ENGINEERING MECHANICS [EE(ME)301]

Online Courseware (OCW)

B.TECH (2nd YEAR – 3rd SEM)

(2022-23)

Prepared by: **Mr. Sourav Majumdar**

Department of Applied Science & Humanities

Guru Nanak Institute of Technology
(Affiliated to MAKUT, West Bengal , Approved by AICTE - Accredited by NAAC – 'A+' Grade)
157/F Nilgunj road, Panihati, Kolkata-700114, West Bengal

Guru Nanak Institute of Technology

Online Courseware-UG Program

Applied Science & Humanities Department

(Section: Mechanical Engineering)

2nd Year-1st Semester (Electrical Engineering)

Paper Name: **Engineering Mechanics**

Paper Code: **EE(ME)301**

Contact hours/week: **3L:0T:0T**

Credit: **3;** Total Lecture: **36**

Full Marks = **100** (**30** for Continuous Evaluation; **70** for End Semester Exam)

Prerequisite: Basic Concept of Physics

Course Outcomes: After successful completion of the course, student will be able to

- **CO1.** To understand representation of force, moments for drawing free-body diagrams and analyze friction-based systems in static condition
- **CO2.** To locate the centroid of an area and calculate the moment of inertia of a section.
- **CO3.** Apply of conservation of momentum & energy principle for particle dynamics and rigid body kinetics
- **CO4.** Understand and apply the concept of virtual work, rigid body dynamics and systems under vibration.

Syllabus:

Module 1: Introduction to Engineering Mechanics 8L
Force Systems Basic concepts, Particle equilibrium in 2-D & 3-D; Rigid Body equilibrium; System of Forces, Coplanar Concurrent Forces, Components in Space – Resultant- Moment of Forces and its Application; Couples and Resultant of Force System, Equilibrium of System of Forces, Free body diagrams, Equations of Equilibrium of Coplanar Systems and Spatial Systems; Vector Mechanics- dot product, cross product, Problems

Module 2: Friction 4L
Types of friction, Limiting friction, Laws of Friction, Static and Dynamic Friction; Motion of Bodies, wedge friction, screw jack & differential screw jack, Problems.

Module 3: Basic Structural Analysis 4L
Equilibrium in three dimensions; Method of Sections; Method of Joints; How to determine if a member is in tension or compression; Simple Trusses; Zero force members; Beams & types of beams; Frames & Machines, Problems.

Module 4: Centroid and Centre of Gravity 4L

Distributed Force: Centroid and Centre of Gravity; Centroids of a triangle, circular sector, quadrilateral, etc., Centroid of simple figures from first principle, centroid of composite sections; Centre of Gravity and its implications, Problems.

Module 5: Moment of Inertia 4L

Area moment of inertia- Definition, Moment of inertia of plane sections from first principles, Theorems of moment of inertia, Moment of inertia of standard sections and composite sections; Mass moment inertia of circular plate, Cylinder, Cone, Sphere, Hook, Problems.

Module 6: Virtual Work and Energy Method 3L

Virtual displacements, principle of virtual work for particle and ideal system of rigid bodies, degreesof freedom. Active force diagram, systems with friction, mechanical efficiency. Conservative forcesand potential energy (elastic and gravitational), energy equation for equilibrium. Applications of energy method for equilibrium. Stability of equilibrium, Problems.

Module 7: Review of particle dynamics 5L

rectilinear motion; Plane curvilinear motion (rectangular, path, and polar coordinates). 3-D curvilinear motion; Relative and constrained motion; Newton's 2nd law (rectangular, path, and polarcoordinates). Work-kinetic energy, power, potential energy. Impulse-momentum (linear, angular); Impact (Direct and oblique), Problems

Module 8: Introduction to Kinetics of Rigid Bodies 4L

Basic terms, general principles in dynamics; Types of motion, Instantaneous center of rotation in plane motion and simple problems; D'Alembert's principle and its applications in plane motion and connected bodies; Work energy principle and its application in plane motion of connected bodies; Kinetics of rigid body rotation, Problems.

Text Books:

1. Irving H. Shames (2006), Engineering Mechanics, 4th Edition, Prentice Hall
2. F. P. Beer and E. R. Johnston (2011), Vector Mechanics for Engineers, Vol I - Statics, Vol II, – Dynamics, 9th Ed, Tata McGraw Hill
3. R.C. Hibbler (2006), Engineering Mechanics: Principles of Statics and Dynamics, Pearson Press.
4. Andy Ruina and Rudra Pratap (2011), Introduction to Statics and Dynamics, Oxford University Press
5. Shanes and Rao (2006), Engineering Mechanics, Pearson Education,
6. Hibler and Gupta (2010), Engineering Mechanics (Statics, Dynamics) by Pearson Education

Reference Books:

1. Reddy Vijaykumar K. and K. Suresh Kumar (2010), Singer's Engineering Mechanics
2. Bansal R. K. (2010), A Text Book of Engineering Mechanics, Laxmi Publications
3. Khurmi R.S. (2010), Engineering Mechanics, S. Chand & Co.
4. Tayal A.K. (2010), Engineering Mechanics, Umesh Publications

Lesson plan

Date	Module No	Lecture No	Topic	Method of Teaching	Reference Book
	1	1	Force Systems Basic concepts, Particle equilibrium in 2-D & 3-D; Rigid Body equilibrium	Demonstration	1. Engineering Mechanics (vol-I & II) – Meriam & Kraig, 5th ed. Wiley India 2.Engineering Mechanics by Nag, Pati & Jana McGraw-Hill Education Pvt. Ltd. (New Delhi) 3. Engineering Mechanics: Statics & Dynamics by I. H. Shames, 4th ed. PHI 4. Engineering Mechanics by Timoshenko, Young and Rao, revised 4th ed. TMH 5. Engineering Mechanics by Basudeb Bhattacharyya – Oxford Unversity
		2	System of Forces, Coplanar Concurrent Forces, Components in Space – Resultant	Demonstration	
		3	Moment of Forces and its Application	Demonstration	
		4	Couples and Resultant of Force System, Equilibrium of System of Forces	Demonstration	
		5	Free body diagrams	Case Study	
		6	Equations of Equilibrium of Coplanar Systems	Case Study	
		7	Spatial Systems	Demonstration	
		8	Vector Mechanics- dot product, cross product, Problems	Demonstration	
	2	9	Types of friction, Limiting friction, Static and Dynamic Friction	Demonstration	
		10	Laws of Friction, Motion of Bodies	Demonstration	
		11	Wedge friction	Case Study	
		12	Screw jack & differential screw jack	Demonstration	
	3	13	Equilibrium in three dimensions; Method of Sections	Demonstration	
		14	Method of Joints; How to determine if a member is in tension or compression;	Case Study	
		15	Simple Trusses; Zero force members; Beams & types of beams	Demonstration	
		16	Frames & Machines, Problems.	Case Study	
	4	17	Distributed Force: Centroid and Centre of Gravity; Centroids of a triangle, circular sector, quadrilateral, etc.	Demonstration	

		18	Centroid of simple figures from first principle	Case Study	Press 6. Engineering Mechanics: Statics & Dynamics by Hibbeler & Gupta 7. Vector Mechanics for engineers: Statics & Dynamics by Beer & Johnston 6th ed. McGraw-Hill 8. Elements of strength of Materials by Timoshenko & Young, 5th ed. E.W.P
		19	centroid of composite sections	Demonstration	
		20	Centre of Gravity and its implications, Problems	Demonstration	
	5	21	Area moment of inertia Definition, Moment of inertia of plane sections from first principles	Demonstration	
		22	Theorems of moment of inertia,		
		23	Moment of inertia of standard sections and composite sections;	Case Study	
		24	Mass moment inertia of circular plate, Cylinder, Cone, Sphere, Hook	Case Study	
	6	25	Virtual displacements, principle of virtual work for particle and ideal system of rigid bodies, Degrees of freedom. Active force diagram, systems with friction, mechanical efficiency.	Demonstration	
		26	Conservative forces and potential energy (elastic and gravitational), energy equation for equilibrium	Demonstration	
		27	Applications of energy method for equilibrium, Stability of equilibrium, Problems.	Case Study	
	7	28	Rectilinear motion; Plane curvilinear motion (rectangular, path)	Demonstration	
		29	Plane curvilinear motion (polar coordinates). 3-D curvilinear motion;	Case Study	
		30	Relative and constrained motion; Newton's 2nd law (rectangular, path, and polar coordinates).	Case Study	
		31	Work-kinetic energy, power, potential energy.Impulse-momentum (linear, angular);	Demonstration	
		32	Impact (Direct and oblique).	Case Study	
	8	33	Basic terms, general principles in dynamics; Types of motion, Instantaneous center of rotation in plane motion and simple problems	Demonstration	
		34	D'Alembert's principle and its applications in plane motion and connected bodies;	Demonstration	
		35	Work energy principle and its application in plane motion of connected bodies	Case Study	

		36	Kinetics of rigid body rotation, Problems	Demonstration	

CO-PO Mapping:

COs	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	3	2	2	-	-	-	-	1	-	-	-
CO2	3	3	2	2	-	-	-	-	1	-	-	2
CO3	3	2	3	2	1	-	-	-	1	-	-	2
CO4	3	3	3	3	-	-	-	-	1	-	2	-

ENGINEERING MECHANICS

Module -1
Lecture 1

BASIC CONCEPTS

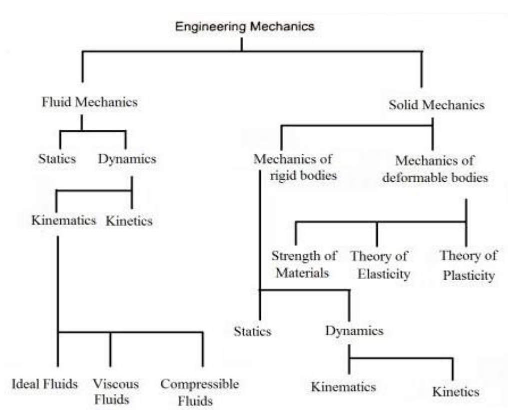
Importance of mechanics in engineering:

Engineering Mechanics is the one of the most basic subjects that is required to study mechanical engineering. The subject will expose you to concepts like **friction, kinetics, kinematics, resolving forces, trusses** etc. All these concepts are required to study subjects like **Strength of Materials, Theory of Machines I, II, Machine Design, Mechanical Vibration** etc. Basically every subject in Mechanical Engineering will use these concepts of engineering mechanics ranging from production technology to design sciences to thermal sciences. There is not a single topic in the engineering world which is out of syllabus for a mechanical engineer.

Without mechanics, you can't design an engine, not even a car, if you have to design a car, which can run at a speed of 140 km/hr on an expressway. In order to do this, you have to find engine power and the forces acting on the car body. Forces will come due to wind resistance, rolling resistance and inertia etc. You won't be able to survive in an industry where you have to prove that you are worth it.

Introduction:-

Mechanics is the combination of physic and mathematics concerned with the behavior of bodies that are acted upon by forces deals with the state of rest or the state of motion.



Statics: is when the force system acting on a body is balanced, the system has no external effect on the body, the body is in equilibrium.

Dynamics: It is also a branch of mechanics in which the forces and their effects on the bodies in motion are studied. Dynamics is sub-divided into two parts: (i) Kinematics and (ii) Kinetics

Kinematics: It deals with the geometry of motion of bodies without and application of external forces.

Kinetics: It deals with the motion of bodies with the application of external forces.

Particle: It can be defined as an object which has only mass and no size.

Rigid body: It can be defined as an object which has mass and definite size. The relative portion of any two particle in it do not change under the action of forces

Lecture 2

VECTOR MECHANICS:

Various quantities used in engineering mechanics may be grouped into scalars and vectors.

Scalar Quantity: A quantity is said to be scalar if it is completely defined by its magnitude alone. Examples of scalar quantities are: Area, length, Volume, Mass, Energy, Power, and Work, Moment of inertia etc.

Vector Quantity: A quantity is said to be vector if it is completely defined only when its magnitude as well as direction are specified.

Examples of vector quantities include: Displacement, Velocity and Acceleration Force, Moment, Momentum etc.

Types of vector 1) free vector 2) Sliding vector 3) bound or fixed vector

1. **Free Vector:** Vectors with no unique line of action and unique point of application are called free Vector. Examples are Couple and displacement vector.

2. **Sliding Vector:** Vectors with unique line of action but without unique point of application is known as sliding vector. Example is force acting on rigid body.

3. **Fixed Vector: F**orce with unique line of action as well as unique point of application is called fixed vector. Example is force acting on a deformable body.

FORCE: Force is defined as the cause of change in the state of motion of a particle or body. The product of mass of the particle and its acceleration is equal to force. It is a vector quantity.

A Force has following basic characteristics

i) Magnitude ii) Direction iii) Point of application iv) Line of action

Magnitude of the force is shown in Fig 10N, direction is 45° with the horizontal in first quadrant, point of application is C and line of action is AB.

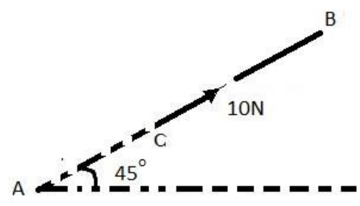


Fig.1.1. Characteristics of a force

SYSTEMS OF FORCES

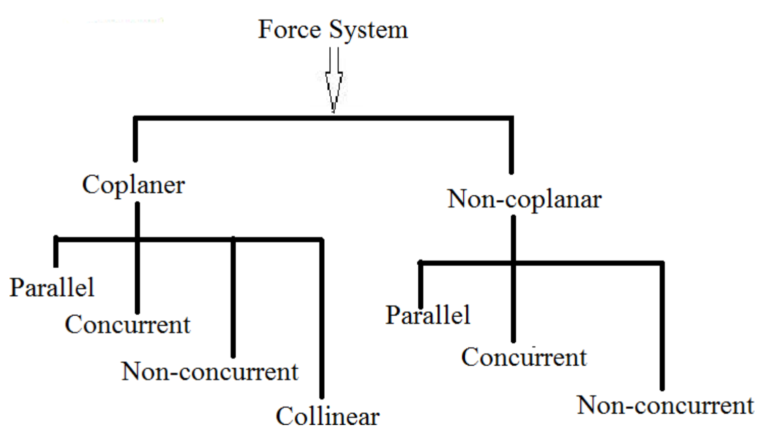
When a system has more than one force acting, it is known as a force system.

Collinear Force System

When the lines of action of all the forces of a system act along the same line, this force system is called collinear force system.

Coplanar Force System

When the lines of action of a set of forces lie in a single plane is called coplanar force system.



Non-Coplanar Force System

When the line of action of all the forces do not lie in one plane, is called Non-coplanar force system

Coplanar and concurrent force system

A force system in which all the forces lie in a single plane and meet at one point, For example, forces acting at a joint of a roof truss.

Coplanar and non-concurrent force system

These forces do not meet at a common point; however, they lie in a single plane, for example, forces acting on a beam.

Concurrent Force System

The forces when extended pass through a single point and the point is called point of concurrency. The lines of actions of all forces meet at the point of concurrency. Concurrent forces may or may not be coplanar.

Non-concurrent Force System

When the forces of a system do not meet at a common point of concurrency, this type of force system is called non-concurrent force system. Parallel forces are the example of this type of force system. Non-concurrent forces may be coplanar or non-coplanar.

Non-coplanar and concurrent force system

In this system, the forces lie in different planes but pass through a single point. Example is forces acting at the top end of an electrical pole

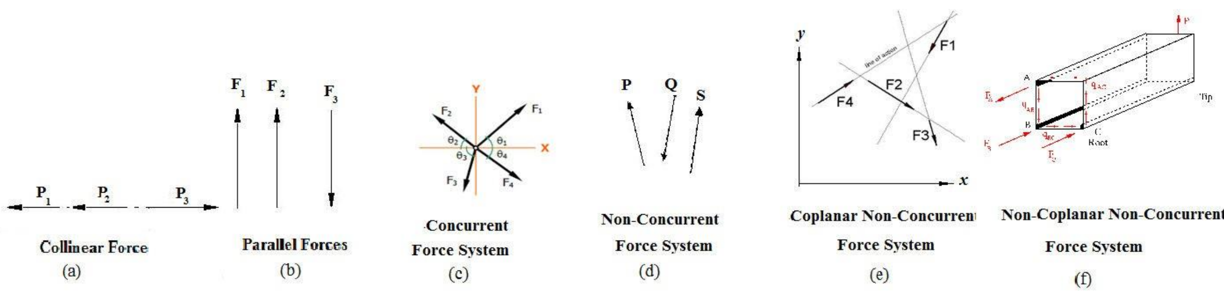


Fig.1.2. Force System

Lecture 3

PRINCIPLE OF SUPERPOSITION OF FORCES

This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces

Consider two forces *P* and *Q* acting at *A* on an object. Let *R* be the resultant of these two forces *P* and *Q*.

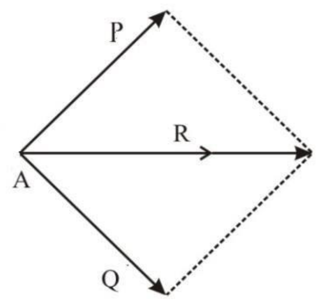


Fig.1.3. Principle of superposition

PRINCIPLE OF TRANSMISSIBILITY OF FORCES

The transmissibility of forces is the point of application of force may be transmitted along its line of action without changing the effect of the force on any rigid body to which it may be applied.

Lecture 4

Resultant of two forces by parallelogram law

$R^2 = P^2 + Q^2 - 2PQCos\beta$

As $\beta = (180° - \theta)$

$R^2 = P^2 + Q^2 + 2PQCos\theta$

$\tan\alpha = \frac{Qsin\theta}{(P+Qcos\theta)}$

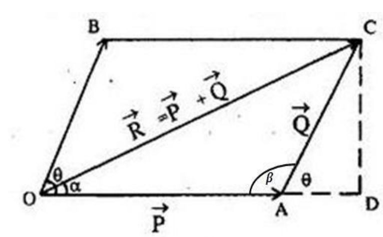


Fig 1.4

Addition and subtraction of vectors (by triangle law)

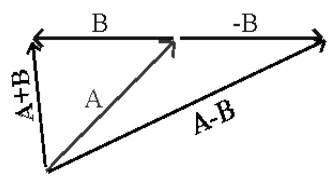


Fig1.5

Addition and subtraction of vectors (by parallelogram **law)**

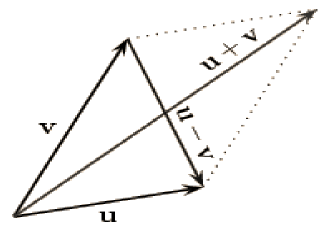


Fig1.6

MOMENT OF A FORCE: Moment of a force is the turning effect. Moment of a force about a point is the product of the magnitude of the force and the perpendicular distance from the moment centre on to the line of action of the force. Moment of the force F about O = M_O

$$M_O = F \times d$$

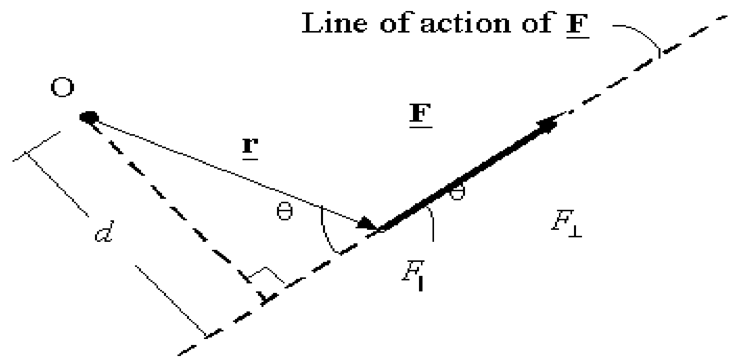


Fig 1.7

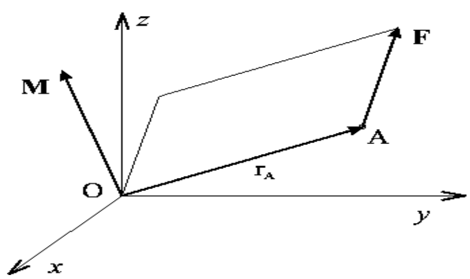
Lecture 5

Moment of a force about a point (vector method)

Moment of force $\vec{\boldsymbol{F}}$ about O $= M$

$M = \vec{\boldsymbol{r_A}} \times \vec{F}\ (vector\ product)\ , \vec{\boldsymbol{r_A}} = position\ vector\ of\ point\ A,$

The cross product $\vec{r_A} \times \vec{F}$ is defined as a vector M that is perpendicular to both $\vec{r_A}$ and $\vec{F}$, with a direction given by the right-hand rule and of magnitude equal to the area of the parallelogram formed by the vectors $\vec{r_A}$ and $\vec{F}$.



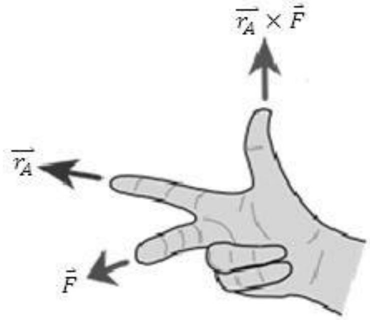


Fig 1.8

COUPLE

A system of two equal parallel forces acting in opposite directions is said to form a couple. shows a couple formed by horizontal, vertical and inclined forces

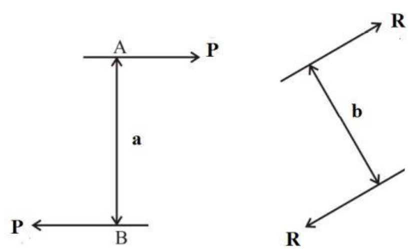


Fig.1.9. Couple System.

Resolution of a Force into Rectangular Components

A given force F can be resolved into (or replaced by) two forces, which together produces the same effects that of force F. These forces are called the components of the force F

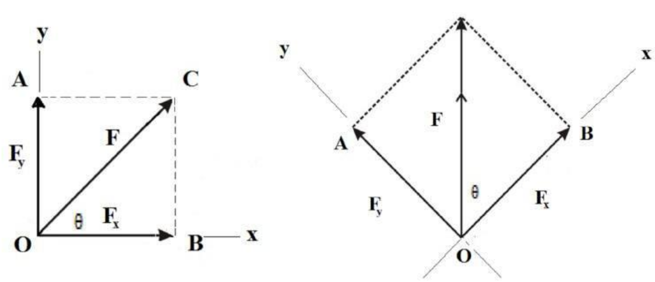


Fig.1.10 Rectangular Force Components

Consider a force F acting on a particle O inclined at an angle Θ as shown in Fig.. Let x and y axes can be the two axes passing through O perpendicular to each other. These two axes are called rectangular axes or coordinate axes. They may be horizontal and vertical or inclined as shown in Fig.

The force F can now be resolved into two components F_x and F_y along the x and y axes and hence, the components are called rectangular components.

Therefore, the two rectangular components of the force F are:

$F_x = F \cos\theta$ and Fy = $F \sin\theta$

Rectangular components of the moment of a force

Three Dimensional Systems

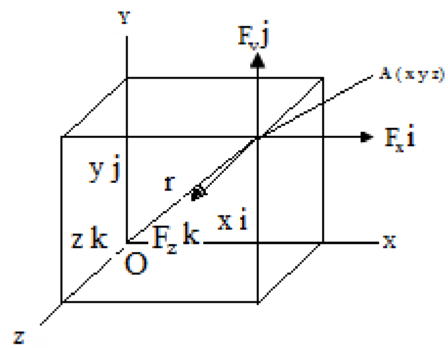


Fig.1.11. Rectangular Moment Component.

The moment of force F about O,

$$\overrightarrow{M_O} = \vec{r} \times \vec{F}, \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\vec{k} \text{ and } \vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\vec{k}$$

$$\overrightarrow{M_O} = M_x\hat{\imath} + M_y\hat{\jmath} + MF_z\vec{k}$$

$$\overrightarrow{M_O} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = (yF_z - zF_y)\hat{\imath} + (zF_x - xF_z)\hat{\jmath} + (xF_y - yF_x)\vec{k}$$

Lecture 6

Scalar product of two vectors applications

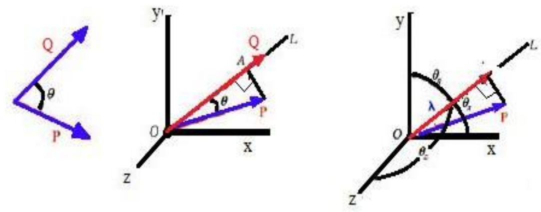


Fig 1.12

$$\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\vec{k}$$
$$\vec{Q} = Q_x\hat{i} + Q_y\hat{j} + Q_z\vec{k}$$
$$\vec{P}.\vec{Q} = PQCos\theta = P_xQ_x + P_yQ_y + P_zQ_z$$
$$Cos\theta = \frac{P_xQ_x + P_yQ_y + P_zQ_z}{PQ}$$

Projection of a vector along any direction (OL)

$$P_{OL} = PCos\theta = projection\ of\ P\ along\ OL$$

$$\vec{P}.\vec{Q} = PQCos\theta$$

$$\frac{\vec{P}.\vec{Q}}{Q} = Pcos\theta = P_{OL} = \vec{P}.\vec{\lambda}\ (where\ \vec{\lambda}\ is\ the\ unit\ vector\ along\ OL)$$

Example 1: A single force P acts at C in r direction perpendicular to the handle BC of the crank shown. Knowing that M_X = + 20 N.m and M_y is equal to - 8.75 N.m and M_Z = - 30 N.m determine the magnitude of p and the values of φ and θ.

Solution:

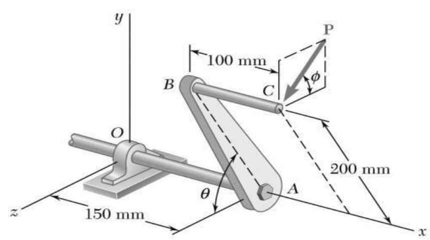


Fig.1.13

$$\vec{r_c} = (0.25m)\hat{i} + (0.2m)sin\theta\,\hat{j} + (0.2m)cos\theta\vec{k}$$
$$\vec{P} = -Psin\emptyset\,\vec{j} + Pcos\emptyset\,\vec{k}$$
$$M_O = \vec{r_c} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \vec{k} \\ 0.25 & 0.2sin\theta & 0.2cos\theta \\ 0 & -Pson\emptyset & Pcos\emptyset \end{vmatrix}$$

Expanding the determinant, we get

$$M_x = (0.2)P(sin\theta cos\emptyset + cos\theta sin\emptyset)$$

$$M_x = 0.2Psin(\theta + \emptyset)$$
$$M_y = -(0.25)Pcos\,\emptyset$$

$$M_z = -(0.25)Psin\,\emptyset$$

$$tan\emptyset = \frac{M_z}{M_y} = \frac{-30}{-8.75}$$

$$\emptyset = 73.74°$$

$$M_y^2 + M_z^2 = (0.25P)^2$$

$$P = 4\sqrt{8.75^2 + 30^2} = 125N$$

Substituting the values of *P and* $\emptyset$ in the above equations, we get $\theta = 53.1°$ (considering positive value for θ)

Lecture 7

Example: 2

The three forces and a couple shown are applied to an angle bracket. Reduce the system to a force and a couple at B.

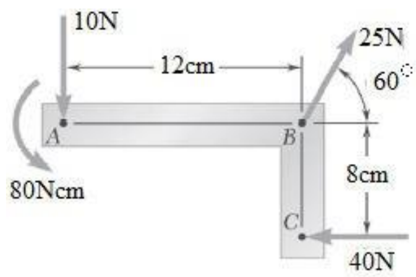


Fig.1.14.

The system to a force and a couple at B is,

$R_x = \Sigma F_x = 25\cos 60^\circ - 40 = -27.5N$ (←)

$R_y = \Sigma F_y = 25\sin 60^\circ - 10 = 11.6506N$ (↑)

$M_B = 80 + 10\times12 - 40\times8 = -120N.cm$ (clockwise)

Resultant Force

$\vec{\mathbf{R}} = -27.5i + 11.65j$

R = 29.86N

Example 3: A Package is supported by three ropes as shown in figure. Determine the weight of the package if the tension in the rope AD is 450N.as shown in figure below.

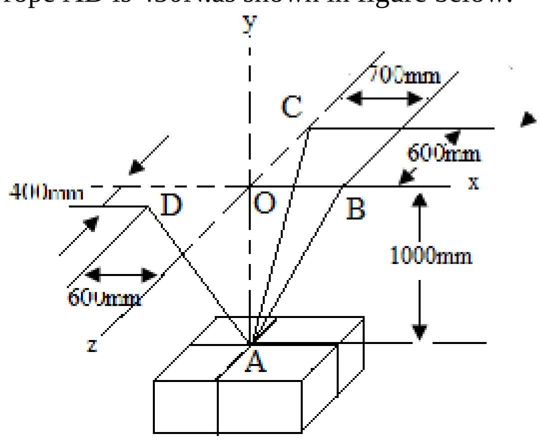


Fig.1.15.

Solution : First we find the co-ordinate of Point, A(0,-1000,0); B(700,0,0); C(0,0, -600); D(-600,0,400)

We know, $\overrightarrow{T_{AB}} = \frac{T_{AB}(700\vec{i}+1000\vec{j})}{\sqrt{700^2+1000^2}} = \overrightarrow{T_{AB}} = T_{AB}(0.573\vec{i}+0.819\vec{j})$

$$\overrightarrow{T_{AC}} = \frac{T_{AC}(1000\vec{j}-600\vec{k})}{\sqrt{1000^2+600^2}} = T_{AC}(0.8574\vec{j}-0.5145\vec{k})$$

$$\overrightarrow{T_{AD}} = \frac{T_{AD}(-600\vec{i}+1000\vec{j}+400\vec{k})}{\sqrt{1000^2+600^2+400^2}} = \overrightarrow{T_{AD}} = \frac{4500(-600\vec{i}+1000\vec{j}+400\vec{k})}{1232.88}$$

$$= T_{AD}(-2190\vec{i}+3650\vec{j}+1460\vec{k})$$

$\overrightarrow{W} = \frac{W(-1000\bar{j})}{\sqrt{1000^2}} = -W(\bar{j})$

For Equilibrium, $\sum F = 0$, $\because$ $\sum F = \overline{T_{AB}} + \overline{T_{AC}} + \overline{T_{AD}} + \overrightarrow{W} = 0$ Equating the coefficient of $\vec{i}, \vec{j}$ *and* $\vec{k}$ equal to zero. T_{AB} = 3822N, T_{AC}= 2823N we can find the package weight from the above equations

Assignment

1.Determine the moment about the origin O of the force $F' = 3\bar{i} + 4\bar{j} - \vec{k}$ which acts at a point A. Assume the position vector of A is $\vec{r} = 3\bar{i} - 3\bar{j} + 4\vec{k}$

2.A force of 50N is acting along the force $\vec{F} = 5\bar{i} + 6\bar{j} - 3\vec{k}$ is applied at point P(1,-1,2). Find the moment of force of 50N about the point O(2,-1,3)

3. Example: Determine the tension in cables AB, AC and AD.

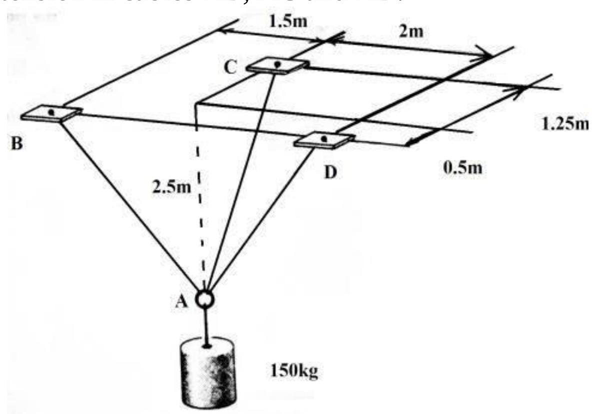


Fig.1.16

Lecture 8

LAMI'S THEOREM

It states that," If three forces acting at a point are in equilibrium each force will be proportional to the sine of the angle between the other two forces."

Suppose the three forces P, Q and R are acting at a point O and they are in equilibrium as shown in figure. Rearrange the forces makes a triangle when equilibrium.

Let α = Angle between force P and Q.

β = Angle between force Q and R.

γ = Angle between force R and P.

Then according to Lami's Theorem,

$$\frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\gamma)}$$

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

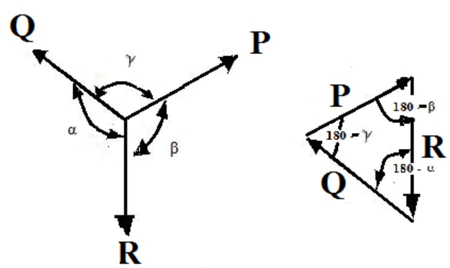


Fig.1.17

Example 4: A small block of weight 100 N is placed on an inclined plane which makes an angle, ϴ = 30° with the horizontal. What is the component of this weight parallel to inclined plane and perpendicular to inclined plane?

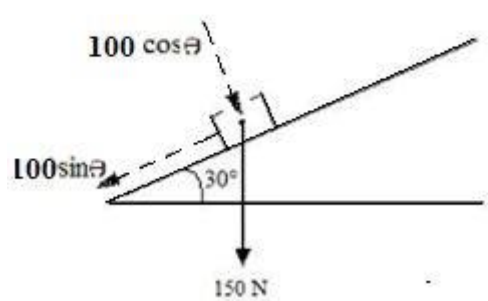


Fig.1.18

Sol^n: 1. Select the axis

x-axis is parallel to inclined plane

y-axis is perpendicular to inclined plane

2. Draw the force diagram,

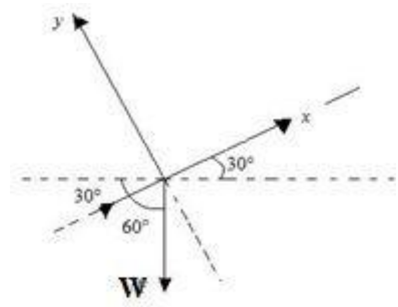


Fig.1.19

3. Find the force components,

F= 100 N

$\alpha = 60°$

$F_x = F \cos \alpha = 100 \cos 60° = 50$ N

$F_y = F \sin \alpha = 100 \sin 60° = 87$ N

Lecture 9

Example 5: A circular roller of weight 450N and radius r=*150mm* hangs by a tie rod AC=300mm and rests on a smooth vertical wall at B. Determine the tension S in the tie rod and force R_b exerted against the roller as shown in Fig-16.

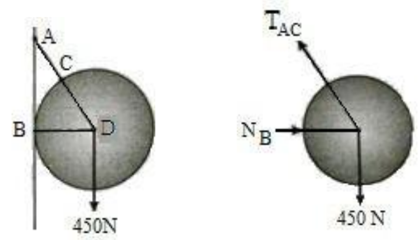


Fig.1.20

From free body diagram of the roller as shown in figure. Observed that the forces are congruent and equilibrium. Thus, we apply Lami's theorem.

$$\frac{T_{AC}}{\sin\ 90} = \frac{\mathbf{450}}{\sin\ \Theta}$$

$$T_{AC} = \frac{\mathbf{450}}{\sin\ \Theta}$$

∴ from figure AD = AC + CD = (300+150)mm= 450mm

∴ $\cos\Theta = \frac{BD}{AD}$

∴ $\cos\Theta = \frac{150}{450}$ = 70.53^{o}

∴ $T_{AC} = \frac{\mathbf{450}}{\sin 70.53^{o}}$ = 477.3N

$T_{AC} = 477.3\ N$ is the required tension

VARIGNON'S THEOREM:

Statement: The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the algebraic sum of the moments of the components with respect to the same centre.

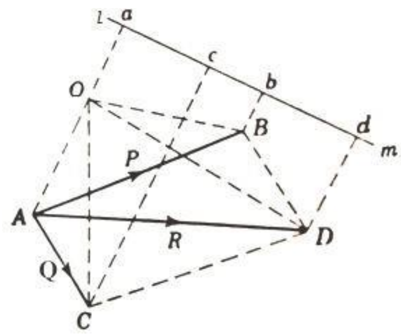


Fig.1.21

Proof : The moments of both forces P and Q with respect to centre 'O' as shown in figure. the moment of resultant force R. Joining OA and extended the line OA, draw the perpendicular *l-m* to the line OA. Joining the perpendiculars Aa, Bb, Cc and Dd

Now the area of ΔOAB = $\frac{1}{2}\,OA$.ab, the area of ΔOAC = $\frac{1}{2}\,OA$.ac, and the area of ΔOAD = $\frac{1}{2}\,OA$.ad,

Since, ad = ab + bd = ab + ac,

we conclude that, area of ΔOAD = area of ΔOAB + area of ΔOAC

Therefore, $\mathbf{P.d_1 + Q.d_2 = R.d}$ where, $\mathbf{d_1}$, $\mathbf{d_2}$ and **d** are the perpendicular distance from forces **P,Q** and **R** from point O **.**

FREE BODY DIAGRAM

1. It is a diagram or sketch of a body.
2. The body is shown completely separated from all other bodies.
3. The action on the body of each body removed in the isolating process is shown as a force or forces on the diagram.

Example 5: A boom of length 30m supports load of 1500N as shown in figure. The string is horizontal and 15m long. Determine the forces in the boom.

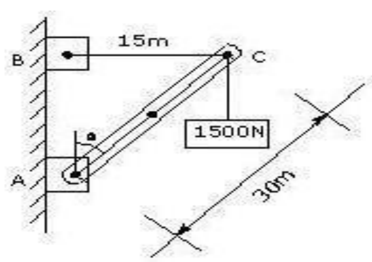


Fig 1.21

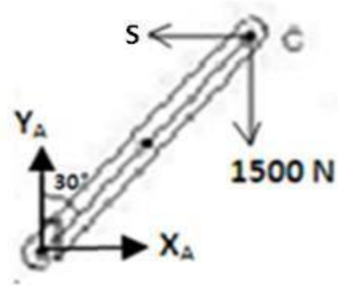
Let S is the tensile force induced in the string

$Sin\ \angle CAB = \frac{15}{30} = \frac{1}{2} \Rightarrow \angle CAB = 30°$

$\sum F_y = 0$

$Y_A - 1500 = 0$

$Y_A = 1500$N



Taking moment of all forces about point A we can write,

Fig 1.22

$M_A = S \times 30Cos30° - 1500 \times 15 = 0$

$S = \frac{1500\times15}{30Cos30°} = 866$ N

$\sum F_x = 0 \Rightarrow X_A - S = 0 \Rightarrow X_A = 866$ N

$R_A = \sqrt{X_A^2 + Y_A^2} = 1732$ N

Lecture 10

Example 6: A right circular roller of weight W rests on a smooth horizontal plane and is held in position by an inclined bar AC. Find the tension S in the bar AC and vertical reaction R_b at B, if there is a horizontal force P acting at C. P=100N and α=30°.

Solution:

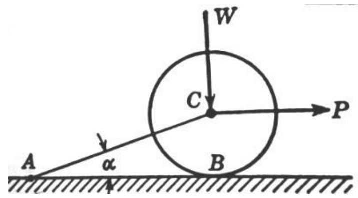


Fig 1.23

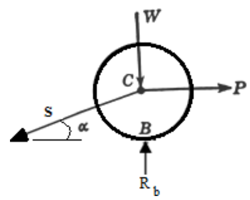


Fig 1.24

From equations of force equilibrium of roller,

$$P = S\cos\alpha$$
$$R_b = W + S\sin\alpha$$

Solving these,

$$S = P\sec\alpha$$
$$R_b = W + P\tan\alpha$$

Example 7:Two smooth circular cylinders, each of weight W=N radius *r*=6cm are connected at the centres by a string *l*=16cm and rest upon a horizontal plane supporting above them a third cylinder of weight Q=200N and radus *r*=6cm, find the force S in the string AB and pressure produced on the floor at D and E.

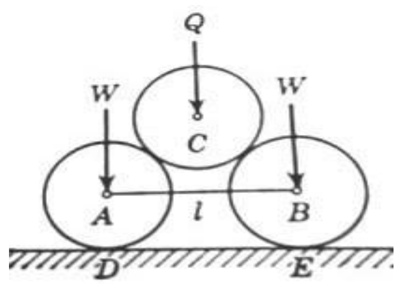


Fig 1.25

Solution, Free Body diagram :

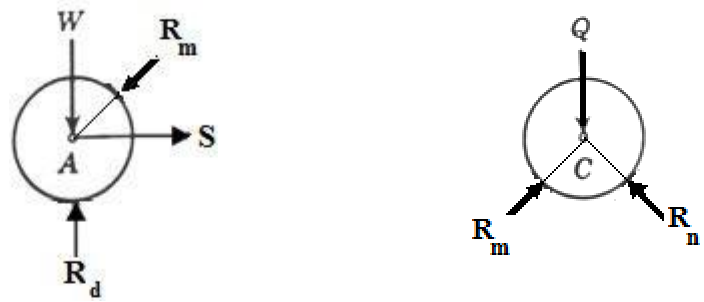


Fig 1.26 (a & b)

From geometry, $\sin\theta = \frac{l/2}{2r} = \frac{8}{2\times 6} = \frac{2}{3}$

$\theta = 41.81^{\circ}$

From symmetry it can be concluded that reactive force at points of contact at M and N are equal say

$R_m = R_n = R$ and $R_D = R_E$

From equilibrium of vertical forces in sphere C, We obtain

$$2R\cos\theta = Q$$

$$R = \frac{200}{2\cos 41.81^{\circ}} = 134.163N$$

From equations of force equilibrium of sphere A

$S = R\sin\theta$

$W + R\cos\theta = R$

or

$S = 134.163\sin 141.81^{\circ}$

$100 + 134.163\cos 41.81^{\circ} = R_D$

$S = 89.44N, R_D = 199.99N = R$

Lecture 11

Example 8:. A ball of weight Q rests in right angled through as shown. Determine the reaction forces of the wall.

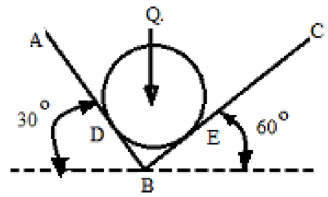


Fig 1.27

Solution…. FBD of Ball:

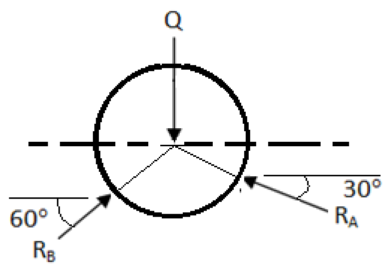


Fig 1.28

Considering Equilibrium we can form those equations,

$\sum F_y = 0 = Q - (R_B \cos 60° + R_A \cos 30°)$, $\because$ $R_B \times 1/2 + R_A \times \sqrt{3}/2 = Q$.......... (i)

$\sum F_x = 0 = (R_B \sin 60° - R_A \sin 30°)$ $\because$ $R_B \sqrt{3}/2 - R_A/2 = 0$… ......(ii)

Solving (i) & (ii)

We get $R_A = \sqrt{3}/2\ Q$; $R_B = Q/2$

Example 9: A 150 kg mass stands on the middle point of a 50kg ladder. Assuming that floor and wall are perfectly smooth and also reaction R_A and R_B at A and B.

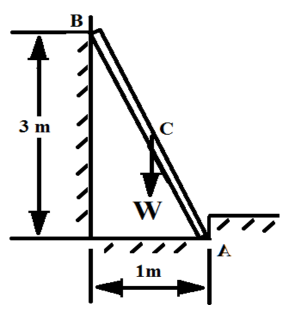


Fig 1.29

Solution:

FBD of Ladder……

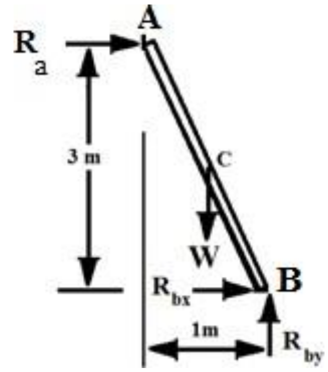


Fig 1.30

Considering Equilibrium we can write those equations…

$\sum F_Y = 0 = R_{BY} - W = 0$, $\because R_{BY} = W = 200$ N;

$\sum F_x = 0 = R_A - R_{BX} = 0$, $\because R_A = R_{BX}$;

$\sum M_A = 0 = W\times 1/2 + R_{BX}\times 3 - R_{BY}\times 1 = 0$;

Solving these equations we get,

$R_{BY} = 200$ N;

$R_{BX} = 100/3$ N;

$R_B = \sqrt{R_{Bx}^2 + R_{By}^2} := 202.76$N

$R_A = 100/3$ N .

Example 10: Determine the forces exerted on the cylinder at B and C by the spanner wrench due to a vertical force of 10N applied to the handle. Neglect friction at B.

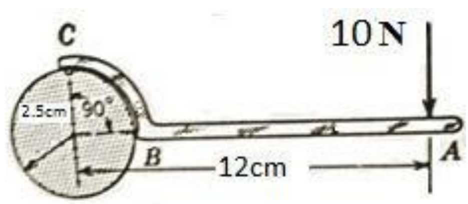


Fig 1.31

Solution, Free Body diagram :

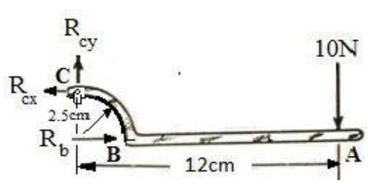


Fig 1.32

Considering Equilibrium we can write those equations and solving,

$\sum F_x = 0 = R_b - R_{cX} = 0$, $\because R_b = R_{cX}$;

$\sum F_Y = 0 = R_{cY} - 10 = 0$, $\because R_{cY} = 10$ N;

$\sum M_c = 0$, $\because R_B\times 2.5 - 10\times 12 = 0$;

$\because$ RB = 48N, $\because R_b = R_{cX} = 48$N

$\because R_c = \sqrt{R_{cx}^2 + R_{cy}^2} = \sqrt{48^2 + 10^2} = 49.03$

Module II

EQUILIBRIUM OF FORCES

Equilibrium is defined as the condition of a body, which is subjected to a force system whose resultant force is equal to zero.

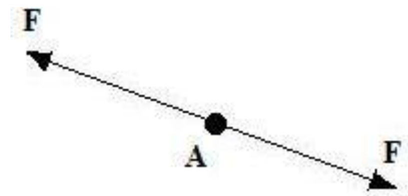


Fig.2.1.

Equations of equilibrium for a concurrent, coplanar force system

The resultant of a concurrent, coplanar force system is a single force through the point of concurrence. When the resultant force is zero, the body on which the force system acts in equilibrium.

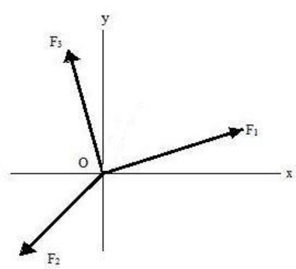


Fig.2.2. Equilibrium of concurrent and coplanar Force system

If the sum of the x components of the forces of the system is equal to zero, and the sum of the y components of the forces of the system is equal to zero is the equations of equilibrium of a concurrent, coplanar force system

i.e $\sum F_x = 0$, $\sum F_y = 0$,

In a similar manner, a third set of independent equations can be shown to be

$\sum M_A = 0$, $\sum M_B = 0$

Example 1: P and Q are two Forces acting parallel to the bar AB is supported by a peg, so that it can remain horizontal. If the length of the bar is *l*. find the position of support O from both sides,

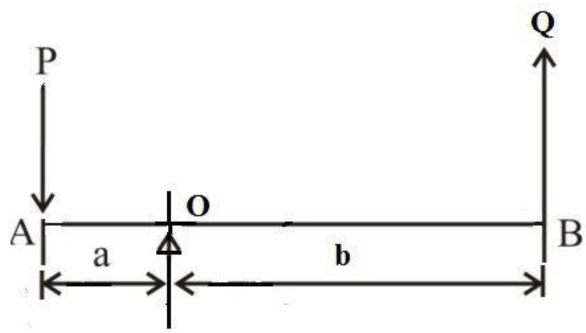


Fig.2.3

PROCEDURE FOR THE SOLUTION OF PROBLEMS IN EQUILIBRIUM

1. Draw a free body diagram of the member or group of members on which some or all of the unknown forces are acting.

2. The number of independent equations of equilibrium of force system.

3. Compare the number of unknowns on the free body diagram with the number of independent equations of equilibrium .

4. Then solve the unknown according to equilibrium equations get the results

Example 2: Determine the value of F and θ so that particle A is in equilibrium.

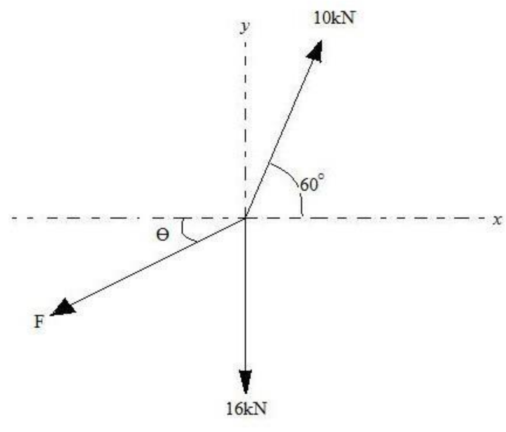


Fig.2.4

Solution:

1. $\sum$ Fx =0

   10 cos 60° + F cos (180° + θ) + 16 cos 270° = 0

   = 5.0– F cos θ – 0

   Therefore, F cos θ = 5

2. $\sum$ Fy = 0

   10 sin 60° + F sin (180° + θ) + 16 sin 270° = 0

   – 7.34– F sin θ = 0

Therefore, F sin θ = –7.34

3. $\frac{\text{F sin}\,\Theta}{\text{F cos}\,\Theta} = \tan\theta = \frac{-7.34}{5} = -1.47$

θ = -55.74°

F cos θ = 5

F = 8.88 N

Example 3: A ball of W rest on a smooth plane as shown in figure. Find the angle α and the normal reaction on the ball for equilibrium.

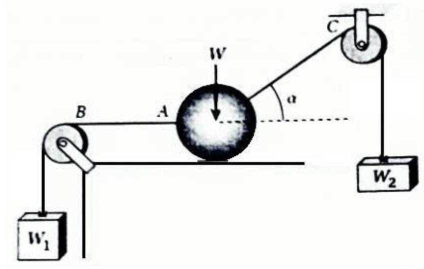


Fig.2.5

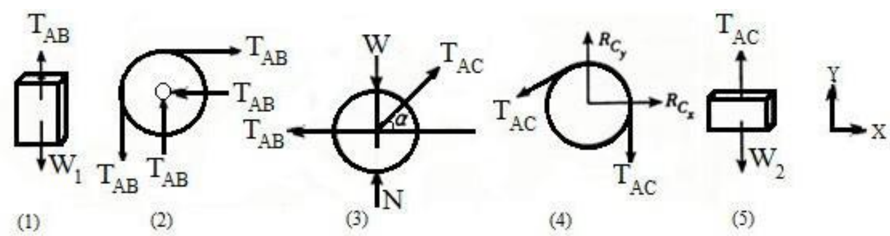


Fig.2.6

Free body diagram (1), (2), (4) and (5)

From The condition of equilibrium $W_1 = T_{AB}$ and $W_2 = T_{AC}$

From free body (3) $\sum Fx = 0$

$T_{AC}\cos\alpha - T_{AB} = 0$ ⇒ $\cos\alpha = \frac{T_{AB}}{T_{AC}}$

$\therefore \cos\alpha = \frac{W_1}{W_2}$ $(W_1 < W_2)$ ⇒ $\alpha = \cos^{-1}(\frac{W_1}{W_2})$; $(W_1 < W_2)$

Again, $\sum F_y = 0$ ⇒ $T_{AC}\sin\alpha + N - W = 0$

⇒ $N = W - T_{AC}\sin\alpha$ ⇒ $N = W - W_2 \frac{\sqrt{W_2^2 - W_1^2}}{W_2}$;

[$\because T_{AC} = W_1$ and $\sin\alpha = \sqrt{1 - \cos^2\alpha} = \frac{\sqrt{W_2^2 - W_1^2}}{W_2}$]

⇒ $N = W - \sqrt{W_2^2 - W_1^2}$ is the required normal reaction exerted by the plane on the ball.

Example 4: A roller of radius r=12cm and Q=500kgf is to be rolled over a curb of height h=6cm by a horizontal force P applied to the end of a string wound around the circumference of the roller. Find the magnitude of P required to start the roller over the curb. Fig

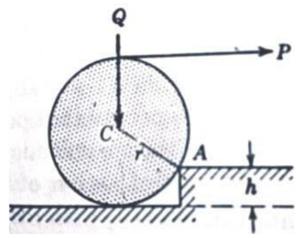
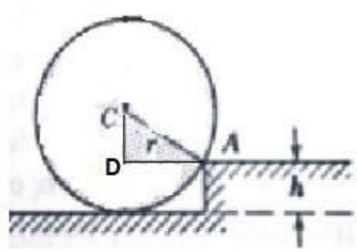


Figure 2.7

Solution: For the condition of rolling over the curb, the roller must be rested on the edge of the curb at A and should be lifted from the floor.



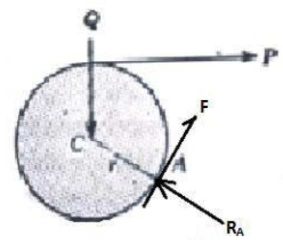


Figure 2.8 **Figure 2.9**

Now by taking moments of all the forces about point A
We can write,
$M_A = Q\times AD - P(2r - h) = 0$
And $AD = \sqrt{AC^2 - CD^2} = 6\sqrt{3}$ cm
Or, $P = (500\times6\sqrt{3})/18 = 288.67$kgf

Lecture 12

Example 5: Two identical rollers each of weight Q=100 N, are supported by an inclined plane and a vertical wall as shown in figure below. Assuming smooth surfaces, find the reactions induced at the point A,B, and C.

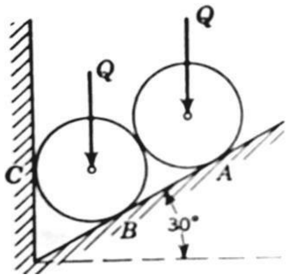


Fig -2.10

Solution:

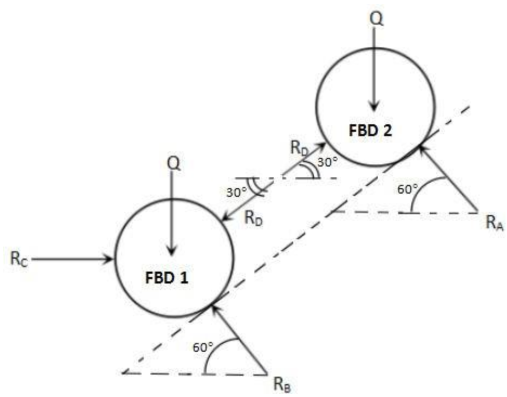


Figure 2.11 (Free Body diagram for both the roller)

Applying conditions of equilibrium for FBD 1

$\sum F_X = 0$, => R_C - R_B Cos60° - R_D Cos30° = 0 --------------- (1)

$\sum F_y = 0$, => R_B Sin60° - Q - R_D Sin30° =0---------------- (2)

Applying conditions of equilibrium for FBD 2

$\sum F_X = 0$, => R_D Cos30° -R_ACos60°= 0 ---------------(1)

$\sum F_y = 0$, => R_D Sin30° + R_ASin 60° = 0---------------- (2)

Solving the above equations we get, RA = 96.6N , RB = 144.33N, RC = 115.46kgfN and RD = 50N

Example 6: A circular roller of weight W=450N rest on a smooth inclined plane and is kept from rolling down by a string AC as shown in figure, find the tension S in the string and the reaction R_B at B. Fig-1.30

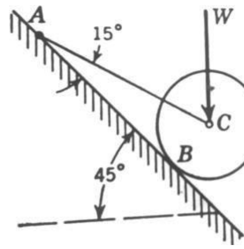


Fig 2.12

Solution: Free Body diagram of the circular roller is shown in figure below

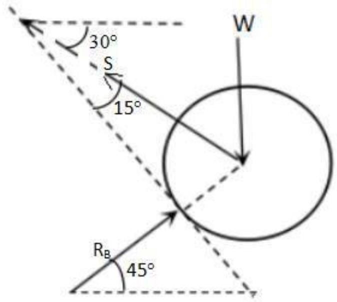


Fig 2.13

Now applying the conditions of equilibrium, we can write

$\sum F_X = 0$, => R_B Cos45° - S Cos30° = 0

$\sum F_X = 0$, => R_B Sin45° + S Sin30° - W = 0

Solving the above equations , we get RB = 416 N and S = 341 N

Problems on Equilibrium System

1. Two smooth spheres, each of raddi r=25cm and weight Q=40kgf rest in a horizontal channel having a vertical wall of distance b=90cms. Find the pressure exerted on the floor at the points of contacts. **Fig.1.25.**

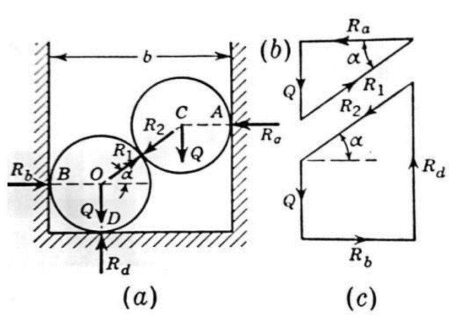


Fig.2.14

2. A bar AB of length *l* is supported as shown in Fig-1. 33. At any point along its length a vertical load Q can be applied. Determine the position of this load for which the tensile force S in the cable BC will be a

maximum and evaluate the same if the various angles are as shown in the figure. In calculation neglect the weight of the bar and the cable.

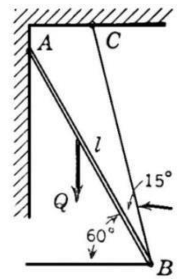


Fig.2.15

3. An electric light fixture of weight 20N hangs from a point, by two straight AC and BC.AC is inclined at 60^0 to the horizontal and BC at 30^0 to the vertical as shown in fig. 1 .Using Lame's theorem determine the forces in the strings AC and BC.

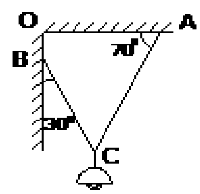


Fig 2.16

4. A string ABCDE whose one end A is fixed and weights W_1 And W_2 are attached at B and C respectively and passing round a smooth peg at D carrying a weight 1000N at the free end E as shown in fig.3 if in a state of equilibrium . BC is horizontal and AB and CD make angles of 150^0 and 120^0 respectively with BC. Determine the tension in AB, BC, CD and DE of the string and the values of W_1 and W_2.

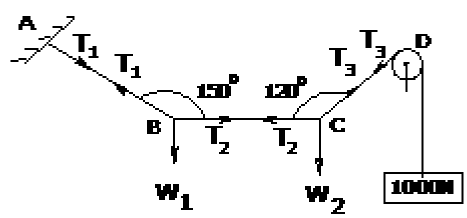


Fig -2.17

Lecture 13

FRICTION

Introduction

The sliding of a solid body in contact with another solid body is always opposed by force of *friction*. Friction acts in the direction opposite to that of relative motion and it is tangential to the surface of two bodies at the point of contact.

Types of Friction:

1. Static friction
2. Kinetic friction
3. Rolling friction
4. Fluid friction

Static Friction

Static friction comes into play when a body is forced to move along a surface but movement does not start. The magnitude of static friction remains equal to the applied external force and the direction is always opposite to the direction of motion. The magnitude of static friction depends upon μ_s (coefficient of static friction) and N (net normal reaction of the body).

Kinetic Friction

Kinetic friction denoted as μ_k comes into play when a body just starts moving along a surface. When external applied force is sufficient to move a body along a surface then the force which opposes this motion is called as kinetic frictional force.

Rolling Friction

Rolling frictional force is a force that slows down the motion of a rolling object. Basically it is a combination of various types of frictional forces at point of contact of wheel and ground or surface.

Fluid Friction

When a body moves in a fluid or in air then there exists a resistive force which slows down the motion of the body, known as fluid frictional force.

Culomb,s Laws of Friction:

1. Limiting friction force is proportional to the normal reaction force (N)
2. Friction force is independent of the area of contact surface; it depends on the roughness and nature of the material.
3. The coefficient of static friction is slightly greater than the coefficient of kinetic friction.
4. Within rather large limits of friction, kinetic friction is independent of velocity.

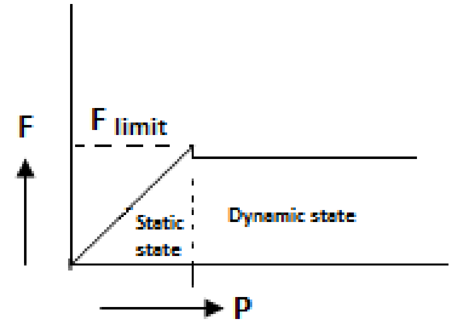


Fig.2.18

Angle of Friction:

Angle of friction is defined as the angle made between the normal reaction force and the resultant force of normal reaction force and friction. $tan\theta = \frac{F}{R} = \frac{\mu R}{R} = \mu$

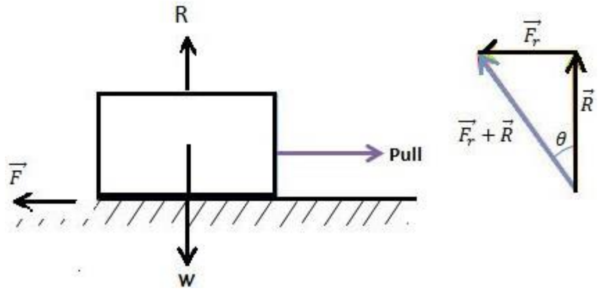


Fig. 2.19

∴ R= Normal reaction, F= Friction, μ=Coefficient of friction

Relation between Angle of Friction and Angle of Repose:

Angle of repose is defined as the minimum angle made by an inclined plane with the horizontal such that an object placed on the inclined surface just begins to slide.
Let us consider a body of mass 'm' resting on a plane.
Also, consider when the plane makes 'θ' angle with the horizontal, the body just begins to move.

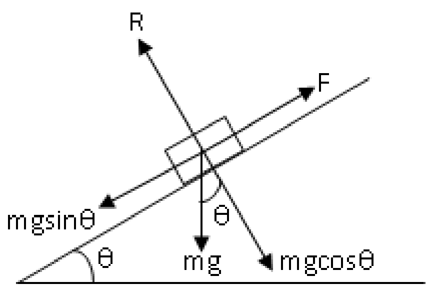


Fig. 2.20

Let ‘R’ be the normal reaction of the body and ‘F’ be the frictional force.

Here,

$F = mg\ sin\theta$..............(i)

$R = mg\ cos\theta$ ....(ii)

Dividing equation (i) by (ii) $\frac{mgsin\theta}{mgcos\theta} = \frac{F}{R}$

$$\frac{F}{R} = \frac{mg\ sin\theta}{mg\ cos\theta}$$

Or, $\tan\theta = \mu$, where 'μ' is the coefficient of friction

Or, $\tan\theta = \tan\alpha$ $(\tan\alpha = \mu)$

Where 'α' is the angle of friction

$\boldsymbol{\theta = \alpha}$

Angle of repose is equal to angle of friction.

Lecture 14

Cone Of Friction

A **cone** in which the resultant force exerted by one flat horizontal surface on another must be located when both surfaces are at rest, as determined by the coefficient of static **friction**.

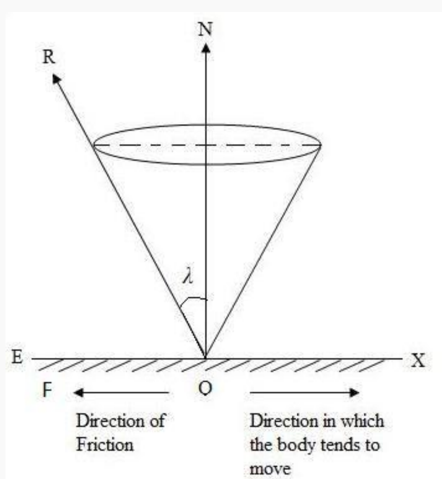


Fig. 2. 21

Lecture 15

Workout Problems:

Example 1: A ladder 5 m long weighing 100 N is resting against a wall at an angle 0f 60° to the horizontal ground. A man weighing 600 N climbs the ladder. At what position along the ladder from bottom does he induce slipping? The coefficient of friction for both the wall and the ground with the ladder is 0.25.

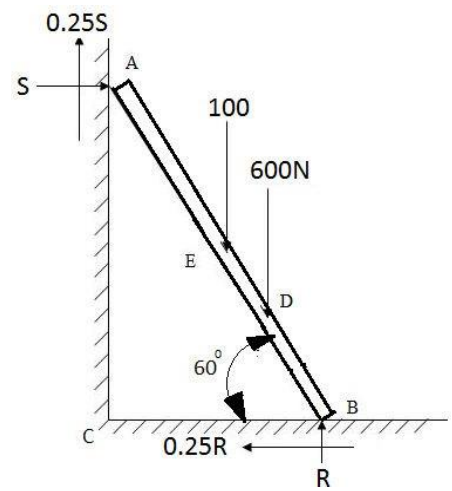


Fig. 2.22

Sol^n: Let the man is at a distance x metres from the foot of the ladder at F. as per figure

BF = x, BE = AE = 2.5m

the normal reactions at the floor and the wall be R and S. Friction at the floor and the wall will be 0.25R and 0.25S respectively.

From equilibrium condition the forces on the ladder horizontally and vertically,

$\sum F_x = 0$, $\sum F_y = 0$

$S = 0.25R$........................... (i)

$R + 0.25S = 700$ N............ (ii)

From equations (i) and (ii), we get $R = 658.82$ N, $S = 164.7$N

Taking moments about B, $\sum M_B = 0$

$100 \times 2.5 \cos 60° + 600 \times x \cos 60° - S \times 5 \sin 60° - 0.25S \times 5 \cos 60° = 0$

$125 + 300x = 4.33\ S + 0.625\ S$

$125 + 300x = 4.955 \times S$

$300x = 816.68 - 125 = 691.68$

$x = 2.30$m

Example 2: A block is weighing 100 kg is placed on a rough surface whose coefficient of friction is 0.30 and inclined force P is applied at its top corner as shown in Fig.2.6 Determine whether the block will tip or slide and the force P required to move the block.

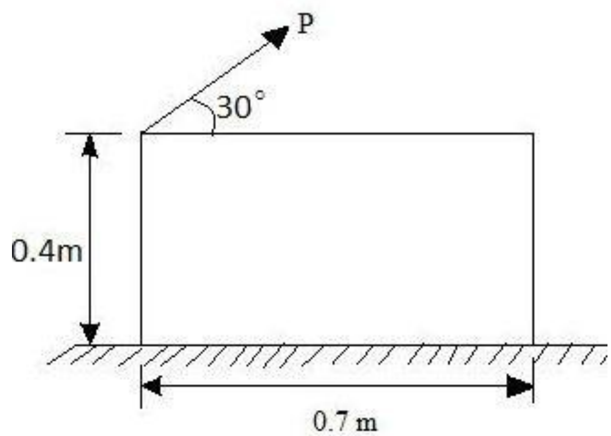


Fig. 2. 23

Sol^n: W = 100 × 9.8 = 980. N,
Considering sliding of the Block

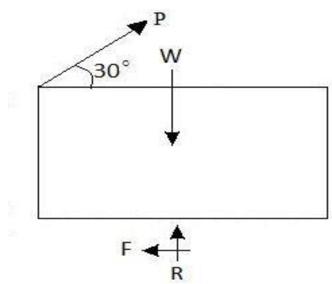


Fig. 2. 24

From free body diagram,.

$R - W + P \sin 30° = 0$

$R = 980 - 0.5\ P$

Hence limiting friction force = $\mu\ R_1$

$= 0.3\ (980 - 0.5\ P)$

$= 294 - 0.15\ P$

From $\sum F_x = 0$, $F = P \cos 30°$

So, $F = 0.866\ P$

Let us assume that the block slides before tipping,

Then $0.866\ P = 294 - 0.15\ P$

$P = 289.37$ N

Checking the tipping of the block,

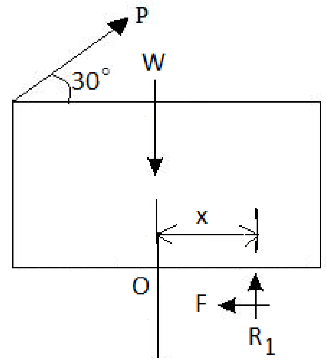


Fig. 2.25

$\sum M_0 = 0$, $(289.37 \sin 30°)(0.35) + (289.37 \cos 30°)(0.4) - R_1(x) = 0$
$50.63 + 100.24 - R_1\, x = 0$
$150.87 = R_1\, x$
By putting the value of R_1, we get
$150.87 = (980 - 0.5\,P)\, x$
$x = 0.18$ m
As $x < 0.35$ m, tipping will not occur.
So block slides with P = 289.37 N

Example 3: A block weighing 10 N is a rectangular prism resting on a rough inclined plane as shown in Fig The block is tied up with a horizontal string which has a tension of 5 N. Find

(a) The frictional force on block
(b) Normal reaction of the inclined plane
(c) The coefficient of friction between the surfaces of contact

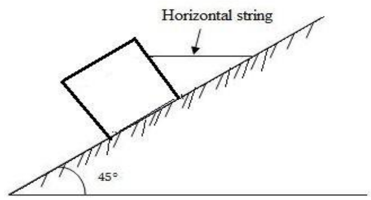


Fig. 2.26

Sol^n: Weight of the block W = 10 N
Tension in the horizontal string T = 5 N
Angle of the inclined plane = 45°

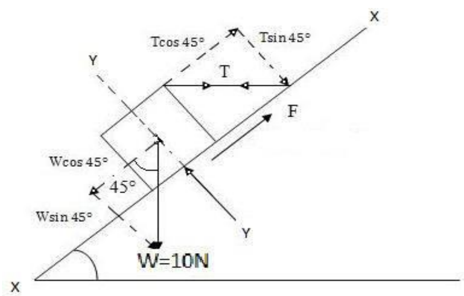


Fig. 2.27

Tangential and normal force components indicated by the dotted arrows. Resolving the force parallel to inclined plane,

(a) From $\sum F_x = 0$, we get,
$T\cos45^0 + F - W\cos45^0 = 0$, $F = \frac{W-T}{\sqrt{2}}$ = <u>3.53N</u>

(b) From $\sum F_y = 0$, we get, $R - T\sin45^0 - W\sin45^0 = 0$, $R = \frac{W+T}{\sqrt{2}}$ = <u>10.66N,</u>

(c)From the laws of friction,
F = μ R
3.53 = μ × 10.66
<u>μ = 0.331</u>

Example 4: A body resting on a rough horizontal plane required to pull 20N inclined at 30° to the plane just to remove it. It was found that a push of 25N inclined at 30° to the plane just removed the body. Determine the weight of the body and the coefficient of friction.

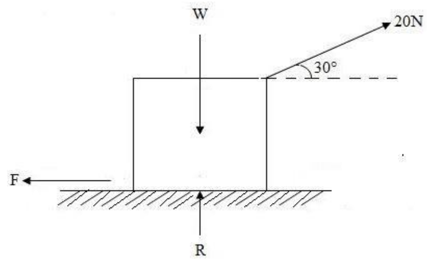


Fig. 2.28

Sol[n]: given, Pull = 20N, Push = 25N and θ = 30°
Let W = weight of the body in N, R = Normal reaction and μ = coefficient of friction
1. Pulling Force acting on the body, we get
F = 20 × 0.866 = 17.32 N
From the vertical forces, we get
R = W - 20 = W – 20 × 0.5 = (W – 10) N
From the laws of friction,
F = μ R
17.32 = μ (W – 10)... (i)

2. The Force acting on the body

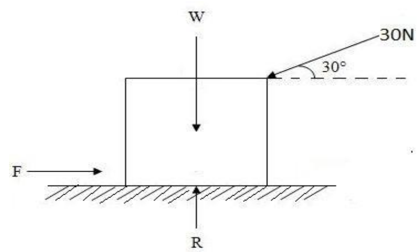


Fig. 2.29

From the horizontal forces, $\sum F_x = 0$,
F = 30 × 0.866 = 25.98 N
From the vertical forces,, $\sum F_y = 0$,
R = W + 30 × 0.5 = (W + 15) N
From the laws of friction,
F = μ R
25.98 = μ (W + 15) ....(ii)
Dividing. (i) to Eq. (ii), we get
W = 60 N
Putting the value of W in Eq. (i) and Eq. (ii), we get μ = 0.34

Assignment On Friction :

1. Two block of weight W_1 and W_2 rest as shown. If the angle of friction of each block is φ, find the magnitude and direction of the least force p applied to the upper block that will induce sliding. Fig-2.9.

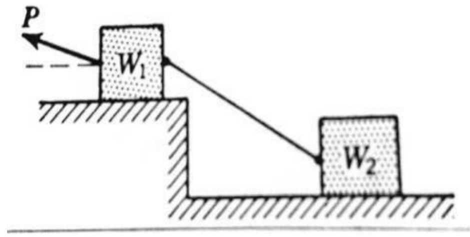


Fig. 2.30

2. Two blocks connected by a horizontal link AB are supported on two rough planes as shown. The co-efficient of friction for block A on the horizontal plane is μ = 0.4. The angle of friction for block B on inclined plane is 15°. What is smallest weight of block A for which equilibrium will exist? Fig-2.10

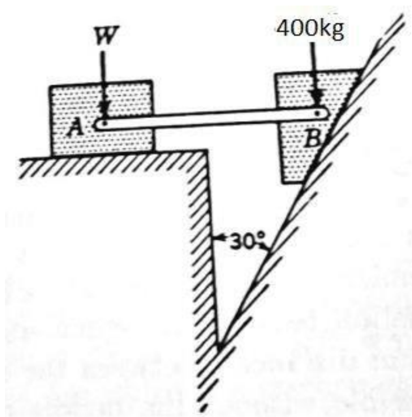


Fig. 2.31

3. Two block of weight W_1 = 200 kg-f & W_2 = 300 kg-f are joined by a cord parallel to plane inclined at an angle α with the horizontal . Find angle α for which the sliding will impend. What is tension in the cord. Given, co-efficient of friction for block A & B are 0.20 & 0.50 respectively. Investigate the case when μ_1= .05 , μ_2=0.2. Fig-2.11

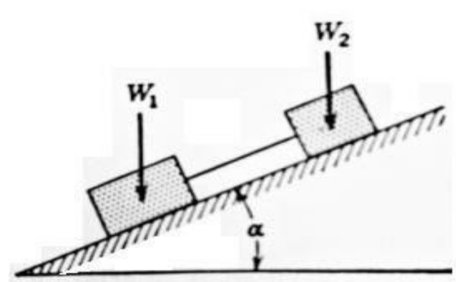


Fig. 2.32

4. A block of weight W_1 = 200 kg-f rests on horizontal surface and support on top of it another block of weight W_2= 50 kg-f. The block W_2 is attached top vertical wall by the inclined string AB . Find magnitude of the horizontal force P applied to the lower block as shown that will be necessary to cause slipping to impend. The coi-efficient of static friction for all contiguous surfaces is μ = 0.3. Fig-2.12

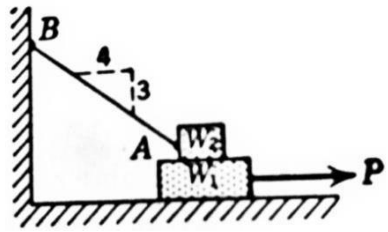


Fig. 2.33

5. Two inclined blocks A & B are connected by a rod & rest against vertical & horizontal planes respectively. If sliding impends when θ = 45°, determine the co-efficient of friction, μ, assuming it to be the same at both floor & wall. Fig-2.13

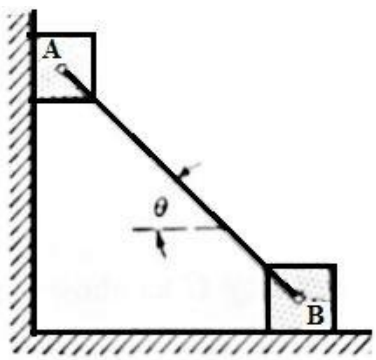


Fig. 2.34

6. A solid right circular cone of altitude h = 12 cm & radius r = 3cm has its c.g. C on its geometric axis at a distance h/4 above the base. The cone rests on the inclined plane AB which makes An angle of 30° with the horizontal and for which co-efficient of friction is 0.5. A horizontal force P is applied in the vertex O of the cone and acts in the vertical plane of figure. Find the maximum and minimum values of P consistent with equilibrium of the cone of weight W = 10kg-f.Fig-2.14

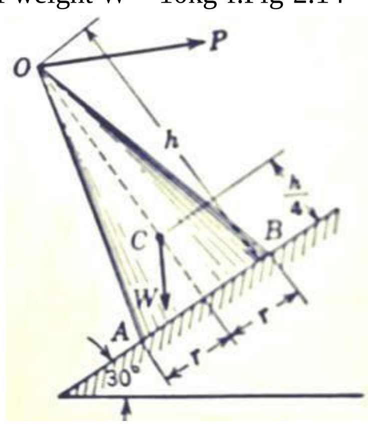


Fig. 2.35

7. A short right circular cylinder of weight W rests in a horizontal V notch having the angle 2α as shown. If the co-efficient of friction is μ , find the horizontal force P necessary to cause slipping to impend. Fig-2.15

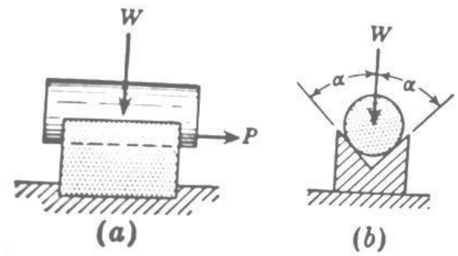


Fig. 2.36

8. A smooth circular cylinder of weight Q and radius r is supported by two semicircular cylinders each of the same radius r and weight Q/2 as shown. Find the maximum value of distance b for which motion will impewnd.Fig-2.16

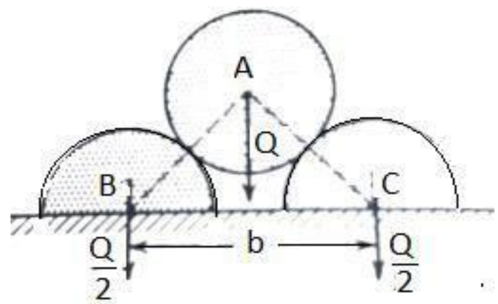


Fig. 2.37

9. In the figure shown, find the minimum value of horizontal force P applied to the lower block that will keep the system equilibrium? Given, coefficients of friction between lower block and floor = 0.25, between the upper block and the vertical wall = 0.30, between the two blocks =0.20.

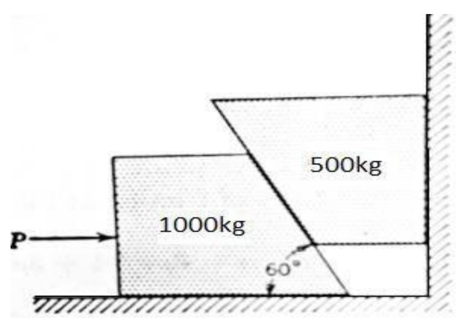


Fig. 2.38

10. A short semicircular right cylinder of radius r and weight W rests on a horizontal surface and is pulled at right angles to its geometric axis by a horizontal force P applied at the middle B of the front edge as shown. Find the angle α that the flat face will make with the horizontal plane just before sliding begins if the coefficient of friction at the line of contact A is μ. The gravity force W must be considered as acting at the centre of gravity C as shown in the figure.

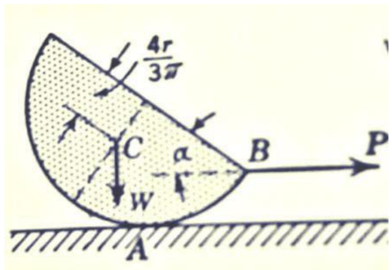


Fig. 2.39

11. A heavy prismatic bar AB of mass M is supported by a circular ring as shown. Length of the bar is such that it subtends an angle of 90° in the ring. If the angle of friction be φ, find greatest angle of inclination θ that the bar can make during equilibrium?

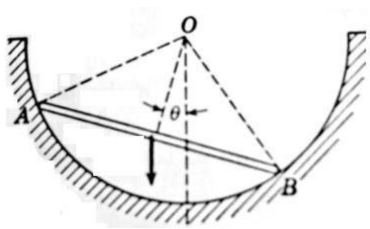


Fig. 2.40

Module 3:

Lecture 16

Centroid: The term centroid is used for two dimensional geometrical figures (areas and curves). For a plane figure it can also be defined as the arithmetic mean ("average") position of all the points in the plane figure. As for example centroid of a rectangle and a triangle are as shown in figure.

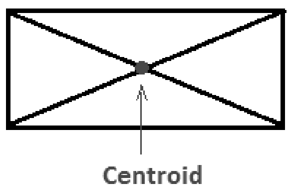


Figure 3.1

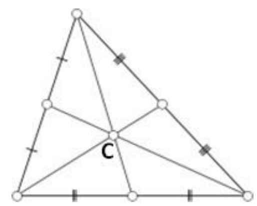


Figure 3.2

Center of gravity:
The center of gravity of a material body is that point through which the resultant of the distributed gravity forces passes, irrespective of the orientation of the body in the space. So, centre of gravity is a term which is applicable only for bodies having the property of weight. It also can be stated as the point through which whole weight of the body acts.

Computation of coordinates of centroid of curves:

Let the equation of curve is given by $y = f(x)$

$$x_c = \frac{\int x dL}{\int dL}$$

$$y_c = \frac{\int y dL}{\int dL}$$

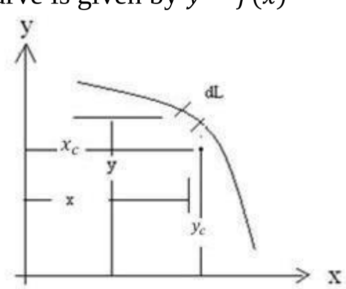


Figure 3.3

x_c and y_c are the coordinates of the centroid of the curve.

Computation of coordinates of centroid of area:

$$x_c = \frac{\int \tilde{x} dA}{\int dA}$$

$$y_c = \frac{\int \tilde{y} dA}{\int dA}$$

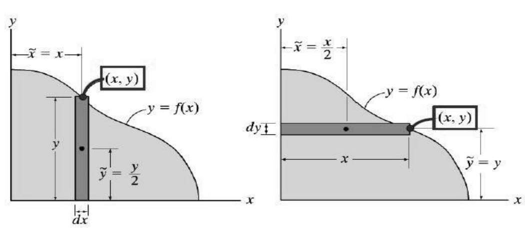


Figure 3.4

x_c, and y_c are the coordinates of the centroid of the area.

Computation of coordinates of centre of gravity of solid:

$$\frac{lV}{V}$$

$$y_c = \frac{\int y dV}{\int dV}$$

$$z_c = \frac{\int z dV}{\int dV}$$

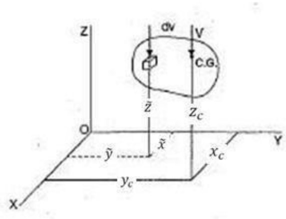


Figure 3.5

x_c, y_c and z_c are the coordinates of the centroid of the solid volume.

Example 1: Locate the centroid of the quadrant AB of the arc of the circle of radius R as shown in figure.

Solution: Elemental length dL can be written as:

$dL = Rd\theta$

Coordinates of the centroid of the elemental length can be written as:

$$\tilde{x} = R\cos\theta$$

$$\tilde{y} = R\sin\theta$$

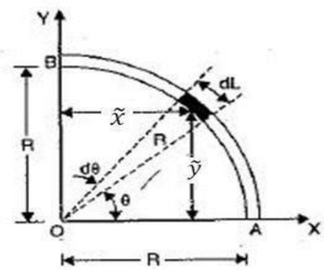


Figure 3.6

Coordinates of the centroid of the arc length AB are:

$$x_c = \frac{\int_0^{\frac{\pi}{2}} (R\cos\theta) R d\theta}{\int_0^{\frac{\pi}{2}} R d\theta} = \frac{2R}{\pi}$$

$$y_c = \frac{\int_0^{\frac{\pi}{2}} (R\sin\theta) R d\theta}{\int_0^{\frac{\pi}{2}} R d\theta} = \frac{2R}{\pi}$$

Example 2:

Determine the coordinates of the centroid of the sector of a circle of radius R and central angle of α

Solution: Elemental area dA can be written as:

$$dA = \frac{1}{2} R.Rd\theta$$

Coordinates of the centroid of the elemental area can be written as:

$$\tilde{x} = R\cos\theta$$

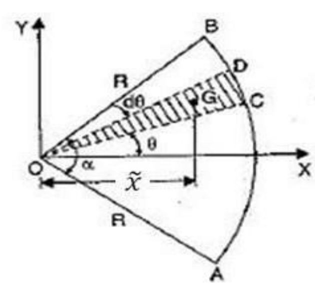


Figure 3.7

Coordinates of the centroid of the sector of the circle are:

$$x_c = \frac{\int_{-\alpha/2}^{\alpha/2} (R\cos\theta)\frac{1}{2}R.Rd\theta}{\int_{-\alpha/2}^{\alpha/2}\frac{1}{2}R.Rd\theta} = \frac{4R}{3\alpha}\sin\frac{\alpha}{2}$$

$y_c = 0$ (Due to symmetry)

Example 3: Find out the coordinates of the centroid of the quadrant of circular area as shown in figure.

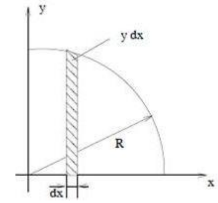


Figure 3.8

Solution: consider an elemental area in the form of a strip of height y and width dx

$$dA = ydx$$

Again, $x^2 + y^2 = R^2$, or $y = \sqrt{R^2 - x^2}$

$$x_C = \frac{\int x dA}{\int dA} = \frac{\int xydx}{\int dA} = \frac{\int_0^R x\sqrt{R^2 - x^2}}{\int_0^R ydx} = \frac{4R}{3\pi}$$

Pappus and Guldinus Theorem:

Theorem 1: The area of the surface generated by rotating a plane curve about a non-intersecting axis in its plane is equal to the product of the length l of the curve and the distance traveled by its centroid.

Surface area of the curve,

$$A = \int dA = \int \tilde{y}\theta \, dl = \theta \int \tilde{y} dl$$

By definition,

$$y_c = \frac{\int \tilde{y} dl}{\int dl}$$

$$\therefore \int \tilde{y} dl = y_c \, l$$

$$A = \theta \int \tilde{y} dl = (y_c \, \theta) \, l$$

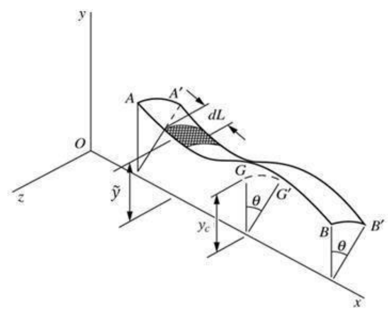


Figure 3.9

Theorem 2: The volume of the solid generated by rotating a plane surface about a non-intersecting axis in its plane is equal to the product of the area A of the Surface and the distance traveled by its centroid.

Volume of the solid,

$$V = \int dV = \int \tilde{y}\theta \, dA = \theta \int \tilde{y} dA$$

By definition,

$$y_c = \frac{\int \tilde{y} dA}{\int dA}$$

$$\therefore \int \tilde{y} dA = y_c \, A$$

$$V = \theta \int \tilde{y} dA = (y_c \, \theta) \, A$$

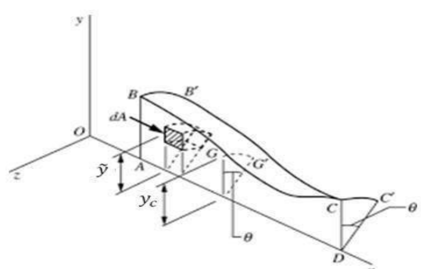


Figure 3.10

Lecture 17

Example 4: Using Pappus and Guldinus theorem find out the coordinates of the centroid of the semicircular area as shown in figure.

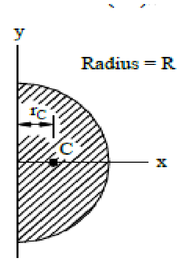


Figure 3.11

Solution: (1) If the semicircular area is revolved around the y $axis$, a sphere of radius R is generated. According to second theorem of Pappus-Guldinus the volume generated is equal to the volume of a sphere of radius R

$$\boldsymbol{V = 2\pi r_C A}$$

Area of the semicircle = $A = \dfrac{\pi R^2}{2}$

And the volume of the sphere = $V = \dfrac{4\pi R^3}{3}$

$$V = \frac{4\pi R^3}{3} = \frac{\pi R^2}{2} 2\pi r_C \text{ or, } r_C = \frac{4R}{3\pi}$$

Example 5: Determine the volume of the cone using Pappus_Guldinus theorem as shown in figure.

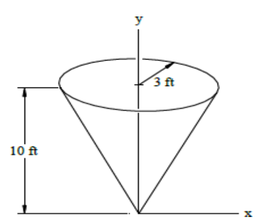
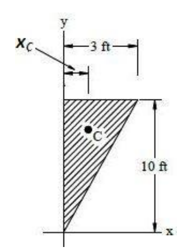


Figure 3.12

Solution: If the triangular area (as shown in figure) is revolved about y axis, the volume generated is equal to the volume of the cone.

According to second theorem of Pappus-Guldinus the volume generated is equal to the volume

of the cone = $\boldsymbol{V = 2\pi x_C A}$ $\quad A = \frac{1}{2}.3.10 = 15ft^2$



$from\ geometry\ we\ can\ write, x_c = 1(ft)$

Figure 3.13

So volume of the cone = $2\pi \times 1 \times 15 = 94.24\ ft^3$

Lecture 18

<u>To find out the coordinates of the centroid of composite area</u>:

The centroid of a plane area can be evaluated by dividing it into a finite number of simple common geometrical shapes such as rectangle, triangle, circle, semicircle, and quadrant etc. For the composite area shown in figure coordinates of centroid C is given by $(\bar{X}, \bar{Y})$

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3} = \frac{\sum Ax}{\sum A}$$

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{\sum Ay}{\sum A}$$

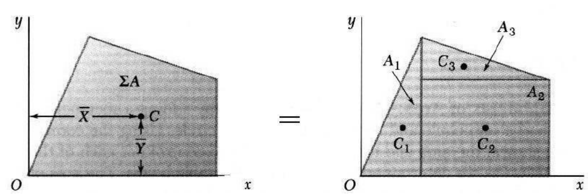


Figure 3.14

Example 6:

Determine the coordinates of the centroid (x_c, y_c) of the shaded area as shown in figure 7.

Solution:

A_1 = Area of quadrant (AOB) = $\frac{\pi 3^2}{4}$ cm^2 = 7.068 cm^2

$x_1 = \frac{4*3}{3\pi}$ = 1.27 cm

$y_1 = \frac{4*3}{3\pi}$ = 1.27 cm

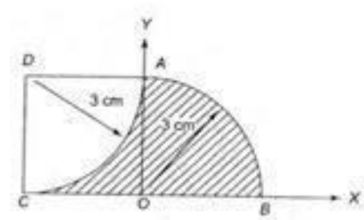


Figure 3.15

$A_2 = area\ of\ the\ square\ ADCO =$ 9 cm^2

x_2 = -1.5 cm, y_2= 1.5 cm

A_3 = area the quadrant (ADC) = = $\frac{\pi 3^2}{4}$ cm^2 = 7.068 cm^2

x_3= - (3- $\frac{4*3}{3\pi}$) cm = -1.73 cm

y_3= (3- $\frac{4*3}{3\pi}$) cm = 1.73 cm

$x_c = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3}$ = 0.856 cm

$y_c = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$ = 1.138 cm

Lecture 19

Example 7: Determine the coordinates of the centroid (x_c, y_c) of the composite area as shown in figure 7.

Solution:

Let area of triangle $A_1 = \frac{1}{2} \times 9 \times 3 = 13.5\ sq\ in$ and the $area\ of\ semicircle =\ A_2 = \frac{\pi \times 3^2}{2} = 14.13\ sq\ in$

Coordinates of centroid of the triangle ($x_1 = \frac{1}{3} \times 3 = 1(inch)$, $y_1 = \frac{2}{3} \times 9 = 6(inch)$ and for semicircle ($x_2 = \frac{3}{2} = 1.5\ (inch), y_2 = 9 + \frac{4 \times 3}{\pi} = 12.81(inch)$

The co-ordinates of the centroid of the whole area (x_C, y_C)

$$x_C = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{13.5 \times 1 + 14.13 \times 1.5}{13.5 + 14.13} = 1.255\ (inch)$$

$$y_C = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{13.5 \times 6 + 14.13 \times 12.81}{13.5 + 14.13} = 9.48\ (inch)$$

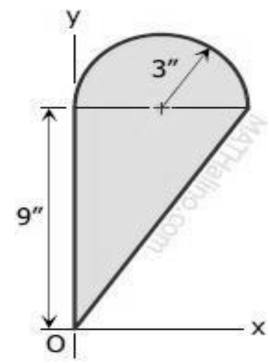


Figure 3.16

Example 8: Determine the coordinates of centroid of the composite area as shown in figure

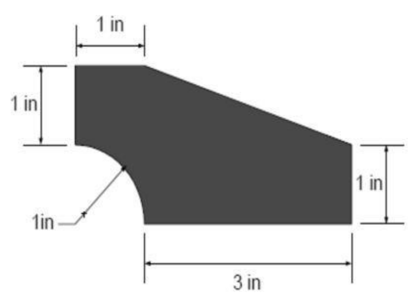


Figure 3.17

Solution:

First choose two coordinate axes as shown in figure and divide the composite area in 4 regular geometrical shapes ($A_1, A_2, A_3, and A_4$)

Let the x coordinates of the centroid of the areas $A_1, A_2, A_3, and A_4$ are $x_1,\ x_2, x_3, and x_4$ and y coordinates of the centroid of the areas $A_1, A_2, A_3, and A_4$ are $y_1,\ y_2, y_3, and y_4$ respectively.

$$\sum \boldsymbol{A} = \boldsymbol{A_1} + \boldsymbol{A_2} + \boldsymbol{A_3} - \boldsymbol{A_4}$$

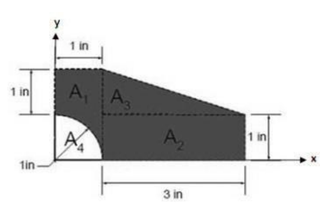


Figure 3.18

Area (sq in)	x ($inch$)	y ($inch$)	Ax	Ay
$A_1 = 2$	$x_1 = 0.5$	$y_1 = 1$	1.00	2.00
$A_2 = 3$	$x_2 = 2.5$	$y_2 = 0.5$	7.50	1.50
$A_3 = 1.5$	$x_3 = 2$	$y_3 = 1.33$	3.00	1.99
$-A_4 = -0.785$	$x_4 = \frac{4\times1}{3\pi} = 0.42$	$y_4 = \frac{4\times1}{3\pi} = 0.42$	-0.33	-0.33
$\sum A = 5.715$			$\sum Ax = 11.17$	$\sum Ay = 5.16$

$x_C = \frac{\sum Ax}{\sum A} = \frac{11.17}{5.715} = 1.95$ (inch)

$y_C = \frac{\sum Ay}{\sum A} = \frac{5.16}{5.715} = 0.903$ (inch)

Example 9: Locate the coordinates of centroid of the composite area as shown in figure

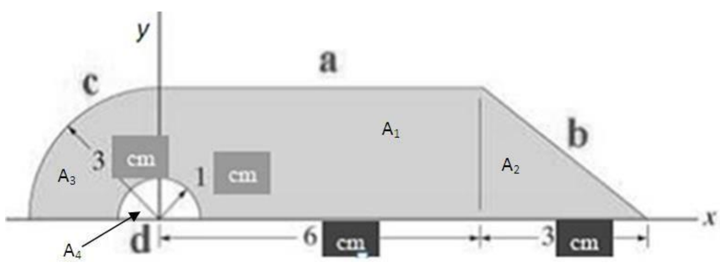


Figure 3.19

Solution:

Area (sq in)	x ($inch$)	y ($inch$)	Ax	Ay
$A_1 = 6\times 3 = 18$	$x_1 = 3$	$y_1 = 1.5$	54	27
$A_2 = \frac{1}{2}\times 3 \times 3 = 4.5$	$x_2 = 6 + \frac{1}{3}\times 3 = 7$	$y_2 = 1$	31.5	4.5
$A_3 = \frac{\pi}{4} \times 3^2 = 7.068$	$x_3 = -\frac{4\times3}{3\pi} = -1.27$	$y_3 = -\frac{4\times3}{3\pi} = -1.27$	-8.97	-8.97
$-A_4 = -\frac{\pi}{2}\times 1^2 = -1.57$	$x_4 = 0$	$y_4 = \frac{4\times1}{3\pi} = 0.42$	0	-0.66
$\sum A = 27.998$			$\sum Ax = 76.53$	$\sum Ay = 21.87$

$$x_C = \frac{\sum Ax}{\sum A} = \frac{76.53}{27.998} = 2.73\ (inch)$$

$$y_C = \frac{\sum Ay}{\sum A} = \frac{21.87}{27.998} = 0.78\ (inch)$$

Example 10: Determine the coordinates of the C.G. of the hemisphere as shown in figure 7.

Solution:

Consider an elemental lamina of radius y and thickness dz and the volume of the element $dV = \pi y^2 dz$

$$z_C = \frac{\int z dV}{\int dV} = \frac{\int z\pi y^2 dz}{\int \pi y^2 \, dz}$$

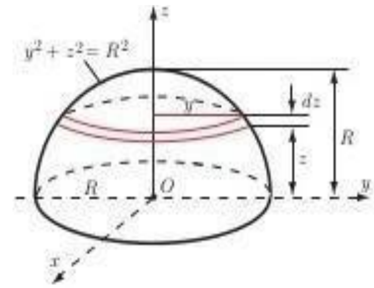


Figure 3.20

From geometry, $y^2 + z^2 = R^2$ or, $y^2 = R^2 - z^2$ or, $y = \sqrt{R^2 - z^2}$

$$z_C = \frac{\int_0^R z\pi(R^2 - z^2)dz}{\int_0^R \pi(R^2 - z^2 dz} = \frac{3R}{8}$$

MCQ Type Questions:

1. The centroid of a quarter circular area of radius **r** is (a) $\frac{2R}{3\pi}$ (b) $\frac{3R}{4\pi}$ (c) $\frac{4R}{3\pi}$ (d) $\frac{3R}{8}$

2. C.G. of hollow hemisphere of radius R is (a) $\frac{R}{2}$ (b) $\frac{3R}{8}$ (c) $\frac{3R}{4}$ (d) $\frac{4R}{9}$

3. C.G. of solid hemisphere of radius R is (a) $\frac{R}{2}$ (b) $\frac{3R}{8}$ (c) $\frac{3R}{4}$ (d) $\frac{4R}{9}$

4. The distance of centroid of a semicircular arc of radius r from base is (a) $\frac{4r}{\pi}$ (b) $\frac{2r}{\pi}$ (c) $\frac{3r}{2\pi}$ (d) None of these

5. Pappus-Guldinus theorem can be applied to calculate (a) Area, volume and centroid (b) Volume and centroid only (c) Area and volume only (d) Area and centroid only

6. The C.G. of a solid right circular cone is (a) h/3 (b) h/4 (c) 2h/3 (d) 4h/3

Assignment:

Problem No 1: Locate the centroid of the shaded area as shown in figure

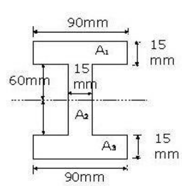


Figure 3.21

Problem No 2: Locate the centroid of the shaded area as shown in figure

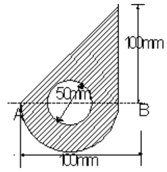


Figure 3.22

Problem No 3: Referring to the figure no 8, determine the coordinates (x_c, y_c) of the centroid of 100mm circular hole cut in a thin plate, so that this point will be the centroid of the remaining part of the plate.

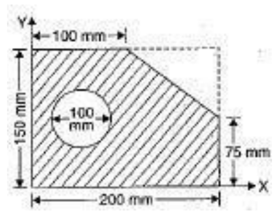


Figure 3.23

Problem No 4: Locate the centroid of the shaded area as shown in figure 9

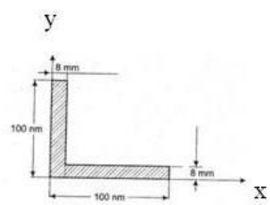


Figure 3.24

Problem No 5: Find the distance from the vertex of the right circular cone to the CG of its volume

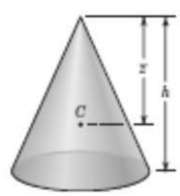


Figure 3.25

Problem No 6: Find out x & y coordinates of centroid of the shaded area as shown in figure.

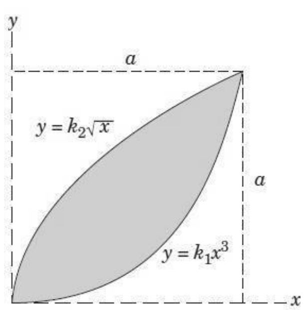


Figure 3.26

Problem No 7: Find out x & y coordinates of centroid of the shaded area as shown in figure.

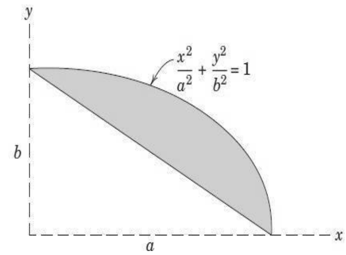


Figure 3.27

Lecture 20
Area moment of inertia

Consider an elemental area dA within a thin lamina of area A as shown in figure

Let x = distance of C.G. of area dA from Y axis

y = distance of C.G. of area $dA$from X axis

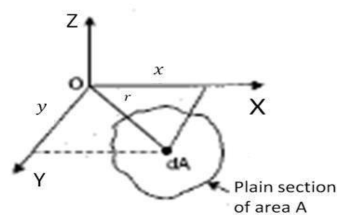


Figure 3.26

Then the moment of the elemental area dA about Y axis = Area × perpendicular distance from axis of rotation (Y axis) =$x.dA$ (this is also known as first moment of area about Y axis)

If the moment of elemental area dA is again multiplied by the perpendicular distance x, then the quantity = dI_y = $x.(x.dA)$ = x^2dA (this is also known as second moment of area, area moment of inertia).

Moment of inertia of the whole area about Y axis = $I_y = \int dI_y = \int x^2dA$

Similarly, moment of inertia of the whole area about X axis = $I_x = \int dI_x = \int y^2dA$

Then moment of inertia of dA about pole O (Z axis) is, $dI_z = r.(r.dA) = r^2dA$

Then moment of inertia of whole area about pole O (Z axis) = $I_z = \int dI_z = \int r^2dA$ (this known as polar moment of inertia.

From geometry we can write, $r^2 = x^2 + y^2$

Therefore, $I_z = \int r^2dA = \int (x^2 + y^2)dA = I_x + I_y$

Parallel Axis Theorem:

The moment of inertia of an area about a non-cetroidal axis may be expressed in terms of the moment of inertia about centroidal axis. In figure $x_0 - y_0$ axes pass through the centroid C of the area parallel to $x - y$ axes.

From definition moment of inertia of the elemental area about x axis can be written as

$$dI_x = (y_0 + dx)^2 dA$$

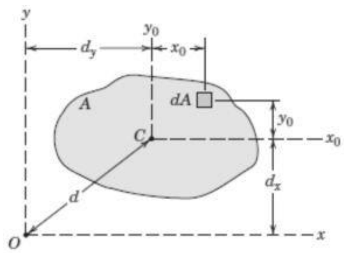


Figure 3.27

So MI of the whole area about x axis will be

$$I_x = \int (y_0 + d_x)^2 dA$$

$$I_x = \int y_0^2 \, dA + 2d_x \int y_0 \, dA + d_x^2 \int dA$$

$$I_x = I_{x_0} + A d_x^2$$

$(as\ here \int y_0 \, dA = 0\ because\ C\ is\ the\ centroid\ and\ also\ origin\ of\ x_0 - y_0\ axes)$

Radius of Gyration: Consider an area A as shown in figure (a) which has rectangular moment of inertia I_x and I_y about O.

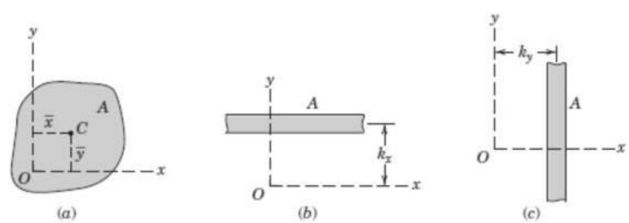


Figure 3.28

Now we will imagine that this area is concentrated into a long narrow strip of area A a distance k_x from x axis as shown in figure (b).

From definition of moment of inertia of the strip about x axis will be the same as that of the original area if, $k_x^2 A = I_x$, then the distance k_x is called the radius of gyration of the area about x axis. Similarly k_y is the radius of gyration of the area about y axis. So we can express k_x and k_y by the relation as given below.

$$k_x = \sqrt{\frac{I_x}{A}} \quad \text{and} \quad k_y = \sqrt{\frac{I_y}{A}}$$

Sample Problem Solved:

Problem No 1:

Determine the moments of inertia of the rectangular area about the centroidal x_0 and y_0 axes.

Moment of inertia of the elemental area dA about about x_0 = dI_{x_0}= y^2dA

I_{x_0}= $\int y^2dA$ = $\int_{-h/2}^{h/2} y^2 . bdy$ = $bh^3/12$

Similarly it also can be written for I_{y_0}= $hb^3/12$

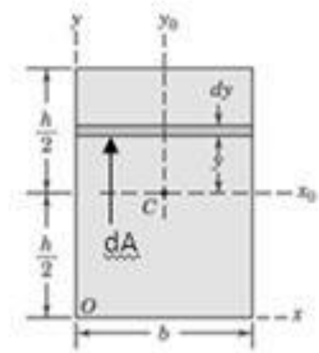


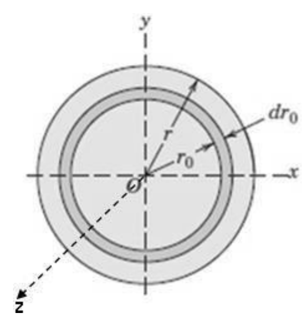
Figure 3.29

Lecture 21

Problem 2:

Calculate the moments of inertia of the area of a circle about a diametral axis and about the polar moment axis through the centre.

Solution: An elemental area in the form of circular ring may be taken for calculation of the moment of inertia about the polar z axis through the centre of the circle O since all elements of the ring are equidistance from O. The elemental area $dA = 2\pi r_0 dr_0$ and thus, $I_z = \int_0 r^2 dA = \int_0^r r_0^2 \, 2\pi r_0 \, dr_0 = \frac{\pi r^4}{2}$



Again, $I_x + I_y = I_z$ and from symmetry we can write, $I_x = I_y$

Thus $I_x = I_y = \frac{\pi r^4}{4}$

Problem 3:

Calculate the moment of inertia of the triangular area about its centroidal axis and which is parallel to base.

Solution:

An elemental area in the form of strip of width dy may be taken for calculation of the moment of inertia about the centroidal axis X parallel to its base.

The elemental $dA = ldy$

From geometry we can write $\frac{l}{b} = \frac{y}{h}$ or $l = \frac{by}{h}$

$$dA = \frac{by.dy}{h}$$

Thus $I_X = \int (\frac{2h}{3} - y)^2 dA = \int_0^h (\frac{2h}{3} - 3)^2 \frac{by}{h} dy = \frac{bh^3}{36}$

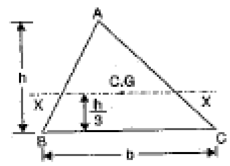


Figure 3.31

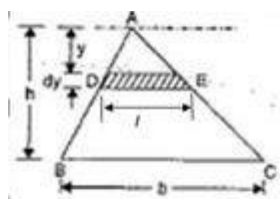


Figure 3.32

Lecture 22

Mass moment of inertia:

Consider a three dimensional body of mass m as shown in figure

Mass moment of inertia of the mass m about O - O axis is $I_{o-o} = \int r^2 dm$

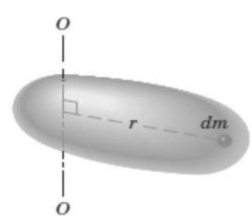


Figure 3.33

Determination of mass moment of inertia of paralleopipped:

Figure shows a paralleopipped of length b, width t, and height d

Consider a small elemental of length b at a distance y from X –X axis.

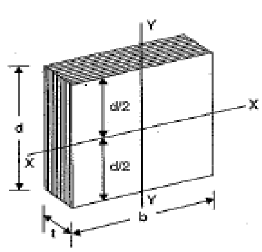
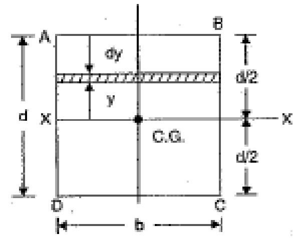


Figure 3.34

X-X axis is a horizontal line passing through the c.g. of the elemental mass

= dm



Here $dm = \rho btdy$ ($\rho = density\ of\ material$)

Moment of inertia of the elemental mass about X-X axis = $I_{X-X} = \int y^2\, dm$

$$I_{X-X} = \int_{-d/2}^{d/2} y^2\, \rho btdy = \rho t \frac{bd^3}{12} = m\frac{d^2}{12}$$

Figure 3.35

(here mass of the paralleopipped = $m = \rho bdt$)

Determination of mass moment of inertia of a cylinder of mass m (having radius R and length L) about its axis:

Solution: Consider an elemental mass of dm

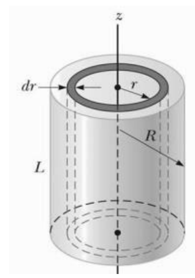


Figure 3.36

$$dm = \rho dV = \rho L(2\pi r)dr$$

$$I_z = \int r^2\, dm = \int_0^R r^2 \{\rho L(2\pi r)dr\}$$

$$I_z = 2\pi\rho L \int_0^R r^3 dr = \frac{\pi\rho L R^4}{2}$$

$$\text{Again } m = \rho\pi R^2 L$$

$$I_z = \frac{mR^2}{2}$$

Problem No 4: Determination of mass moment of inertia of a cylinder of mass m (having radius r and length L) about its centroidal axis x_C and y_C axes.

Solution: Consider an elemenatal mass $dm = \rho\pi r^2 dz$

Moment of inertia of the elemental mass about its own centroidal axis (parallel to y_C axis)

= $(\rho\pi r^4/4)dz$

M I of the elemental mass about its y_C axis (using parallel axis theorem) = $(\rho\pi r^4/4)dz + (\rho\pi r^2 dz)z^2$

M I of the whole cylinder about its y_C axis = $\int_{-L/2}^{L/2} \{(\rho\pi r^4/4)dz + (\rho\pi r^2 dz)z^2\} = m\left(\frac{r^2}{4} + \frac{L^2}{12}\right)$

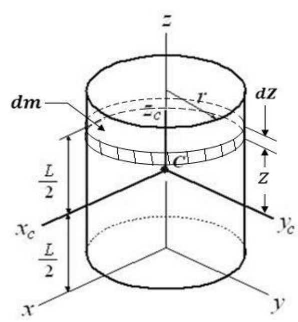


Figure 3.37

Assignment:

Problem No 1: Determine the moment of inertia of the T section about the horizontal and vertical axes, passing through the centroid of the section.

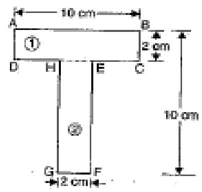


Figure 3.38

Problem No 2: Determine the moment of inertia of the I section about the horizontal and vertical axes, passing through the centroid of the section.

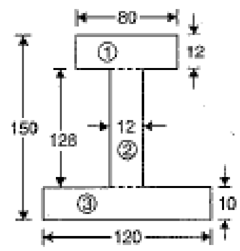


Figure 3.39

Problem No3: Determine the moment of inertia of the semicircular area about the centriodal axes x_0 and about an axis x parallel to x' at a distance 15mm from x' axis.

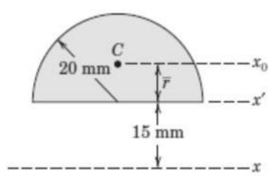


Figure 3.40

Problem No 4: Determine the moment of inertia of the square section about the diagonal $(x' axis)$

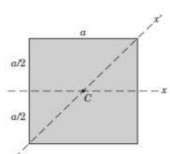


Figure 3.41

MCQ Type Questions:

1. The polar moment of inertia is mathematically expressed by (a) $I_{ZZ} = I_{XX}\ I_{YY}$ (b) $I_{ZZ} = I_{XX} / I_{YY}$ (c) $I_{ZZ} = I_{XX} - I_{YY}$ (d) $I_{ZZ} = I_{XX} + I_{YY}$
2. The moment of inertia of a rectangular area of base b, height h about base is given by (a) $bh^3/12$ (b) $bh^3/36$ (c) $bh^3/4$ (d) $bh^3/3$
3. The moment of inertia of a triangular area of base b and height h about base is (a) $bh^3/12$ (b) $bh^3/36$ (c) $bh^3/4$ (d) $bh^3/3$
4. The moment of inertia of a triangle with respect to centroidal axis parallel to the base is (a) $bh^3/12$ (b) $bh^3/36$ (c) $bh^3/4$ (d) $bh^3/3$
5. The moment of inertia of a semicircular area of radius r about centroidal x-x axis is (a) $0.22r^3$ (b) $0.11\ r^3$ (c) $0.11r^4$ (d) $0.15r^4$
6. The moment of inertia of a rectangular area of base b, height h about a centroidal axis parallel to its base is given by (a) $bh^3/12$ (b) $bh^3/36$ (c) $bh^3/4$ (d) $bh^3/3$
7. The moment of inertia of a quadrant circle of radius r about centroidal x-x axis is (a) $0.055r^4$ (b) $0.11r^4$ (c) $0.044r^4$ (d) r^4
8. The moment of inertia of an ellipse of major axis a and minor axis b about its major axis is (a) $\pi(\frac{a^3b}{4})$ (b) $\pi(\frac{b^3a}{4})$ (c) $\frac{\pi a^2b^2}{4}$ (d) $\frac{\pi a^3b}{3}$
9. The mass moment of inertia of sphere of radius r and mass M is (a) $2M^{r2}/5$ (b) $M^{r2}/2$ (c) $4M^{r2}/5$ (d) none of these
10. The moment of inertia of a square lamina of side a about its diagonal (a) $a^4/12$ (b) $a^4/16$ (c) $a^3/12$ (d) $a^4/24$

Lecture 23

Virtual work:

Consider a particle is in static equilibrium under the action of external forces acting on it. Any assumed and arbitrary small displacement dr $a$way from this natural equilibrium position and consistent with the geometrical configuration of the system is called a virtual displacement. The term virtual is to indicate that the displacement does not exist in reality but only in an assumption so that we can compare various possible equilibrium configuration of the system.

If any force F acts on a particle and a virtual displacement of dr is assumed to be given on the particle then virtual work done by the force F is $dU = \vec{F}.\overrightarrow{dr}$ = $F\ dr\ cos\alpha$

Where α is the angle between force (F) and virtual displacement (dr).

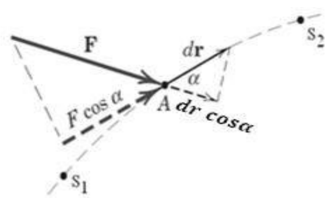


Figure 3.42

Principle of virtual work:

The virtual work done by external forces on an ideal mechanical system in equilibrium will be equal to zero.

Mathematically,

$$\boldsymbol{dU = 0}$$

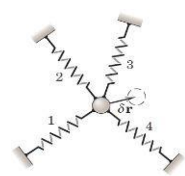


Figure 3.43

Examples:

Problem No 1: Using the principle of virtual work find the magnitude of force P in terms of *a, b, and W* required for the equilibrium of the bell crank *ABC*. The pin at B is frictionless. Neglect the weight of the bell crank.

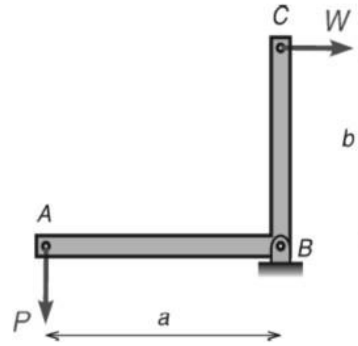


Figure 3.44

Solution: Let a virtual angular displacement of $\delta\theta$ is given to the system as shown in figure below.

Virtual displacements:

$\delta r_C = b\delta\theta, \delta r_A = a\delta\theta$

Virtual work $\delta U = 0$, $P\delta r_A - W\delta r_C = 0$

Or, $Pa\delta\theta - Wb\delta\theta = 0$

$$P = \frac{bW}{a}$$

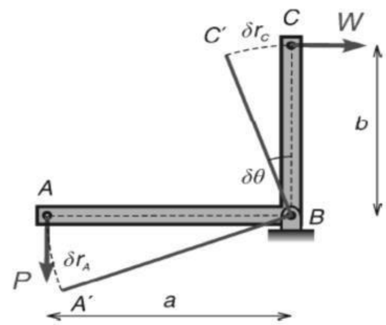


Figure 3.45

Lecture 24

Problem No 2: The thin rod of weight W rest against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium for a given angle θ

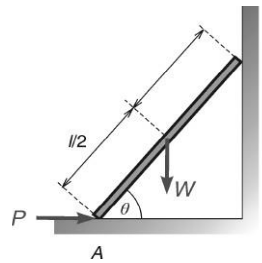


Figure 3.46

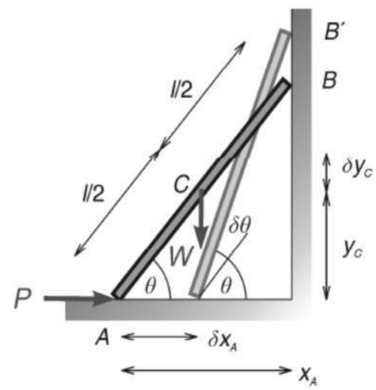


Figure 3.47

Solution: let a virtual angular displacement of $\delta\theta$ is given o the rod from its equilibrium position

Virtual displacements:

$$\delta y_C + y_C = \frac{l}{2}\sin(\theta + \delta\theta)$$

$$\delta y_C = \frac{l\delta\theta\cos\theta}{2}$$

Similarly, $\delta X_A + X_A = l\cos(\theta + \delta\theta)$

$\delta X_A = -l\delta\theta\sin\theta$ (as θ increase, X_A decrease)

Now applying principle of virtual work we can write, $\delta U = 0$

$P\delta X_A - W\delta y_C = 0,\ Pl\delta\theta\sin\theta - W\frac{l}{2}\delta\theta\cos\theta = 0$

or, $P = \frac{1}{2}W\cot\theta$

Lecture 25

Problem No 3: Determine the magnitude of the couple M required to maintain the equilibrium of the mechanism.

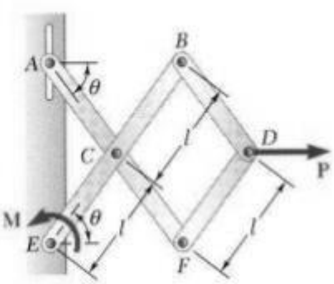


Figure 3.48

Solution: let a virtual displacement of δX_D is given at point D of the mechanism

Applying principle of virtual work

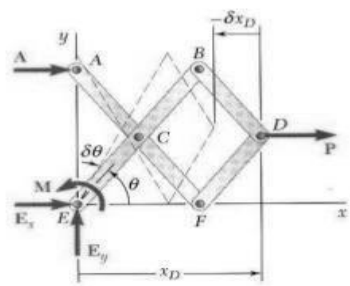


Figure 3.49

$$X_D = 3l\cos\theta$$

$$\delta X_D = -3l\sin\theta\,\delta\theta$$

Again, $\delta U = 0 = M\delta\theta - P\delta X_D = 0$

Or, $M\delta\theta - 3Pl\sin\theta\,\delta\theta = 0$

Or, $M = 3Pl\sin\theta$

MCQ type questions

1. The virtual work principle can be best suited in
   (a) Rigid body (b) Connected several rigid bodies
   (c) Particle (d) None of these
2. The virtual work concept is very useful in solving problems related to
   (a) Static (b) Dynamics (c) Equilibrium (d) none of these

Assignment:

Problem No 1: A frictionless double-inclined plane with angle α_1 and α_2 as shown in figure below carries a set of sliding weights W_1 and W_2 connected with an inextensible string and passing over a frictionless pulley. By using the principle of virtual work obtain the relationship between α_1 and α_2 in terms of W_1 and W_2 and hence determine α_2 , if $\alpha_1 = 30^0$ and $W_1 = 2W_2$.

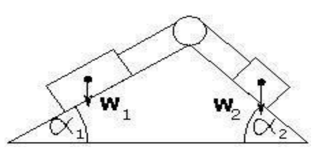


Figure 3.50

Problem No 2: Using principle virtual work find the value of the angle θ defining the configuration of equilibrium of the system shown in figure below The ball D and E can slide freely along the bars AC and BC, but the string DE connecting the balls is inextensible.

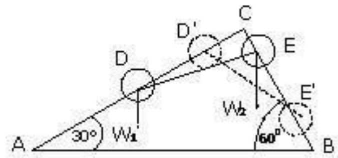


Figure 3.51

Problem No 3: Determine the expression for θ and for the tension in the spring which correspond to the equilibrium position of the spring. The unstretched length of the spring is h and the spring constant is k. neglect the weight of the mechanism.

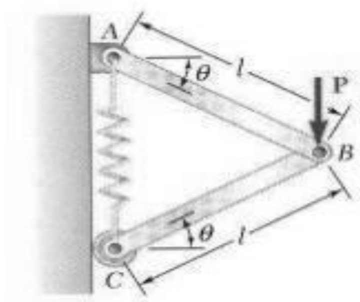


Figure 3.52

Dynamics

Lecture – 26

Topic: Introduction to Dynamics

Dynamics is that branch of mechanics which deals with the motion of bodies under the action of the forces. Dynamics has two distinct parts: kinematics, which is the study of motion without reference to the forces which cause motion, and kinetics, which relates the action of forces on bodies to their resulting motions.

The beginning of a rational understanding of dynamics is credited to Galileo (1564-1642), who made careful observations concerning bodies in free fall, motion on an inclined plane, and motion of the pendulum.

Sir Isaac Newton (1642-1727), was guided by Galileo's work, was able to make an accurate formulation of the laws of motion and, hence, to place dynamics on a sound basis. He stated the three laws of motion for governing the motion of a particle. In addition to this he was the first to correctly formulate the law of universal gravitation. Following Newton's time, important contributions to mechanics were made by Euler, D'Alembert, Lagrange, Laplace, Poinsot, Coriolis, Einstein, and others.

Basic Concepts:

Space: Space is the geometric region occupied by bodies.

Position: Position in space is determined relative to some geometric reference system by means of linear and angular measurements.

Frame of reference: The basic frame of reference for the laws of Newtonian mechanics is the primary inertial system or astronomical frame of reference, which is an imaginary set of rectangular axes assumed to have no translation or rotation in space with respect to the surface of the earth.

Time: Time is a measure of the succession of events and is considered an absolute quantity in Newtonian mechanics.

Mass: Mass is the quantitative measure of the inertia or resistance to change in motion of a body and also it can be defined as the property that gives rise to gravitational attraction.

Force: A **force** is any interaction that, when unopposed, will change the motion of an object.

Particle: A particle is a body of negligible dimensions.

Rigid body: A rigid body is a body whose changes in shape are negligible compared with the overall dimensions of the body or with the changes in the position of the body as a whole.

Newton's laws:

First law: A particle remains at rest or continues to move in straight line with a constant velocity if there is no unbalanced force acting on it.

Second law: The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

Third law: The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

These laws have been verified by countless physical measurements. The first two laws hold for measurements made in an absolute frame of reference, but are subject to some correction when the motion is measured relative to a reference system having acceleration, such as one attached to the earth's surface.

Kinematics: The relation among displacement ($x(t)$), velocity ($\dot{x}(t)$) and acceleration ($\ddot{x}(t)$),

$\dot{x} = \frac{dx(t)}{dt}$ and $\ddot{x} = \frac{d^2x(t)}{dt^2}$

These are without reference to applied force.

Velocity is defined as the rate of change of displacement with respect to time. Velocity may be a constant quantity for a particle and again it may be a variable also. SI unit of velocity is m/s.

Acceleration is defined as the rate of change of velocity with respect to time. Acceleration also, like velocity, may be a constant as well as a variable. SI unit of acceleration is m/s^2.

Kinetics: the relation between force ($F(t)$) and acceleration ($x(t)$),
$\sum(F_a + F_r) = mx$, where F_a is applied force and F_r is reaction force.

Newton's Law of Universal Gravitation

Newton's law of universal gravitation states that a particle attracts every other particle in the universe using a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = G\frac{m_1 m_2}{r^2}$$

Where:

F is the force between the masses;

G is the gravitational constant (6.674×10^{-11} N · (m/kg) ^{2});

m_1 is the first mass;

m_2 is the second mass;

r is the distance between the centers of the masses.

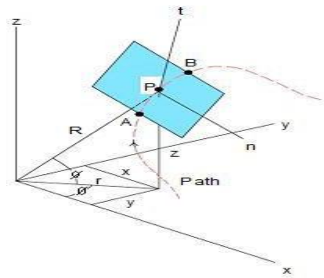
Lecture - 27

Topic: **Kinematics of Particles**

Rectangular Coordinates $\vec{r}(x, y, z)$

Cylindrical Coordinates $\vec{r}(r, \theta, z)$

Spherical Coordinates $\vec{r}(R, \theta, \phi)$



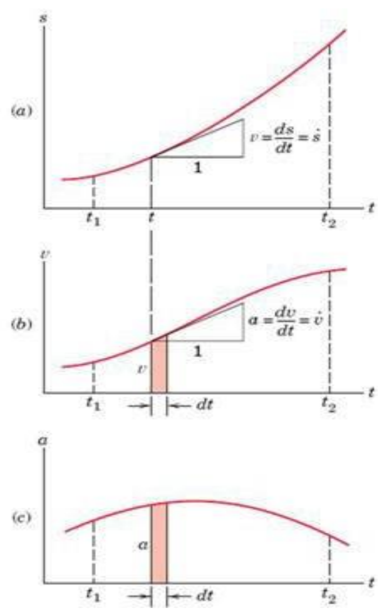
To analyze the kinematics of particles a reference frame is required. The chosen reference frame may be rectangular Cartesian coordinates, cylindrical coordinates, spherical coordinates or any other. The choice depends upon the requirement. In most of the cases, rectangular Cartesian coordinate system is used and also this coordinate system is very easy to use. And if the problem becomes 2-D, then the solution becomes simpler.

Construction of s-t, v-t and a-t graphs

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt$$

$$\int_{v_1}^{v} dv = \int_{t_1}^{t_2} a\,dt$$

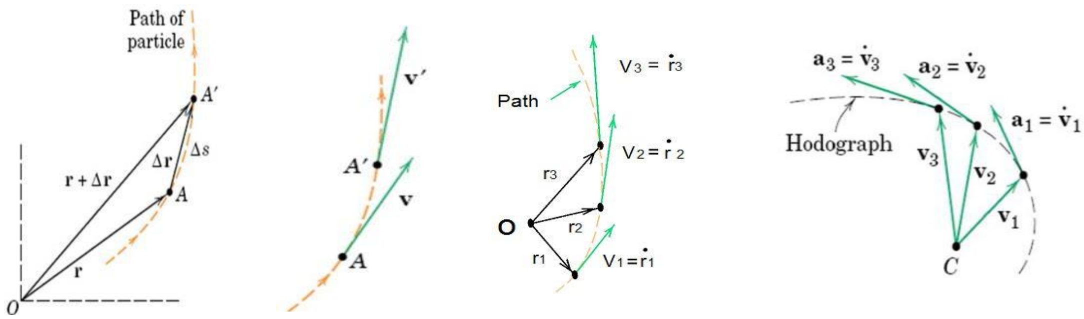
$$\int_{x_1}^{x_2} dx = \int \dot{x}\,dt = \int_{t_1}^{t_2} \left(\int_{t_1}^{t_2} \ddot{x}\,dt \right) dt$$



For the integrals to be performed the velocity, acceleration etc. should be known as a function of time and if they are constant quantities the integration becomes straightforward. For the construction of the graphs of s-t, v-t and a-t the displacement, velocity and acceleration should be known as a function of time.

Lecture – 28

Velocity and Acceleration Vector



Hodograph: A curve the radius vector of which represents in magnitude and direction the velocity of a moving object.

$$\dot{r} = \frac{d\vec{r}}{dt} \equiv \vec{v} \qquad\qquad \ddot{r} = \frac{d\dot{r}}{dt} \equiv \vec{a}$$

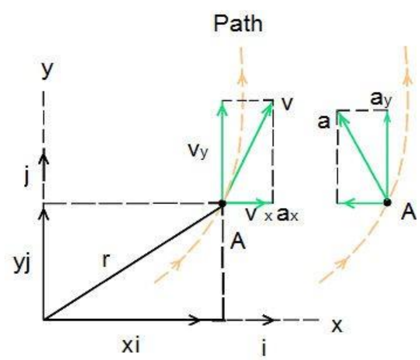
Rectangular Coordinates

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\dot{r} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

$$\ddot{r} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

$$r = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \dot{r} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \qquad \ddot{r} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}$$



Rectilinear Motion of a Particle:

The well known equations for rectilinear motion are

(a) $v = u + at$

(b) $s = ut + \frac{1}{2}at^2$

(c) $v^2 = u^2 + 2as$, where

u is initial velocity
v is final velocity
a is acceleration
t is time
s is displacement

Example: 1

A particle is moving with constant acceleration a. It covers initial distance of 16 m in 10 seconds. What time it will take to cover entire distance of 400m? What will be its final velocity? Assume that the particle has started from rest.

Solution:

$$s = \frac{1}{2}at^2 \Rightarrow a = \frac{2s}{t^2} = \frac{2\times16}{10^2} = 0.32\,m/s^2$$

Again $s = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times400}{0.32}} = 50\,s$

For final velocity, $V = at = 0.32\times50 = 16m/s$

Example: 2

The acceleration of a particle at any point A is expressed by the relation $a = 200x(1+kx^2)$, where a and x are in m/s^2 and meters respectively and k is a constant. If the velocity of the particle at A is $v_A = 2.5m/s$ when x=0 and $v_A = 5m/s$ when x=0.15 m, find the value of k.

Solution:

Given $a = 200x(1+kx^2)$

$$\therefore \frac{dv}{dt} = 200x(1+kx^2) \Rightarrow \frac{dv}{dx}\frac{dx}{dt} = 200x(1+kx^2) \Rightarrow v\frac{dv}{dx} = 200x(1+kx^2)$$

$$vdv = 200x(1+kx^2)dx \Rightarrow \int vdv = \int 200x(1+kx^2)dx$$

$$\Rightarrow \frac{v^2}{2} = \frac{200x^2}{2} + \frac{200kx^4}{4} + C_1$$

$$\therefore c_1 = \frac{2.5^2}{2} = 3.125 \quad [\because when\ x = 0; v_A = 2.5m/s]$$

Again $\frac{5^2}{2} = \frac{200\times0.15^2}{2} + \frac{200k\times0.15^4}{4} + 3.125$

Solving, we get $281m^{-2}$

Example: 3

The vertical acceleration of a certain solid-fuel rocket is given by $a = ke^{-bt} - cv - g$, where k, b, c are constants, v is the vertical component of velocity acquired, and g is the acceleration due to gravity that remains approximately constant over the trajectory of an atmospheric flight. The exponential term represents the effect of a decaying thrust as fuel is burned, and the term –cv approximates the retardation due to atmospheric resistance. Determine v as a function of t.

Solution:

Given $a = ke^{-bt} - cv - g$

$$\Rightarrow \frac{dv}{dt} = ke^{-bt} - cv - g \Rightarrow \frac{dv}{dt} + cv = ke^{-bt} - g \Rightarrow e^{ct}\frac{dv}{dt} + cve^{ct} = ke^{ct}e^{-bt} - ge^{ct}$$

$$\frac{d}{dt}(ve^{ct}) = ke^{(c-b)t} - ge^{ct} \Rightarrow \int d(ve^{ct}) = \int[ke^{(c-b)t} - ge^{ct}]dt$$

$\Rightarrow ve^{ct} = \frac{ke^{(c-b)t}}{(c-b)} - \frac{ge^{ct}}{c} + A$ where A is a constant of integration.

Now, using the given initial condition, we can write

$$A = \frac{g}{c} - \frac{k}{(c-b)} \left[\because v = 0 \; at \; t = 0\right]$$

$$\therefore v = \frac{g}{c}\left(e^{-ct} - 1\right) + \frac{k}{(c-b)}\left(e^{-bt} - e^{-ct}\right)$$

Problems:

(1) A ball is projected upwards with a velocity of 24 m/s. Two seconds later, a second ball is vertically projected upwards with a velocity of 18 m/s. At what point above the surface of earth will they meet?
(2) A body in rectilinear motion is found to travel 15 m in 5^{th} second and 25 m in the 10^{th} second. What distance will it travel in 15 seconds from the starting point if its acceleration is constant throughout?
(3) Water drips from a faucet at a uniform rate of n drops per second. Find the distance x between any two adjacent drops as a function of the time t that the trailing drop has been in motion.
(4) A train accelerates from rest with constant acceleration of a_1 to acquire a maximum velocity of v_{max} and immediately starts decelerating with constant deceleration of a_2 so as to come to rest. If the total duration of the travel is T, prove that $v_{max} = \frac{a_1 a_2}{a_1 + a_2} T$.
(5) The position of a particle describing rectilinear motion can be described by $x = t^3 - 9t^2 + 15t + 18$, x is expressed in meter and t is in second. Determine the time, displacement, and acceleration of the particle when its velocity is zero.

Multiple Choice Questions:

(1) Acceleration is a
(i) scalar (ii) vector (iii) tensor (iv) none of these
(2) The SI unit of time rate of change of acceleration is
(i) m/s (ii) m/s^2 (iii) m/s^3 (iv) kg/m^3
(3) A particle starts from rest and moves along a straight line with constant acceleration. The particle attains a velocity of 10 m/s after a distance of 25 m, the acceleration is
(i) 2.50 m/s (ii) 2.25 m/s (iii) 2.00 m/s (iv) 1.75 m/s
(4) The velocity of a particle is described as $x = \frac{1}{2}at^2 \; where \; a = 10\, m/s^2$. The displacement of the particle when t=5 s is
(i) 208.33 m (ii) 108.57 m (iii) 236.74 m (iv) 182.83 m
(5) A particle has straight line motion according to the equation $x = t^3 - 3t^2 - 5$, where x is in meter and t is in second. The change in position of the particle is when its velocity changes from 8 m/s to 40 m/s
(i) 46.1 m (ii) 43.7 m (iii) 45.8 m (iv) 41.6 m

Lecture – 29

Topic: Plane Curvilinear Motion of Particles

So far we have considered motion of particles along a straight line. But there are situations where the particles move in curved path. This happens only when the direction of initial velocity and direction of force are different. If the curved path lies in a plane, the resulting motion is called the plane curvilinear motion.

Rectangular Components of Velocity: Let θ be the angle made by the direction of the velocity u with the horizontal. Then the horizontal component and vertical component of velocity are $u\cos\theta$ *and* $u\sin\theta$. If R is the range (horizontal distance travelled by the particle) and if t is the time taken by the particle to move through a distance of R, then $R = u\cos\theta \times t = ut\cos\theta$. The vertical displacement becomes 0 when the particle reaches the range R. So by using the formula $s = ut + \frac{1}{2}at^2$, we can write $0 = u\sin\theta \times t - \frac{1}{2}gt^2$. Therefore, $t = 0, \frac{2u\sin\theta}{g}$. The reason for two numbers of answers is that vertical displacement is zero for two times. When the particle reaches to the maximum height, the vertical component of velocity becomes 0. Therefore, by using the formula $v^2 = u^2 + 2as$, we can write $0^2 = (u\sin\theta)^2 - 2gh_{\max} \Rightarrow h_{\max} = \frac{u^2\sin^2\theta}{2g}$.

Rectangular Components of Motion

As the direction of the velocity of a particle in curvilinear motion changes continuously, so it is convenient to deal with its components v_x *and* v_y along x and y axes respectively.

As the particle moves, the position vector r changes and so also the velocity v changes.

Since $\vec{r} = x\hat{i} + yj$

$$\therefore \vec{v} = \frac{d\vec{r}}{dt} = \dot{r} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \dot{x}\hat{i} + \dot{y}\hat{j}$$

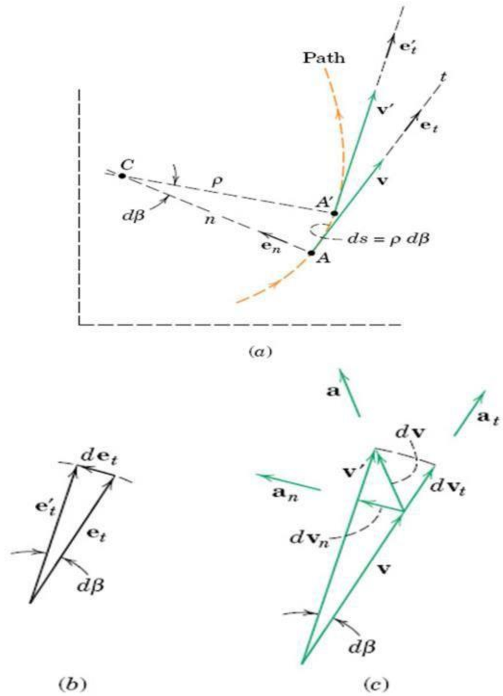
Similarly the acceleration a can be expressed as following.

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v} = \ddot{r} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

Lecture - 30

Topic: Normal and Tangential Components

$$\frac{d\vec{r}}{dt} \equiv \vec{v} = |\vec{v}|\hat{e}_t = v\hat{e}_t$$



Time derivative of a vector

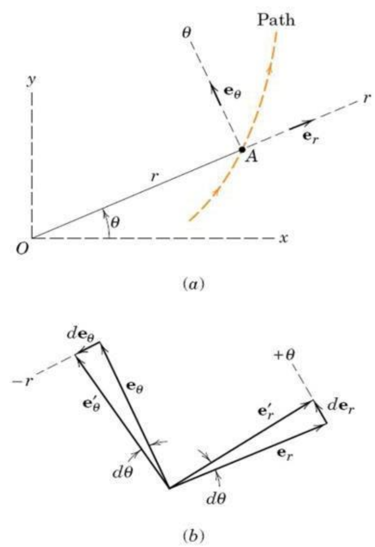
$$r = \frac{dr}{dt}$$

$$= \dot{v}\hat{e}_t + v\dot{e}_t$$

$$= \dot{v}\hat{e}_t + v\frac{v}{\rho}\hat{e}_n$$

ρ : *radius of curvature*

Time Derivative of the Unit Vectors in Polar (Cylindrical) Coordinates (2D)



$$\frac{de_r}{d\theta} = e_\theta \qquad \frac{de_\theta}{d\theta} = -e_r$$

$$\dot{e}_r = \frac{de_r}{dt} = \frac{d\theta}{dt}\frac{de_r}{d\theta} = \dot{\theta}e_\theta$$

$$\dot{e}_\theta = \frac{de_\theta}{dt} = \frac{d\theta}{dt}\frac{de_\theta}{d\theta} = -\dot{\theta}e_r$$

Lecture - 31

Example: 1

A particle moves along the path $y = \frac{1}{3}x^2$ with a constant velocity of 8 m/s. What are the x and y components of the velocity when x=3? What is the corresponding acceleration? Note that x and y are expressed in metres.

Example: 2

A particle moves with constant speed v along a parabolic path $y = kx^2$, where k is a constant. Find the maximum acceleration of the particle.

Example: 3

A car is moving along a curved path with 150 m radius with a uniform velocity of 90 km/hr. Find the normal and tangential acceleration of the car.

Problems:

(1) The coefficient of friction between the road and the wheels of a car is found to be 0.2. At what constant velocity should the car move so as to avoid skidding, if the radius of the curve is 240 m. Assume that the road is leveled.

(2) The equations of motion of a particle undergoing curvilinear motion can be described by $x = 2t^2 + 8t$ and $y = 4.9t^2$, where x and y are expressed in metres and t is in seconds. Determine the velocity and acceleration at the end of 4 seconds.

(3) A motorist is moving along a curved path with a 300 metre radius at a speed of 72 km/hr. He suddenly applies brake that causes its speed to decrease to 40 km/hr at a constant rate in 10 seconds. Calculate the tangential and normal components of acceleration immediately after the application of brake and 6 seconds after that.

(4) A car starts from rest on a curved road of 250 m radius and accelerates at a constant tangential acceleration of 0.6 m/s^2. Determine the distance and the time for which the car should travel before the magnitude of the total acceleration attained by the car becomes 0.75 m/s^2.

Multiple Choice Questions

(1) The tangential component of acceleration of a particle in a curvilinear motion is defined by

(a) $a_t = \frac{dv}{dt}$ (b) $a_t = \frac{dx}{dt}$ (c) $a_t = \frac{dy}{dt}$ (d) $a_t = \frac{v^2}{\rho}$

(2) The maximum velocity of a car following a circular motion of radius r and having coefficient of friction as μ to avoid skidding is

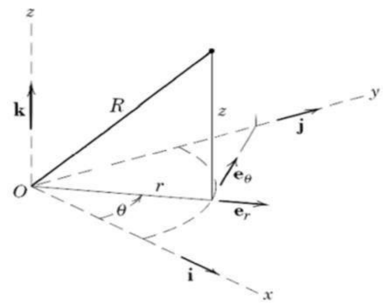
(a) μgr (b) $\frac{1}{2}ugr$ (c) $\sqrt{\mu gr}$ (d) $\frac{1}{2}\sqrt{\mu gr}$

(3) The normal component of acceleration of a particle in a curvilinear motion is defined by

(a) $a_n = \frac{dv}{dt}$ (b) $a_n = \frac{dx}{dt}$ (c) $a_n = \frac{dy}{dt}$ (d) $a_n = \frac{v^2}{\rho}$

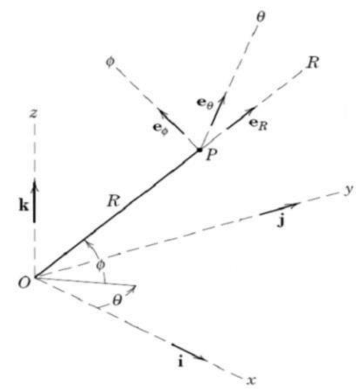
Cylindrical Coordinates (3D)

$$\vec{r} = r\hat{e}_r + z\hat{e}_z$$
$$\dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r + \dot{z}\hat{e}_z + z\dot{\hat{e}}_z$$
$$= \dot{r}e_r + r\dot{\theta}e_\theta + \dot{z}\hat{e}_z$$
$$\ddot{\vec{r}} = \ddot{r}\hat{e}_r + \dot{r}\dot{\hat{e}}_r + \dot{r}\dot{\theta}e_\theta + r\ddot{\theta}e_\theta + r\dot{\theta}\dot{e}_\theta + \ddot{z}e_z$$
$$= \left(\ddot{r} - r\dot{\theta}^2\right)e_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)e_\theta + \ddot{z}e_z$$



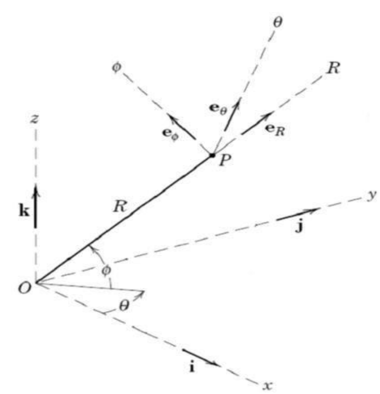
Spherical Coordinates

$$r = Re_r$$
$$\dot{r} = \dot{R}e_r + R\dot{e}_r$$
$$\omega = \dot{\theta}e_z + \dot{\phi}e_\theta$$
$$= \dot{\theta}\left(\sin\phi\, e_r - \cos\phi\, e_\phi\right) + \dot{\phi}e_\theta$$
$$= \dot{\theta}\sin\phi\, e_r + \dot{\phi}e_\theta - \dot{\theta}\cos\phi\, e_\phi$$
$$\dot{\vec{e}}_r = \omega \times \hat{e}_r$$
$$= 0 + \dot{\phi}e_\phi + \dot{\theta}\cos\phi\, e_\theta$$
$$\therefore \dot{\vec{r}} = \dot{R}e_r + R\dot{\theta}\cos\phi\, e_\theta + R\dot{\phi}e_\phi$$



Velocity and Acceleration in Spherical Coordinates

$$\dot{r} \equiv \vec{v} = v_R e_R + v_\theta e_\theta + v_\phi e_\phi$$
$$v_R = \dot{R}$$
$$v_\theta = R\dot{\theta}\cos\phi$$
$$v_\phi = R\dot{\phi}$$
$$\ddot{r} \equiv \vec{a} = a_R e_R + a_\theta e_\theta + a_\phi e_\phi$$
$$a_r = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$
$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}\left(R^2\dot{\theta}\right) - 2R\dot{\theta}\dot{\phi}\sin\phi$$
$$a_\phi = \frac{1}{R}\frac{d}{dt}\left(R^2\dot{\phi}\right) + R\dot{\theta}^2\sin\phi\cos\phi$$



Module V

Lecture - 32

Topic: Kinetics of Particles

Newton's laws:

First law: A particle remains at rest or continues to move in straight line with a constant velocity if there is no unbalanced force acting on it.

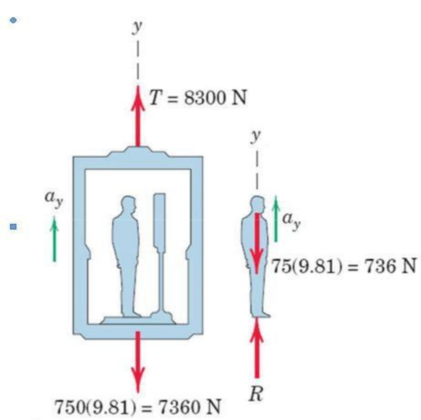
Second law: The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.
The mathematical form of Newton's second law of motion:

$\vec{F} = m\vec{a}$ where $\vec{F}$ is the force causing motion, m is the mass of the particle and a is the resulting acceleration.

Third law: The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.
These laws have been verified by countless physical measurements. The first two laws hold for measurements made in an absolute frame of reference, but are subject to some correction when the motion is measured relative to a reference system having acceleration, such as one attached to the earth's surface.

Examples:

(1) A 75-kg man stands on a spring scale in an elevator. The tension T in the hoisting cable is 8300 N.Find the reading R of the scale in Newton and the velocity v of the elevator after 3 seconds. The total mass of the elevator, man, and scale is 750kg.

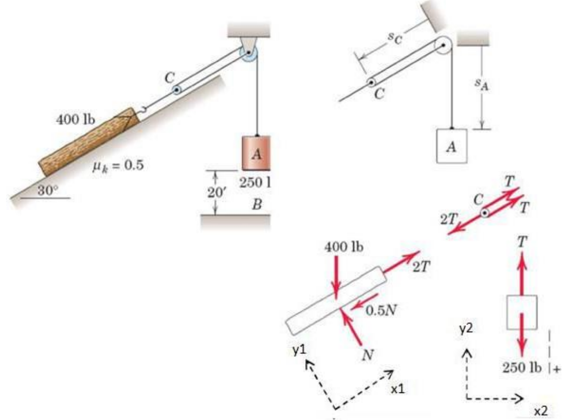


$$\sum F_y = ma_y$$
$$\Rightarrow T - m_1 g = m_1 \ddot{y}$$
$$\Rightarrow 8300 - 7360 = 750 a_y$$
$$\Rightarrow a_y = 1.257\, m/s^2$$

$$\sum F_y = ma_y$$
$$\Rightarrow R - m_2 g = m_2 \ddot{y}$$
$$\Rightarrow R - 736 = 75(1.257)$$
$$\Rightarrow R = 830\, N$$

$$\Delta v = \int a\,dt$$
$$\Rightarrow v - 0 = \int_0^3 1.257\,dt$$
$$\Rightarrow v = 3.77\, m/s$$

(2) The 250-lb concrete block *A* is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at *B*.



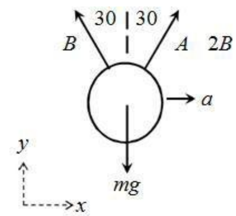
$$N - m_1 g\cos\theta = m_1\ddot{y}_1 = 0$$

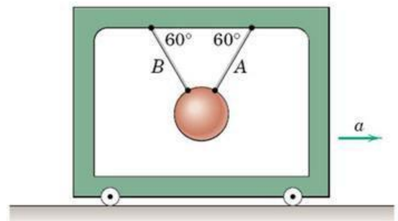
$$\Rightarrow -\mu N + T - m_1 g\sin\theta = m_1\ddot{x}_1$$

$$\Rightarrow -m_2 g + T = m_2\ddot{y}_2$$

$$\Rightarrow 2x_1 + y_2 = \text{constant}$$

(3) The steel ball is suspended from the accelerating frame by the two cords A and B. Determine the acceleration of the frame which will cause the tension in A to be twice that in B.





$$\sum F_x = ma_x$$

$$\Rightarrow 2B\sin 30^\circ - B\sin 30^\circ = m\ddot{x}$$

$$\sum F_y = 0$$

$$2B\cos 30^\circ + B\cos 30^\circ - mg = m\ddot{y} = 0$$

Eliminate B and get $\ddot{x} = a = \dfrac{g}{3\sqrt{3}}$

Let ρ =mass/length

$$F = \mu N = \mu g\rho(L-b)$$

$$\Sigma T_0 - \mu g\rho(L-b)$$

Solve to obtain: $b = \dfrac{\mu L}{1+\mu}$ $[\because T_0 = \rho g b]$

$$\sum F = ma$$

$$T - \mu g\rho(L-x) = \rho(L-x)a$$

$$\rho g x - T = \rho x a$$

Eliminate B to obtain:

$$a = \ddot{x} = \frac{g}{L}[x(1+\mu) - \mu L]$$

$$v\,dv = \ddot{x}\,dx$$

$$\int_0^v v\,dv = \int_b^L \frac{g}{L}[x(1+\mu) - \mu L]\,dx$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{g}{L}\left[\frac{x^2}{2}(1+\mu) - \mu L x\right]_b^L$$

Substitute b and simplify:

$$v = \sqrt{\frac{gL}{1+\mu}}$$

Another approach:

$$\rho x g - \rho(L-x)g\mu = \rho L\ddot{x}$$

$$xg - \mu g(L-x) = L\ddot{x}$$

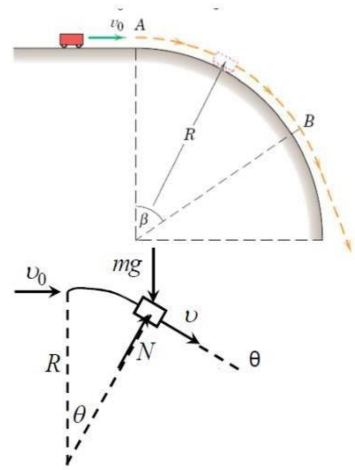
$$\ddot{x} = \frac{g}{L}(x(1+\mu) - \mu L)$$

$$\ddot{x} - \frac{g(1+\mu)}{L}x = \mu g$$

$$x = e^{\lambda t},\ \lambda^2 - a^2 = 0\ a = \sqrt{\frac{g(1+\mu)}{L}}$$

$$x = c_1 e^{at} + c_2 e^{-at} + \frac{\mu L}{1+\mu}$$

(4) A small vehicle enters the top A of the circular path with a horizontal velocity v_0 and gathers speed as it moves down the path. Determine an expression for the angle β which locates the point where the vehicle leaves the path and becomes a projectile. Evaluate your expression for $v_0 = 0$. Neglect friction.



$$\sum F_\theta = ma_\theta \quad \Rightarrow mg\sin\theta = ma_\theta \quad \Rightarrow a_\theta = g\sin\theta$$

$$\int v\,dv = \int a_\theta\,ds\ ,\ \int_{v_0}^{v} v\,dv = \int_0^\theta g\sin\theta\,(R d\theta)$$

$$v^2 = v_0^2 + 2gR(1-\cos\theta)$$

$$\sum F_r = ma_r\ ,\ -mg\cos\theta + N = -\frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv_0^2}{R} - 2mg(1-\cos\theta)$$

$$= mg\left(3\cos\theta - 2 - \frac{v_0^2}{gR}\right)$$

When $N = 0,\quad so\quad 3\cos\beta = 2 + \frac{v_0^2}{gR} \quad \Rightarrow \beta = \cos^{-1}\left(\frac{2}{3} + \frac{v_0^2}{3gR}\right)$

For $v_0 = 0,\quad \beta = \cos^{-1}\left(\frac{2}{3}\right) = 48.2^\circ$

Lecture -33

Work, Power and Energy

Work can be defined as transfer of energy. In physics we say that work is done on an object when you transfer energy to that object. If one object transfers energy to a second object, then the first object does work on the second object.

Work is the application of a force over a distance. Lifting a weight from the ground and putting it on a shelf is a good example of work. The force is equal to the weight of the object, and the distance is equal to the height of the shelf ($dU = \vec{F}.\vec{dr}$).

Work-Energy Principle --The change in the kinetic energy of an object is equal to the net work done on the object.

Energy can be defined as the capacity of doing work. The simplest case of mechanical work is when an object is standing still and we force it to move. The energy of a moving object is called kinetic energy. For an object of mass m, moving with velocity of magnitude v, this energy can be calculated from the formula $E = \frac{1}{2}mv^2$. Energy is a scalar quantity and SI unit of energy is joule (J).

Potential energy is the stored energy of an object. It is the energy by virtue of an object's position relative to other objects. Potential energy is often associated with restoring forces such as a spring or the force of gravity. The action of stretching the spring or lifting the mass is performed by an external force that works against the force field of the potential. This work is stored in the force field, which is said to be stored as potential energy. If the external force is removed the force field acts on the body to perform the work as it moves the body back to the initial position, reducing the stretch of the spring or causing a body to fall.

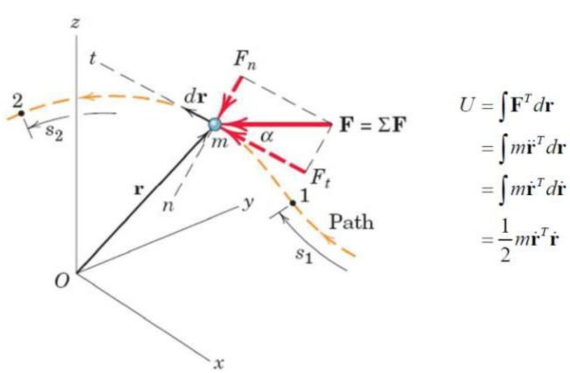
Power is defined as the time rate of doing work. Power is a scalar quantity and SI unit of power is watt (W).

$$U_{1-2} = \int_1^2 F.dr = \int_1^2 \left(-mg\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right)$$

$$= -mg\int_{y_1}^{y_2} dy = -mg\left(y_2 - y_1\right)$$

$$U_{1-2} = \int_1^2 \vec{F}.d\vec{r} = \int_1^2 \frac{-Gm_e m}{r^2}\hat{e}_r . dr\hat{e}_r$$

$$= -Gm_e m\int_{r_1}^{r_2} \frac{dr}{r^2}$$



Lecture –34

Example: 1

A block of weight W is thrown with an initial velocity of v_0 along a rough horizontal plane and is brought to rest by friction in a distance x. Determine the coefficient of friction.
Solution:
Let the coefficient of friction between the block and the floor be μ.
Thus the work done by the frictional force is $U = Fx = \mu Nx = \mu mgx$
Change in kinetic energy (K.E.)=$\frac{1}{2}mv_0^2 - 0 = \frac{1}{2}mv_0^2$
Since the work done is equal to the change in K.E., it therefore follows that

$$\mu mgx = \frac{1}{2}mv_0^2 \Rightarrow \mu = \frac{v_0^2}{2gx}$$

Example: 2

A particle of mass m moves linearly along x axis under the action of force F=kx, where k is a constant. Find the velocity v as a function of displacement x if the initial conditions of motion are $x_0 = 0$ *and* $\dot{x}_0 = v_0$.
Solution:

$$F = kx = m\ddot{x} = m\frac{dv}{dt} \Rightarrow mdv = kxdt = \frac{kxdx}{\frac{dx}{dt}} = \frac{kxdx}{v} \Rightarrow mvdv = kxdx$$

$$m\int_{v_0}^{v} vdv = k\int_{0}^{x} xdx \Rightarrow v = \sqrt{v_0^2 + kx^2/m}$$

Lecture –35

D'Alembert's Principle:

From Newon's second law of motion we know that $\vec{F} = m\vec{a}$. This can be written as $\vec{F} - m\vec{a} = 0$ $\Rightarrow \vec{F} + \left(-m\vec{a}\right)$. The force $\left(-m\vec{a}\right)$is called inertia force. Therefore, vector summation of applied force and inertia force is zero. This is called D'Alembert's principle. The basic philosophy behind this is that the problem of dynamic equlibrium is equivalently converted the problem of static equlibrium.

Principle of Conservation of Energy:

The energy can neither be created nor be destroyed. This is the principle of conservation of energy. That means total energy of a system remains constant.

Power:

Power is defined as time rate of doing work.

Efficiency:

Efficiency is defined as the ratio of output energy and input energy.

Example: 1
A train of mass 100 ton is moving uniformly along an incline of 1 in 200 having frictional resistance as 6N/kN. If the power produced by the engine is 120000 W, find the speed of the train.
Solution:

Frictional resistance = $\dfrac{6\times100\times1000\times9.81}{1000}N$ =5886 N

For the inclined plane, $\tan\theta = \dfrac{1}{200} \approx \sin\theta$

In the absence of any acceleration, force balance along the incline gives

$$\sum F = 0 \Rightarrow W\sin\theta + F_f = F_T$$

$$\therefore F_T = 100\times1000\times9.81\times\frac{1}{200} + 5886\ N = 10791\ \text{N}$$

Again, $P = F_T v \Rightarrow v = \dfrac{p}{F_T} = \dfrac{120000}{10791} = 11.12\ m/s$

Lecture –36

Exercise

(2) A particle is projected to move along a parabola $y^2 = 4x$. At a certain instant, when passing through a point P(4,4), its speed is 5 m/s and the rate of increase of its speed is $3m/s^2$ along the path. Express the velocity and acceleration in terms of rectangular coordinates.

(4) An aircraft moving horizontally at 400 km/hr accidentally loses a rivet when it is at a height of 1800 m from a point A on the ground level. Determine the location of point B with respect to A where the rivet will land.

(5) A shell is fired horizontally from cliff 45 m above the sea level with an initial velocity of 300 m/s to hit a target at sea level. Determine (a) the time taken by the shell to hit the target, (b) the horizontal distance of cliff from the target, and (c) the velocity with which the shell strikes the target.

(6) A body of mass M moves in outer space with velocity V. The body breaks into two parts so that mass of one part is 1/10 th of the total mass. After explosion, the heavier part comes to rest while the lighter part continues to move along the original direction of motion. Determine the velocity of smaller part in terms of velocity V.

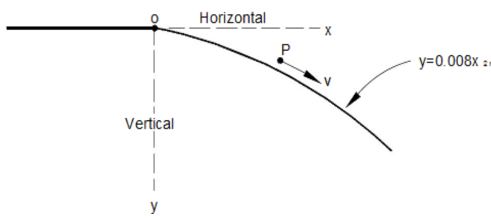
(7) A pile driving hammer of mass 250 kg falls 3 m from rest on a pile of mass 1200 kg. There is no rebound and the pile is driven 20 cm into the ground. Make calculations for the common velocity after impact and the average resistance offered by the ground.

(8) A spring of unstretched length 100 mm extends to a length of 125 mm when a load is applied to it. When the load is removed, the spring has to do 2 J of work in returning to its original position.

(9) A particle is projected at an angle such that the horizontal range is three times the maximum height attained. Find the angle of projection.

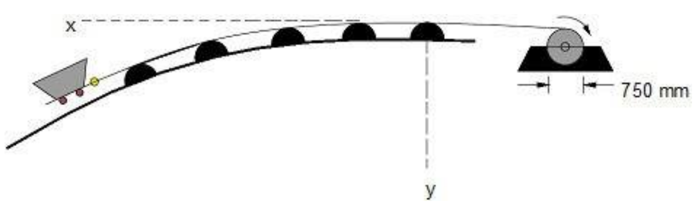
(10) Two seconds after projection, a projectile is moving at 30° above the horizontal. After one more second, it is moving horizontally. Find the magnitude and direction of its velocity.

(11) A small particle P starts from point O with a negligible speed and increases its speed to a value $v = \sqrt{2gy}$ where y is the vertical drop from O. When x=15 m, determine the n-component of acceleration of the particle.

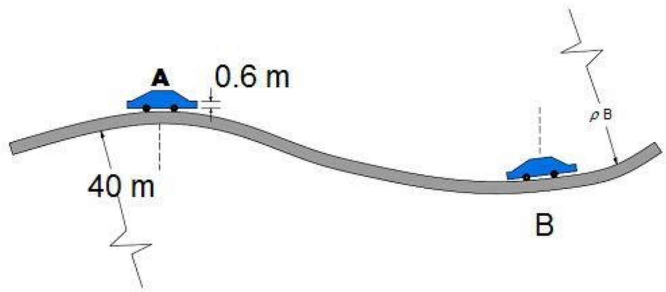


(12) The mine skip is being hauled to the surface over the curved track by the cable wound around the 750-mm drum which turns at the constant clockwise speed of 120 rpm. The shape of the track is designed so that $y = x^2/16$, where x and y are in meters. Calculate the magnitude of the total acceleration of the skip as it reaches a level of 1 m below the top. Neglect the dimensions compared with those of the path.

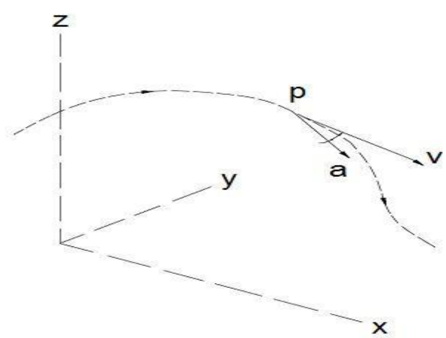
Recall that the radius of curvature is given by $\rho = \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$.



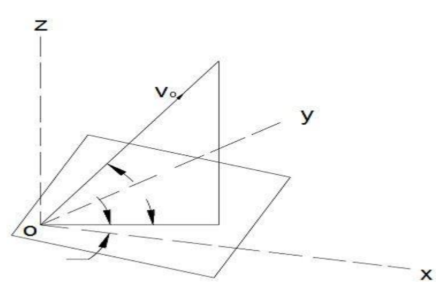
(13) The speed of a car increases uniformly with time from 50 km/h at A to 100 km/h at B during 10 seconds. The radius of curvature of the hump at A is 40 m. If the magnitude of the total acceleration of the car's mass center is the same at B as at A, compute the radius of curvature ρ_B of the dip in the road at A. The mass center of the car is 0.6 m from the road.



(14) The particle P moves along the space curve and has a velocity $\vec{v} = 4\hat{i} - 2\hat{j} - \hat{k}\ m/s$ for the instant shown. At the same instant the particle has an acceleration a whose magnitude is $8\ m/s^2$. Calculate the radius of curvature ρ of the path for this position and the rate $\dot{v}$ at which the magnitude of the velocity is increasing.



(15) A projectile is launched from point O with an initial speed $v_0 = 150\ m/s$ directed as shown in the figure. Compute the x-, y-, and z-components of position, velocity, and acceleration 20 seconds after launch. Neglect aerodynamic drag.



(16) An aircraft takes off at A and climbs at a steady angle with slope of 1 to 2 in the vertical y-z plane at a constant speed $v = 400 km/h$. The aircraft is tracked by radar at O. For the position B, determine the value of $R, \theta, and\ \phi$.

(17) A mountain climber has a mass of 80 kg. Determine his loss of absolute weight in going from the foot of Mount Everest at an altitude of 8848 m. Mount Everest has latitude of $28^{\circ} N$, and the mean radius of the earth is 6371 km.

(18) The displacement of a particle which moves along s-axis is given by $s = (-2 + 3t)e^{-0.5t}$, where s is in meters and t is in seconds. Plot the displacement, velocity, and acceleration versus time for the first 20 seconds of motion. Determine the time at which the acceleration is zero.

(19) A rocket is fired vertically up from rest. If it is designed to maintain a constant upward acceleration of 1.5g, calculate the time t required for it to reach an altitude of 30 km and its velocity at that position.

(20) A ball is thrown vertically up with a velocity of 30 m/s at the edge of a 60 m cliff. Calculate the height h to which the ball rises and the total time t after release for the ball to reach the bottom of the cliff. Neglect air resistance and take the downward acceleration to be 9.81 m/s^2 .