GURU NANAK INSTITUTE OF TECHNOLOGY

An Autonomous Institute under MAKAUT

2022

MATHEMATICS-III

M301

TIME ALLOTTED: 3HR **FULL MARKS:70**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable

GROUP – A

(Multiple Choice Type Questions)

Answer any ***ten*** from the following, choosing the correct alternative of each question: **10×1=10**

			Marks	CO No
1.	i)	The function f(z)=$\bar{z}$ is a) an essential singularity at z=0 b) a pole of order 4 at z=0 c) a simple pole at z=0 d) no singularity at z=0	1	CO4
	ii)	The value of the integral $\int_C \frac{e^z}{(z-2)^2} dz$ where $C: \lvert z-2\rvert = 1$ is traversed in the counter clockwise sense, is a) $\pi i e^2$ b) $-\pi i e^2$ c) $2\pi i e^2$ d) $2\pi i.$	1	CO3
	iii)	If the random variable X has the Binomial distribution with parameters n and p, then the mean and variance are respectively a) np and np(1-p) b) n and np c) np and np d) np and np^2	1	CO2
	iv)	The distribution for which mean and variance are equal is a) Poisson b) Binomial c) Normal d) None	1	CO4
	v)	The order of the pole $z=0$ of the function $f(z)=\frac{\cos z}{z^3}$ is a) 1 b) 2 c) 3 d) 4	1	CO2

vi) If $f(x)$ is an odd function, then $F(f(x))$ is given by 1 CO2

a) $F(s)=2F_s(s)$

b) $F(s)=2iF_s(s)$

c) $F_s(s)=2F(s)$

d) $F_s(s)=2iF(s)$

vii) The function $f(z)=\frac{e^{z^2}}{z^4}$ has 1 CO1

a) an essential singularity at $z=0$.

b) a pole of order 4 at $z=0$.

c) a simple pole at $z=0$.

d) no singularity at $z=0$.

viii) The Fourier coefficient b_n of the function $f(x)=x^2, -\pi \le x \le \pi$ is 1 CO5

a) $\frac{2\pi^3}{3}$

b) $(-1)^n \frac{4}{3}$

c) 0

d) none of these

ix) The period of the function $f(x)=\sin 2\pi x$ is 1 CO1

a) $\frac{1}{2}$

b) 1

c) 0

d) $\frac{1}{3}$

x) If two events A and B are independent then 1 CO2

a) $P(A \cup B) = P(A) + P(B)$

b) $P(A \cup B) = P(A) \,.\, P(B)$

c) $P(A \cap B) = P(A) + P(B)$

d) $P(A \cap B) = P(A) \,.\, P(B)$

xi) X has normal distribution with s.d. 2 ad z is standard normal variate. If X=9.6 implies z=-0.9, then the mean of X is 1 CO5

a) 11

b) 11.4

c) 12

d) 13

xii) If X is normally distributed with zero mean and unit variance, then $E(X^2)$ is — 1 — CO4

a) 1
b) 2
c) 8
d) 20

GROUP – B
(Short Answer Type Questions)

(Answer any *three* of the following) — **3 x 5 = 15**

		Marks	CO No
2.	Expand f(x)=x in Fourier Series on the interval $-\pi \le x \le \pi$.	5	CO5
3.	Prove that $\lim_{z\to 0}\frac{\bar{z}}{z}$ does not exist.	5	CO4
4.	If a random variable has a Poisson distribution such that P(1)=P(2), find i) mean of the distribution ii) standard deviation. iii) P(X=4)	5	CO2
5.	A variable X has the following density function $f(x) = \frac{x}{2}, \quad 0 \le x \le 1$ $= \frac{1}{2}, \quad 1 < x \le 2$ Find the mean and variance and also find $E\{(X-1)^2\}$	5	CO4
6.	Solve by method of separation of variable $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial t}$ where $u(0,t) = 8e^{-3t}$.	5	CO3

GROUP – C
(Long Answer Type Questions)

(Answer any *three* of the following) — **3 x 15 = 45**

		Marks	CO No
7. a)	Find the Fourier series of $f(x)=\lvert x\rvert, -\pi < x < \pi$. Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\ldots=\frac{\pi^2}{8}$.	9	CO5
b)	Find Fourier sine transform of $\frac{e^{-ax}}{x}$	6	CO4
8. a)	Evaluate $\int_0^{2\pi}\frac{dt}{1+a^2+2a\cos t}, 0 < a < 1.$	5	CO3
b)	If $f(z)$ is analytic, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$	5	CO3
c)	Evaluate: $\oint_C \frac{e^z}{(z^2+\pi^2)^2}dz$, where C is the circle $\lvert z\rvert = 4$	5	CO2

9. a)	Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, subject to the conditions u(0,y)=u(l,y)=u(x,0)=0, u(x,a)= $\sin\frac{\pi x}{l}x$	9	CO4
b)	A random variable X has the following probability mass function X: 0 1 2 3 4 5 6 7 P(X):0 k 2k 2k 3k k^2 $2k^2$ $7k^2+k$ i) Determine k. ii) Find P(X > 5), P(2 < X < 6) iii) Obtain the probability distribution F(x)	6	CO3, CO4
10. a)	Show that the function u=cos x cos hy is harmonic function.	4	CO4
b)	If the weekly wages of 10000 workers in a factory follows normal distribution with mean and s.d. Rs. 70 and Rs. 5 respectively, find the expected number of workers whose weekly wages are i)between Rs. 66 and Rs. 72. Ii) less than Rs. 66 iii) more than Rs. 72.	7	CO4
c)	Find the mean and variance of a Binomial variate X with parameter μ.	4	CO3
11. a)	Expand the function $f(z)=(z^2-1)/(z+2)(z+3)$ when i) IZI<2 ii) 2< IZI<3 iii) IZI>3	8	CO3
b)	Given $\int_0^\infty f(x)\cos ax\,dx = 1-a\,, 0<a<1$ $= 0$, a>1 Find f(x). Hence evaluate $\int_0^\infty \frac{sin^2 x}{x^2}dx$	7	CO4