2 = \sqrt{\varepsilon_{r2}}$.

$$k_1 \sin\theta_1 = k_2 \sin\theta_2$$