Autonomy
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B.TECH/CSE/ECSE/IT/ECE/EE/FT/ODD/SEM-I/M101/R21/2022-23

GURU NANAK INSTITUTE OF TECHNOLOGY

An Autonomous Institute under MAKAUT

2022-2023

MATHEMATICS I

M101

TIME ALLOTTED: 3Hours **FULL MARKS:70**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable

GROUP – A

(Multiple Choice Type Questions)

Answer any *ten* from the following, choosing the correct alternative of each question: **10×1=10**

	Question	Marks	CO No.
1. i)	$\int_2^3 \int_4^6 dx\, dy=$ a. 1 b. 12 c. 2 d. None of these	1	CO2
ii)	$\sum u_n$ and $\sum v_n$ be two series of positive real numbers , such that $lim \frac{u_n}{v_n} = 5$, and $\sum v_n$ is divergent then $\sum u_n$ is a. convergent b. divergent c. oscillatory d. None	1	CO1
iii)	$\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ be a homogeneous function of degree a. 0 b. 2 c. 1/2 d. None	1	CO2
iv)	If $\lambda^3 - 6\lambda^2 + 9\lambda = 4$ is the characteristic equation of a square matrix A, then A^{-1} is a. $A^2 - 6A + 9I$ b. $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}I$ c. $\frac{1}{4}A^2 - \frac{3}{2}A + \frac{9}{4}$ d. $A^2 - 6A + 9$	1	CO2

v) Which of the following theorem can be applied on f(x)=IxI in the interval [-1, 1] 1 CO2
a. Rolle's theorem
b. Lagrange Mean value theorem
c. Cauchy Mean value theorem
d. None of these

vi) The sum of the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots \ldots \ldots \ldots$ is 1 CO2
a. 1
b. 0
c. 1/2
d. Does not exist

vii) 1 CO2

$$\int_0^1 \int_y^{\sqrt{y}} dxdy =$$

a. 1/2
b. 1/6
c. 2/3
d. 4/3

viii) Sum of two homogeneous functions is homogeneous if 1 CO1
a. they have same degree of homogeneity
b. they are both continuous
c. they are both derivable
d. none of these

ix) The critical point of the function f(x,y)=xy 1 CO2
a. (1,1)
b. (1,-1)
c. (-1,1)
d. (0,0)

x) 1 CO2

The eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ are

a. 0,0,1
b. 1,2,3
c. 2,3,6
d. None of these

xi) The value of m such that 1 CO1
$\vec{A}=(mxy-z^3)\hat{\imath}+(m-2)x^2\hat{\jmath}+(1-m)xz^2\hat{k}$ is irrotational is
a. 0
b. 4
c. -4
d. None

		Marks	CO No.
xii)	If a function f(x,y) has minimum or maximum value at the points (2,3) then $f_x(2,3)$= a. 1 b. 0 c. Any non zero value d. None	1	CO2

GROUP – B
(Short Answer Type Questions)
(Answer any *three* of the following) **3 x 5 = 15**

		Marks	CO No.
2.	Find maxima and minima of the function f(x,y)=$x^3 + y^3 - 12x - 27y + 10$, also find the saddle points.	5	CO2
3.	Test the convergence of the power series $\frac{x}{1} + \frac{1}{2}\cdot\frac{x^2}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \frac{1\cdot 3 1.5}{214\cdot 6}\cdot\frac{x^7}{7} + \cdots\infty$	5	CO3
4.	Diagonalize the matrix, if possible $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$	5	CO2
5.	Evaluate the double integral $\int_0^{\pi/2} \int_{\pi/2}^{\pi} e^x \cos(y-x)\, dy\, dx$	5	CO2
6.	If x=rsin θ cos φ, y=$r\sin\theta\sin\varphi$, z=$r\cos\theta$, then find $J\left(\frac{x,y,z}{r,\theta,\varphi}\right)$.	5	CO2

GROUP – C
(Long Answer Type Questions)
(Answer any *three* of the following) **3 x 15 = 45**

		Marks	CO No.
7. a)	Reduce to Normal form the following matrix $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$ and find its rank.	7	CO1
b)	Find the eigen values and the corresponding eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	8	CO1
8. a)	If u= $\cos^{-1}\left(\frac{x^3+y^3}{\sqrt{x+y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-5}{2}\cot u$.	4	CO3
b)	If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{zx}\right)$ show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$	5	CO3
c)	If $u = \sin^{-1}\left\{\frac{x^{1/3}+y^{1/3}}{\sqrt{x}+\sqrt{y}}\right\}^{1/2}$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$	6	CO3

9. a)	Evaluate by Green's theorem $\oint_c \{(\cos x \sin y - xy)dx + (\sin z \cos y)dy\}$ where C is the circle $x^2 + y^2 = 1$.	7	CO3
b)	Evaluate $\iiint_V z(x^2 + y^2)\, dzdydx$ where V is the volume of the cylinder $x^2 + y^2 = 1$ intercepted by the planes z=2 and z=3.	8	CO4
10. a)	Verify Cauchy's Mean Value Theorem for the following pair of functions $f(X) = \sqrt{x},\ g(x) = 1/\sqrt{x};\ x \in [1, 2]$	5	CO2
b)	State Lagrange's Mean Value Theorem. Write Taylor's formula for the function $f(x) = \log(1+x), -1 < x < \infty$ about $x = 2$ with Lagrange's form of remainder after 3 terms.	6	CO4
c)	Test the convergence of the series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2}$	4	CO2
11. a)	Use Cayley-Hamilton theorem to find A^{-1}, where $A=\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$	5	CO3
b)	In what direction from the point (1,2 ,3), the directional derivative of $f(x, y, z) = x^2 - y^2 + 2z^2$ is a maximum? Also find the value of this maximum directional derivative.	6	CO4
c)	Show that $A = \frac{1}{2}(\hat{i}x^2 + \hat{j}y^2 + \hat{k}z^2)$ is irrotational.	4	CO2